# On the Resolution of Time Paradoxes by Altered Quantum Probabilities 

Andrew Gray<br>London, UK<br>agray592@aol.com

April 13, 2021


#### Abstract

The consequences of the ability to send information back in time, albeit with an error rate, are considered. It is shown that time paradoxes are avoided due to the error rate, but that in the process quantum probabilities within the time machine are altered. It is then shown that by setting up appropriate causal loops we may alter external quantum probabilities. Further, it is shown that probabilities can be altered in chaotic classical systems as well


## 1 Introduction

We will suppose that it is possible to send information back in time, with an error rate, as discussed here [1]. We will consider what happens under various time paradox and causal loop scenarios. We will find that potential time paradoxes are avoided by the alteration of quantum probabilities from their normal values. Thus the improbable can be made probable, and one can view this as improbability generation, a notion that will be defined more precisely below.

## 2 Quantum improbability

We will consider various scenarios in which information is sent back in time and acted on in order to create causal loops. Normally one would expect paradoxes to arise when doing this, but we will show that instead probabilities are altered at the quantum level, and no paradoxes occur.

First we must discuss the capabilities of our time machine. We will consider a device that can send a single bit of information back in time, with probability $p$ for that bit to be received incorrectly at the earlier time. We will send the value 0 or 1 back in time. Thus if we send 1 back, the probability of 1 being received at the earlier time is $1-p$, and 0 has chance $p$ to be received. We assume the error rate is the same whether we are sending back a 0 or a 1 . A key point is that as the time machine operates by using quantum effects (weak measurement $[3,4]$ ), there must always be an error rate, even if very small, so $p>0$ always.

Secondly, how do we analyse such situations? With information travelling back in time, using standard forward evolving state vectors is going to be inadequate. We will instead turn to the interpretation of quantum mechanics described here [2]. This assigns probabilities to entire histories, rather than outcomes; so it does not matter that some outcomes happen in the past as a result of a future action, and vice versa. We can just look at the entire history of events and assign probabilities to them. Standard quantum mechanics can be derived from this approach. Also, it explicitly allows backward causation at the microscopic level, with this normally hidden from us by decoherence, thus it is well suited to studying time travel problems.

### 2.1 Consequences of ensuring a prediction is wrong

Firstly we will consider the case where the time machine sends information back in time about a random future event. For example suppose a photon is emitted, sent to a half-reflecting mirror and ends up in one
of two detectors, labelled A and B. If A is triggered then we send back 0 , if $B$ then 1 . We consider the set of possible histories, each defined by the set of events in it. The probability for each history is simply calculated by multiplying the probabilities of the classical events in it (such as measurements, and sending signals), followed by normalisation if necessary, without regard for the arrow of time. The error rate of the time machine is $p$, which is the probability that we receive at the earlier time the opposite of what we tried to send back, and the fact that this is backward causation does not affect the overall history probability calculation. We will assume detection at either A or B is equally likely with probability of $\frac{1}{2}$ in each case. The time machine receives information at time $t_{r}$, sends it back in time at time $t_{s}$, and the photon is detected at time $t_{d}$ with $t_{r}<t_{d}<t_{s}$. We construct a table of the possible histories and their probabilities, as shown in Table (1).

Table 1: Reporting on the future

| History | Received | Detector | Sent back | $P$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 0 | A | 0 | $\frac{1}{2}(1-p)$ |
| 2 | 0 | A | 1 | 0 |
| 3 | 0 | B | 0 | 0 |
| 4 | 0 | B | 1 | $\frac{1}{2} p$ |
| 5 | 1 | A | 0 | $\frac{1}{2} p$ |
| 6 | 1 | A | 1 | 0 |
| 7 | 1 | B | 0 | 0 |
| 8 | 1 | B | 1 | $\frac{1}{2}(1-p)$ |

We will consider the derivation of some of these. History 2 has a probability of 0 , as if A occurs we will send back 0 , not 1 . In fact, as people and machines can make mistakes, this probability is not exactly zero, but as the decision to send back 0 if A occurs, and the implementation thereof, is best described by macroscopic classical physics, we assume the mechanism is highly reliable. Therefore the probability of error is negligible compared to the error rate of the time machine itself, so to a good approximation we can treat it as zero.

History 4 has a probability of $\frac{1}{2} p$. This is the product of the probability of two events: the probability for B to occur, which is $\frac{1}{2}$, and the probability for the value 1 to be incorrectly sent back in time, which is $p$. All the other probabilities are calculated similarly to these two cases.

