Quantum reconciliation via the Planck constant

D.B. Libby

One of the long-standing conundrums of science is the exact mathematical relationship of quantum mechanics to Newtonian physics, as well as to relativistic physics. Herein it is shown to be possible to use the Planck constant as represented by the quantum formula for photon energy, $E = hc/\lambda$, where h is the Planck constant, and using an approach similar to Bohr's classical mechanical solution for the atom, to construct equations that show the mathematical relationship among all three in fundamental terms. This has the possibility of showing how relativity applied to Newtonian physics produces a quantum mechanical result, and may lead to a better understanding of quantum mechanics from a non-statistical approach, as well as possibly allowing a direct calculation of quantum gravity.

Although Niels Bohr's classical model of atomic structure is commonly regarded today as obsolete, it nevertheless laid the foundation for determining electron shell numbers and electron spin configurations, and included not only circular orbits, but elliptical ones as well, and the unit of the Bohr radius is still useful. His model, while functioning perfectly for less complex atoms, became too cumbersome for higher atomic numbers and was replaced with matrix methodologies, but the foundational approach remains valid. A similar approach to the nature of photonic energy does not suffer from the complications of spectral lines in atoms of higher atomic numbers which gave rise to quantum considerations and its consequent complexities.

Using the quantum representation for photon energy, the resulting plot is instantly recognizable as a two-body energy curve. To construct a simple Newtonian mathematical model for photonic energy, it is logical to begin with an oppositely-charged lepton pair, specifically the electron and positron, as mutually-orbiting bodies, since under certain conditions in quantum electrodynamics, the two particles annihilate to create a single photon. As the particles are assumed at this point to be of opposite and equal charge:

$$F = \frac{ke^2}{(2r)^2} = \frac{ke^2}{4r^2}$$

where F is the attractive electromagnetic force, k is Coulomb's constant, e is the elementary charge and r (radius) is an arbitrarily selected value to produce wavelengths throughout the electromagnetic spectrum. Then the orbital velocity for each particle is:

$$v = \sqrt{\frac{0.5 * Fr}{m}} = \sqrt{\frac{ke^2}{8rm}}$$

Since there are two particles:

$$T = \frac{1}{2} \cdot \frac{2\pi r}{v} = \frac{\pi r}{v}$$

The wavelength of the system then can be calculated as simply:

$$\lambda = cT$$

which by using the preceding formulas, is:

$$\lambda = \frac{\pi r c}{\sqrt{\frac{ke^2}{8rm}}}$$

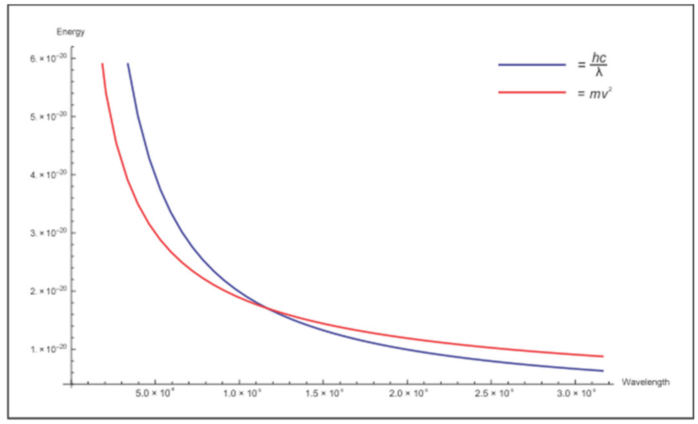
Then, solving for the radius to calculate the velocity from the wavelength yields:

$$r = \left(\frac{\lambda^2 ke^2}{8\pi^2 m\, c^2}\right)^{\frac{1}{3}}$$ **(See Supplement A)**

To plot the expected energy in quantum mechanical terms, as well as the kinetic energy of the model, the following formulas are used:

$$E' = \frac{hc}{\lambda} \quad \text{and} \quad E'' = 2 \cdot \left(\frac{1}{2} mv^2\right) = mv^2$$

The graph to the right shows the two results:



The results differ slightly, but the point at which they are equal can be calculated, and a corrective factor constructed.

Starting with:

$$mv^2 = \frac{hc}{\lambda}$$

and substituting from the previous equations:

$$m\left(\sqrt{\frac{ke^2}{8rm}}\right)^2 = hc\left(\frac{\sqrt{\frac{ke^2}{8rm}}}{\pi rc}\right)$$

then solving for the radius at the point of equality:

$$\boldsymbol{r_{eq} = \frac{8h^2}{\pi^2 ke^2 m}}$$

Using the CODATA 2014 [1] values, the result for $\mathbf{r_{eq}}$ is approximately equal to $1.69336707 \times 10^{-9}$. This corresponds to a mid-IR wavelength of 11.66 μm at an energy of $1.7030253 \times 10^{-20}$ Joules or 0.1063 eV.

It was also realized that $r_{eq} = 32a_0$, where a_0 is the Bohr radius. **(See Supplement B)**

It was then found that the difference in the two results at other radii could be exactly corrected by:

$$\left(\frac{v}{v_{eq}}\right) mv^2$$

Further, it was found, more conveniently, that for any radius $\boldsymbol{r}$, the correction factor was also:

$$\frac{v}{v_{eq}} = \frac{\sqrt{\frac{ke^2}{8rm}}}{\sqrt{\frac{ke^2}{8r_{eq}m}}} = \sqrt{\frac{\frac{ke^2}{8rm}}{\frac{ke^2}{8r_{eq}m}}} = \sqrt{\frac{\boldsymbol{r_{eq}}}{\boldsymbol{r}}}$$

Therefore:

$$\left(\sqrt{\frac{r_{eq}}{r}}\right) mv^2 = \frac{hc}{\lambda}$$

The equality now contains three terms: $\sqrt{\frac{\boldsymbol{r_{eq}}}{\boldsymbol{r}}}$, which is the correction factor; $\boldsymbol{mv^2}$, which is the classical term; and $\frac{\boldsymbol{hc}}{\boldsymbol{\lambda}}$, which is the quantum expression.

It can be seen that the corrective factor is the result of the difference between a circular orbit and an elliptical orbit, where, at the point of equality, the semi-major axis and the semi-minor axis are equal.

According to Kepler's Third Law, for a given major semi-axis the orbital period does not depend on the eccentricity, but rather the orbital period is equal to that for a circular orbit with the radius equal to the semi-major axis of an ellipse. One can start with the orbital period of a circular orbit with a radius equal to the semi-major axis of the ellipse, and then use the perimeter of the elliptic orbit (utilizing the simplest equation for the elliptical perimeter) divided by circumference of the circular orbit to find the elliptical velocity, which is exactly what the preceding correction factor does.

So the calculation for velocity of a single particle must now incorporate the square root of the perimeter of the ellipse divided by the circumference of the circle:

$$\sqrt{\frac{2\pi\sqrt{\frac{\left(r_{eq}+\sqrt{r_{eq}n}\right)^2+\left(r_{eq}-\sqrt{r_{eq}n}\right)^2}{2}}}{2\pi\,(r_{eq}+n)}} = \sqrt{\frac{\mathrm{r_{eq}}}{\sqrt{\mathrm{r_{eq}^2}+\mathrm{r_{eq}}n}}} \qquad \textbf{(See Supplement C)}$$

This result is equal to the square root of the correction factor $\sqrt{\frac{r_{eq}}{r}}$, where r is any radius, represented by $r_{eq}+n$. **(See Supplement D)**