We can simplify Table (1) if we exclude histories with no chance to occur. Also it does not matter how it is decided whether to send a 1 or 0 back in time, just that it is somehow randomly decided with probability $\frac{1}{2}$ in each case. Thus we can omit the detector column and only consider the four histories shown in Table (2).

Table 2: Simplified reporting on the future

| History | Received | Sent back | $P$ |
| :---: | :---: | :---: | :---: |
| 1 | 0 | 0 | $\frac{1}{2}(1-p)$ |
| 2 | 0 | 1 | $\frac{1}{2} p$ |
| 3 | 1 | 0 | $\frac{1}{2} p$ |
| 4 | 1 | 1 | $\frac{1}{2}(1-p)$ |

So far there is nothing paradoxical; however if we make our decision on what value to send back in some way dependent on what was received earlier, then we have created a causal loop, and things start to get interesting.

The simplest example of this is to decide to send back the opposite of what we earlier received. In this case Table (3) shows the possible histories and their probabilities.

Here $P_{u}$ is the unnormalised probability from multiplying the component event probabilities, and $P$ is the properly normalised probability. There is no factor of $\frac{1}{2}$ in the $P_{u}$ column as we are no longer driving the send back decision off a random event; instead we just have to consider the single probability for the event

Table 3: Paradox attempt

| History | Received | Sent back | $P_{u}$ | $P$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 0 | 1 | $p$ | $\frac{1}{2}$ |
| 2 | 1 | 0 | $p$ | $\frac{1}{2}$ |

of unsuccessfully sending back the bit. This may seem a bit puzzling at first: although it seems obvious that both histories are equally likely, where exactly has a random event occurred to generate a 0 or 1 ? In this case, presented with two equally likely histories, the universe randomly picks one; the dice are thrown at the level of the entire history, rather than at the level of events within it.

As we can see, the probability for the time machine to correctly send back the bit is no longer $p$ but 0 , (or very close to zero). Thus the probability for the correct sending back of the bit, based on quantum probabilities of events inside the time machine, has been altered.

So far we have just altered a probability inside our time machine to stop it working, which is perhaps not very useful. Is it possible to alter the probability of something external to the time machine?

### 2.2 Manipulating the probability of a single quantum event with causal loops

We will again create a photon, after receiving our 1 or 0 from the future, and have it detected in detector A or B. This time we will consider the more general case where the probability of detection at A is $a$, and at B is $1-a$. Again the probability of the time machine to incorrectly send a bit back in time is $p$. We will create a causal loop as follows: if a 0 is received then send back 0 if A happens, 1 if B happens; if a 1 is received then send back 1 if A happens, 0 if B happens. Table (4) lists the possible histories and their probabilities in this scenario.

Table 4: Biasing event A

| History | Received | A/B | Sent back | $P_{u}$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 0 | A | 0 | $a(1-p)$ |
| 2 | 0 | A | 1 | 0 |
| 3 | 0 | B | 0 | 0 |
| 4 | 0 | B | 1 | $(1-a) p$ |
| 5 | 1 | A | 0 | 0 |
| 6 | 1 | A | 1 | $a(1-p)$ |
| 7 | 1 | B | 0 | $(1-a) p$ |
| 8 | 1 | B | 1 | 0 |

The probabilities $P_{u}$ are not normalised; to get normalised probabilities we must multiply them by a normalising factor $N$ where

$$
\begin{align*}
N & =\frac{1}{\sum P_{u}} \\
& =\frac{1}{2 a(1-p)+2 p(1-a)} \tag{1}
\end{align*}
$$

We can now calculate the probability of A by adding the normalised probabilities of all the histories in which A occurs. This gives us

$$
\begin{equation*}
P(A)=\frac{a(1-p)}{a(1-p)+p(1-a)} . \tag{2}
\end{equation*}
$$

Clearly this is not the same as $a$, so coupling the photon and detectors apparatus to our time machine in a causal loop has changed the probability of A . We thus have two probabilities for A to occur: the raw
probability, which is the normal probability in the absence of a causal loop, that we will denote by $a_{r}$; and the coupled probability, which is the altered version, that we will denote by $a_{c}$. Using this notation we can write Eq. (2) as

$$
\begin{equation*}
a_{c}=\frac{a_{r}(1-p)}{a_{r}(1-p)+p\left(1-a_{r}\right)} . \tag{3}
\end{equation*}
$$

## 2.3 n :1 notation

Clearly the probability of A has changed, but it is not simple to understand what Eq. (3) means. We will now show that we can express Eq. (3) much more simply if we use $n: 1$ probability notation, instead of using a single number between 0 and 1 to represent a probability. The two notations are related as follows: For a probability $P$

$$
\begin{equation*}
P=n: 1=\frac{1}{n+1} . \tag{4}
\end{equation*}
$$

The $n: 1$ notation is in common usage, so most readers should be familiar with it already. We will now express Eq. (3) in $n: 1$ notation using the following substitutions:

$$
\begin{align*}
& p=n: 1  \tag{5}\\
&=\frac{1}{n+1}  \tag{6}\\
& a_{r}=m_{r}: 1  \tag{7}\\
&=\frac{1}{m_{r}+1} \\
& a_{c}=m_{c}: 1
\end{align*}=\frac{1}{m_{c}+1} .
$$

Thus we can rewrite Eq. (3) as

$$
\begin{align*}
\frac{1}{m_{c}+1} & =\frac{\frac{1}{m_{r}+1}\left(1-\frac{1}{n+1}\right)}{\frac{1}{m_{r}+1}\left(1-\frac{1}{n+1}\right)+\frac{1}{n+1}\left(1-\frac{1}{m_{r}+1}\right)} \\
\Rightarrow \quad \frac{1}{m_{c}+1} & =\frac{n}{n+m_{r}} \\
\Rightarrow \quad m_{c} & =\frac{m_{r}}{n} . \tag{8}
\end{align*}
$$

Thus the event A with raw probability $m_{r}: 1$ of occurring, now has a new coupled probability of $\frac{m_{r}}{n}: 1$ of occurring, and the meaning of this is much clearer than Eq. (3).

This result is of course the result when we are using our time machine to increase the chance for A to occur. We could also use it to decrease the chance, which indeed is what is happening to event B. Similar to the A case, we denote the raw probability for B as $b_{r}$, and the coupled probability as $b_{c}$. We express these in the $n: 1$ notation with

$$
\begin{align*}
b_{r} & =l_{r}: 1=\frac{1}{l_{r}+1},  \tag{9}\\
b_{c} & =l_{c}: 1=\frac{1}{l_{c}+1} . \tag{10}
\end{align*}
$$

Clearly $b_{c}$ is given by

$$
\begin{aligned}
b_{c} & =1-a_{c} \\
& =1-\frac{a_{r}(1-p)}{a_{r}(1-p)+p\left(1-a_{r}\right)}
\end{aligned}
$$

$$
\begin{align*}
& =\frac{p\left(1-a_{r}\right)}{a_{r}(1-p)+p\left(1-a_{r}\right)} \\
& =\frac{p b_{r}}{\left(1-b_{r}\right)(1-p)+p b_{r}} . \tag{11}
\end{align*}
$$

Converting to $n: 1$ notation this becomes

$$
\begin{align*}
\frac{1}{l_{c}+1} & =\frac{\frac{1}{n+1} \frac{1}{l_{r}+1}}{\left(1-\frac{1}{l_{r}+1}\right)\left(1-\frac{1}{n+1}\right)+\frac{1}{n+1} \frac{1}{l_{r}+1}} \\
\Rightarrow \quad \frac{1}{l_{c}+1} & =\frac{1}{n l_{r}+1} \\
\Rightarrow \quad l_{c} & =n l_{r} . \tag{12}
\end{align*}
$$

Thus the event B with raw probability $l_{r}: 1$ of occurring, now has a coupled probability of $n l_{r}: 1$ of occurring: i.e. it has been made more unlikely by a factor of $n: 1$.

### 2.4 Definition of improbability

Clearly by altering probabilities we are in some sense generating improbability. A precise definition of improbability would therefore be useful. Several possibilities exist, but the one which seems most useful, and which will be used for the rest of the paper is as follows:
"An improbability generator generating an improbability of $n: 1$ alters the probability of a random quantum event with a prior $m: 1$ chance of occurring to have a $\frac{m}{n}: 1$ chance to occur, when the improbability is being used to increase the chance of that event."

According to this definition, an improbability of $n: 1$ is equivalent to an improbability of $\frac{1}{n}: 1$, since the latter results in a prior $m: 1$ probability becoming a $n m: 1$ probability, which is also achieved by having a $n: 1$ generator acting to make an event less likely, rather than more likely.

Also note that an improbability of $1: 1$ will not alter the probability of anything, so this level of improbability can be called normality.

### 2.5 Cooperating improbability generators

We will now consider what happens if we have two improbability generators both cooperating to increase the likelihood of the same event. As before each receives a 1 or 0 , then the random event A occurs with raw probability $a_{r}$, and then a 1 or 0 is sent back in time. Both generators will use the rule that if 0 is received send back 0 if A happens, else send back 1 ; if 1 is received send back 1 if A happens, else send back 0 . The first generator has a raw probability $p_{1}$ to incorrectly send back the bit, and the second $p_{2}$. The possible histories are shown in Table (5).