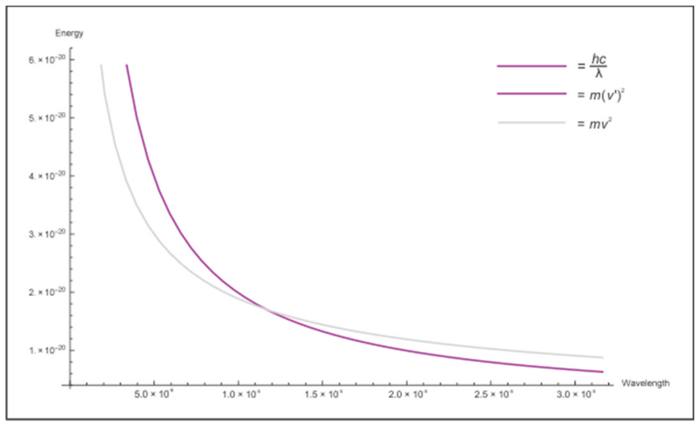
Therefore:

$$v' = v\left(\frac{\mathrm{r_{eq}}}{\mathrm{r}}\right)^{\frac{1}{4}}$$

The final form of the left-hand side of the equality is:

$$\boldsymbol{E'' = m(v')^2}$$

Graphing $\boldsymbol{E''}$ and, as previously,



The results are shown in the graph to the right.

It can be seen that the results agree exactly, validating that, except for the point of equality, the system is elliptic.

The equality $\boldsymbol{\frac{hc}{\lambda} = m(v')^2}$ can now be fully proven. **(See Supplement E)**

The relativistic term can now be constructed with only slight modification, using:

$$\gamma' = \frac{1}{\sqrt{1-\frac{(\mathrm{v}')^2}{\mathrm{c}^2}}}$$

and also modifying the terms of length ($\boldsymbol{\lambda}$) and mass ($\boldsymbol{m}$) in the quantum mechanical and Newtonian terms. Then the relativistic equation for kinetic energy: $KE = \gamma mc^2 - mc^2$ can be finalized in the following form:

$$E''' = mc^2\left(1-\frac{1}{\gamma^2}\right) \qquad \textbf{(See Supplement F)}$$

Due to the nature of the relativistic equation, this is not an exact solution, but it is within 0.0000003% across a range of 151 nm to 3164 nm, or 8.2 eV to 0.39eV. This inaccuracy can be seen to result from the stair-stepped nature of the resulting relativistic curve when observed on a small scale at lower velocities, which implies that relativity itself may indeed produce quantum effects.

The final quantum mechanical/Newtonian/relativistic equality is now:

$$\frac{hc}{\lambda} = m(v')^2 = mc^2\left(1 - \frac{1}{(\gamma')^2}\right)$$

In the preceding equation, the reduction to equality is simple between the relativistic and Newtonian cases, but not as obvious for quantum mechanics.

Although historically it has been thought that a disparity existed between Newtonian physics and quantum mechanics, it can be seen from this purely mathematical model that the disparity only exists between Newtonian physics and relativity, as has long been accepted. The relationship of this model to reality may be generative or substantive in nature, but neither is claimed, and although the Newtonian and relativistic models can be equated, they differ substantially in function. Furthermore, in the Newtonian and the relativistic models, the basis of the elliptical orbits is undetermined, although it may be supposed since this is a two-dimensional model, the three-dimensional reality of the elliptical curves may be explained by precessing orbits. In any case, the mathematical relationship among the three branches of physics as outlined here should be of fundamental interest, while pointing the way to a non-statistical approach to quantum mechanics. Furthermore, as indicated above, the stair-stepped nature of the relativistic curve at small scales and lower velocities is a previously unrecognized phenomenon which may shed light on how relativity could produce quantum effects, and finally, the resulting equivalence should make calculating quantum gravity a simple exercise.

References

1. **Mohr, Peter J., Newell, David B. and Taylor, Barry N.** CODATA recommended values of the fundamental physical constants:2014. *Zenodo.* [Online] 2015. http://doi.org/10.5281/zenodo.22826.