Table 5: Cooperating improbability generators

| History | R 1 | R 2 | A | $P_{u}$ |
| :---: | :---: | :---: | :--- | :--- |
| 1 | 0 | 0 | 0 | $\left(1-a_{r}\right) p_{1} p_{2}$ |
| 2 | 0 | 0 | 1 | $a_{r}\left(1-p_{1}\right)\left(1-p_{2}\right)$ |
| 3 | 0 | 1 | 0 | $\left(1-a_{r}\right) p_{1} p_{2}$ |
| 4 | 0 | 1 | 1 | $a_{r}\left(1-p_{1}\right)\left(1-p_{2}\right)$ |
| 5 | 1 | 0 | 0 | $\left(1-a_{r}\right) p_{1} p_{2}$ |
| 6 | 1 | 0 | 1 | $a_{r}\left(1-p_{1}\right)\left(1-p_{2}\right)$ |
| 7 | 1 | 1 | 0 | $\left(1-a_{r}\right) p_{1} p_{2}$ |
| 8 | 1 | 1 | 1 | $a_{r}\left(1-p_{1}\right)\left(1-p_{2}\right)$ |

Here the R1 and R2 columns show what value the two generators 1 and 2 received respectively. The A column shows 1 if event A occurred, 0 otherwise. The sent back columns are not shown as the sent back
value follows from the received value and whether A occurred. Also, showing all possible sent back values would result in many pointless rows where the probability is obviously 0 .

To normalise the probabilities in this table we again multiply by a normalising factor $N$, this time given by

$$
\begin{equation*}
N=\frac{1}{4\left[\left(1-a_{r}\right) p_{1} p_{2}+a_{r}\left(1-p_{1}\right)\left(1-p_{2}\right)\right]} \tag{13}
\end{equation*}
$$

Thus the overall probability for A to occur is given by

$$
\begin{equation*}
a_{c}=\frac{a_{r}\left(1-p_{1}\right)\left(1-p_{2}\right)}{\left(1-a_{r}\right) p_{1} p_{2}+a_{r}\left(1-p_{1}\right)\left(1-p_{2}\right)} \tag{14}
\end{equation*}
$$

We will now express this in $n: 1$ notation using the following substitutions:

$$
\begin{align*}
& p_{1}=n_{1}: 1=\frac{1}{n_{1}+1}  \tag{15}\\
& p_{2}=n_{2}: 1=\frac{1}{n_{2}+1}  \tag{16}\\
& a_{r}=m_{r}: 1=\frac{1}{m_{r}+1}  \tag{17}\\
& a_{c}=m_{c}: 1=\frac{1}{m_{c}+1} \tag{18}
\end{align*}
$$

This gives us

$$
\begin{align*}
\frac{1}{m_{c}+1} & =\frac{\frac{1}{m_{r}+1}\left(1-\frac{1}{n_{1}+1}\right)\left(1-\frac{1}{n_{2}+1}\right)}{\left(1-\frac{1}{m_{r}+1}\right) \frac{1}{n_{1}+1} \frac{1}{n_{2}+1}+\frac{1}{m_{r}+1}\left(1-\frac{1}{n_{1}+1}\right)\left(1-\frac{1}{n_{2}+1}\right)} \\
\Rightarrow \quad \frac{1}{m_{c}+1} & =\frac{n_{1} n_{2}}{m_{r}+n_{1} n_{2}} \\
\Rightarrow \quad m_{c} & =\frac{m_{r}}{n_{1} n_{2}} . \tag{19}
\end{align*}
$$

By comparison with Eq. (8), we can see that this is the same as a single improbability of $n_{1} n_{2}: 1$ acting to increase the chance of event A. Thus two improbability generators, generating improbabilities $n_{1}: 1$ and $n_{2}: 1$ respectively, generate an overall improbability of $n_{1} n_{2}: 1$ when cooperating to increase the probability of event A.

### 2.6 Competing improbability generators

Now suppose the first generator is acting to increase the probability of A, but the second is acting to decrease it. To achieve this, the first generator uses the rule that if 0 is received send back 0 if A happens, else send back 1 ; if 1 is received send back 1 if A happens, else send back 0 . The second generator uses the rule that if 0 is received send back 1 if A happens, else send back 0 ; if 1 is received send back 0 if A happens, else send back 1. The history probabilities in this case are given by Table (6), where everything has the same meaning as in the previous example.