Supplement A

$$\lambda = \frac{\pi \, r \, c}{\sqrt{\frac{k \, e^2}{8 \, r \, m}}}$$

$$\lambda^2 = \frac{\pi^2 \, r^2 \, c^2}{1} \cdot \frac{8 \, r \, m}{k \, e^2}$$

$$\lambda^2 = \frac{8 \, \pi^2 \, r^3 \, m \, c^2}{k \, e^2}$$

$$r^3 = \frac{\lambda^2 \, k \, e^2}{8 \, \pi^2 \, mc^2}$$

$$r = \left(\frac{\lambda^2 \, k \, e^2}{8 \, \pi^2 \, mc^2} \right)^{\frac{1}{3}}$$

Supplement B

$$r_{eq} = \frac{8\,h^2}{\pi^2\,k\,e^2\,m} \quad ; \quad r_{eq} = 32\,a_0 \quad ; \quad a_0 = \frac{h}{2\,\pi\,m\,c\,\alpha} \quad ; \quad \alpha = \frac{ke^2}{\hbar\,c} \quad ; \quad \hbar = \frac{h}{2\,\pi}$$

$$\alpha = \frac{k\,e^2}{c} \cdot \left(\frac{2\,\pi}{h}\right) = \frac{2\,\pi\,k\,e^2}{h\,c}$$

$$a_0 = \frac{h}{2\,\pi\,m\,c\,\frac{2\,\pi\,k\,e^2}{h\,c}} = \frac{h}{2\,\pi\,m\,c} \cdot \left(\frac{h\,c}{2\,\pi\,k\,e^2}\right) = \frac{h^2}{4\,\pi^2\,k\,e^2\,m}$$

$$32 \cdot \left(\frac{h^2}{4\,\pi^2\,k\,e^2\,m}\right) = \frac{8\,h^2}{\pi^2\,k\,e^2\,m}$$

$$\frac{8\,h^2}{\pi^2\,k\,e^2\,m} = \frac{8\,h^2}{\pi^2\,k\,e^2\,m}$$

Supplement C

$$\sqrt{\frac{2\pi\sqrt{\frac{\left(r_{eq}+\sqrt{r_{eq}\,n}\right)^2+\left(r_{eq}-\sqrt{r_{eq}\,n}\right)^2}{2}}}{2\pi\,(r_{eq}+n)}} = \sqrt{\frac{r_{eq}}{\sqrt{r_{eq}{}^2+r_{eq}\,n}}}$$

$$\frac{\sqrt{\frac{\left(r_{eq}+\sqrt{r_{eq}\,n}\right)^2+\left(r_{eq}-\sqrt{r_{eq}\,n}\right)^2}{2}}}{(r_{eq}+n)} = \frac{r_{eq}}{\sqrt{r_{eq}{}^2+r_{eq}\,n}}$$

$$\frac{\frac{\left(r_{eq}+\sqrt{r_{eq}\,n}\right)^2+\left(r_{eq}-\sqrt{r_{eq}\,n}\right)^2}{2}}{(r_{eq}+n)^2} = \frac{r_{eq}{}^2}{r_{eq}{}^2+r_{eq}\,n}$$

$$\frac{\frac{r_{eq}{}^2+r_{eq}\sqrt{r_{eq}\,n}+r_{eq}\sqrt{r_{eq}\,n}+r_{eq}\,n+r_{eq}{}^2-r_{eq}\sqrt{r_{eq}\,n}-r_{eq}\sqrt{r_{eq}\,n}+r_{eq}\,n}{2}}{(r_{eq}+n)^2} = \frac{r_{eq}{}^2}{r_{eq}{}^2+r_{eq}\,n}$$

$$\frac{\frac{2\,r_{eq}{}^2+2\,r_{eq}\,n}{2}}{(r_{eq}+n)^2} = \frac{r_{eq}{}^2+r_{eq}\,n}{(r_{eq}+n)\,(r_{eq}+n)} = \frac{r_{eq}{}^2}{r_{eq}{}^2+r_{eq}\,n}$$

$$\frac{r_{eq}(r_{eq}+n)}{(r_{eq}+n)\,(r_{eq}+n)} = \frac{r_{eq}(r_{eq})}{r_{eq}(r_{eq}+n)}$$

$$\frac{r_{eq}}{r_{eq}+n} = \frac{r_{eq}}{r_{eq}+n}$$

Supplement D

$$\sqrt{\frac{r_{\text{eq}}}{\sqrt{r_{\text{eq}}{}^2+r_{\text{eq}}\, n}}} = \sqrt{\sqrt{\frac{r_{\text{eq}}}{r}}}$$

$$\frac{r_{\text{eq}}}{\sqrt{r_{\text{eq}}{}^2+r_{\text{eq}}\, n}} = \sqrt{\frac{r_{\text{eq}}}{r}} \quad ; \quad \mathrm{r} = r_{\text{eq}} + n$$

$$\frac{r_{\text{eq}}{}^2}{r_{\text{eq}}{}^2+r_{\text{eq}}\, n} = \frac{r_{\text{eq}}}{r_{\text{eq}}+n}$$

$$\left(\frac{r_{\text{eq}}}{r_{\text{eq}}}\right) \cdot \frac{r_{\text{eq}}}{r_{\text{eq}}+n} = \frac{r_{\text{eq}}}{r_{\text{eq}}+n}$$

Supplement E

$$\mathrm{v} = \sqrt{\frac{k\,e^2}{8\,r\,m}} \quad ; \quad \mathrm{v}' = \mathrm{v}\sqrt{\sqrt{\frac{r_{eq}}{r}}} \quad ; \quad r_{eq} = 32\,a_0 \quad ; \quad a_0 = \frac{h^2}{4\,\pi^2\,k\,e^2\,m}$$

$$(v')^2 = v^2\sqrt{\frac{32\,a_0}{r}}$$

$$(v')^2 = \left(\frac{k\,e^2}{8\,r\,m}\right)\cdot\sqrt{\frac{32\,a_0}{r}}$$

$$(v')^4 = \left(\frac{k^2\,e^4}{64\,r^2\,m^2}\right)\cdot\left(\frac{32}{r}\right)\cdot\left(\frac{h^2}{4\,\pi^2\,k\,e^2\,m}\right)$$

$$(v')^4 = \left(\frac{h^2\,k\,e^2}{8\,\pi^2\,r^3\,m^3}\right) \quad ; \quad \mathrm{r} = \left(\frac{\lambda^2\,k\,e^2}{8\,\pi^2\,m\,c^2}\right)^{\frac{1}{3}}$$

$$(v')^4 = \left(\frac{h^2\,k\,e^2}{8\,\pi^2\,m^3}\right)\cdot\left(\frac{8\,\pi^2\,m\,c^2}{\lambda^2\,k\,e^2}\right)$$

$$(v')^4 = \frac{h^2\,c^2}{\lambda^2\,m^2}$$

$$(v')^2 = \left(\frac{h\,c}{\lambda}\right)\cdot\left(\frac{1}{m}\right)$$

$$\mathrm{m}\,(v')^2 = \frac{h\,c}{\lambda}$$

Supplement F

$$\gamma' = \frac{1}{\sqrt{1-\frac{(v')^2}{c^2}}}$$

$$\mathrm{KE}_e = \gamma'\mathrm{m}c^2 - \mathrm{mc}^2 \quad ; \quad \mathrm{KE}_p = \frac{(-\mathrm{mc}^2)}{\gamma'} - \left(-\mathrm{mc}^2\right)$$

$$\mathrm{KE}_{\mathrm{total}}=\gamma'\mathrm{m}c^2 - \frac{\mathrm{mc}^2}{\gamma'}$$

$$\mathrm{KE}_{\mathrm{total}} = \mathrm{mc}^2\left(\gamma' - \frac{1}{\gamma'}\right)$$

$$(E')_{\mathrm{rel}} = \frac{\gamma' h c}{\lambda}$$

$$(E'')_{\mathrm{rel}} = \gamma' m(v')^2$$

$$(E''')_{\mathrm{rel}} =\mathrm{mc}^2\left(\gamma' - \frac{1}{\gamma'}\right)$$

$$E''' = \frac{\mathrm{mc}^2\left(\gamma'-\frac{1}{\gamma'}\right)}{\gamma'}$$

$$E''' = \frac{\gamma'\,\mathrm{mc}^2\left(1-\frac{1}{(\gamma')^2}\right)}{\gamma'}$$

$$E''' = \mathrm{mc}^2\left(1 - \frac{1}{(\gamma')^2}\right)$$