This time we have a normalisation factor $N$ of

$$
\begin{equation*}
N=\frac{1}{4\left[a_{r}\left(1-p_{1}\right) p_{2}+\left(1-a_{r}\right) p_{1}\left(1-p_{2}\right)\right]} \tag{20}
\end{equation*}
$$

Thus the overall probability of event A is given by

$$
\begin{equation*}
a_{c}=\frac{a_{r}\left(1-p_{1}\right) p_{2}}{a_{r}\left(1-p_{1}\right) p_{2}+\left(1-a_{r}\right) p_{1}\left(1-p_{2}\right)} . \tag{21}
\end{equation*}
$$

Table 6: Competing improbability generators

| History | R1 | R2 | A | $P_{u}$ |
| :---: | :---: | :---: | :--- | :--- |
| 1 | 0 | 0 | 0 | $\left(1-a_{r}\right) p_{1}\left(1-p_{2}\right)$ |
| 2 | 0 | 0 | 1 | $a_{r}\left(1-p_{1}\right) p_{2}$ |
| 3 | 0 | 1 | 0 | $\left(1-a_{r}\right) p_{1}\left(1-p_{2}\right)$ |
| 4 | 0 | 1 | 1 | $a_{r}\left(1-p_{1}\right) p_{2}$ |
| 5 | 1 | 0 | 0 | $\left(1-a_{r}\right) p_{1}\left(1-p_{2}\right)$ |
| 6 | 1 | 0 | 1 | $a_{r}\left(1-p_{1}\right) p_{2}$ |
| 7 | 1 | 1 | 0 | $\left(1-a_{r}\right) p_{1}\left(1-p_{2}\right)$ |
| 8 | 1 | 1 | 1 | $a_{r}\left(1-p_{1}\right) p_{2}$ |

Converting to $n: 1$ notation, with the same substitutions as in the cooperating case, we find

$$
\begin{align*}
\frac{1}{m_{c}+1} & =\frac{\frac{1}{m_{r}+1}\left(1-\frac{1}{n_{1}+1}\right) \frac{1}{n_{2}+1}}{\frac{1}{m_{r}+1}\left(1-\frac{1}{n_{1}+1}\right) \frac{1}{n_{2}+1}+\left(1-\frac{1}{m_{r}+1}\right) \frac{1}{n_{1}+1}\left(1-\frac{1}{n_{2}+1}\right)} \\
\Rightarrow \quad \frac{1}{m_{c}+1} & =\frac{n_{1}}{n_{1}+m_{r} n_{2}} \\
\Rightarrow \quad m_{c} & =\frac{n_{2} m_{r}}{n_{1}} . \tag{22}
\end{align*}
$$

Again, by comparison with Eq. (8), we see that this is equivalent to an improbability of $\frac{n_{1}}{n_{2}}: 1$ acting to increase the probability of A . If $n_{2}>n_{1}$ the probability of A will actually decrease. The more powerful of the two generators wins, but is blunted somewhat by the weaker one.

## 3 Classical improbability

We have seen that we can manipulate quantum probabilities, but can we also manipulate classical probabilities? For example can we alter the chance to roll a pair of sixes, or to have a coin toss come up heads? We will see that we can influence classical probabilities in chaotic systems, where a small perturbation to initial conditions leads to large changes in the system later (this is often called the butterfly effect).

We will influence such systems by using a time machine to send back in time $n$ bits, rather than the single bit we have been considering up to now. We receive $n$ bits at the earlier time and they correspond to a binary number between 0 and $2^{n}-1$. We apply a perturbation or set of perturbations to our chaotic system dependent on the value of this number. The details of how we convert the number into a set of perturbations are unimportant as long as each number generates a unique set of perturbations, i.e. no duplicate perturbations amongst the possibilities. We then allow the system to evolve for long enough for the perturbations to have magnified into a large effect, and then look at the result. Supposing we desired a certain outcome, then if that outcome occurs we send back in time the number we originally received, otherwise we invert all the bits and send that back instead. We will suppose our time machine to be very accurate, in which case histories that do not contain the desired outcome have their probabilities greatly reduced, and thus the probability for histories to occur which do contain the desired outcome are increased. Of course if the desired outcome has low raw probability and we only use a small number of bits, it may be that none of the perturbations produces the desired outcome, so the more improbable it is, the more bits we should use to give it a good chance of occurring.

### 3.1 Classical improbability equations

We will now derive the mathematics behind this concept. We send back in time $n$ bits, each with probability $p$ to be corrupted. We want an event A to happen, which has a raw probability to occur of $a_{r}$. There are $2^{n}$ perturbations possible, leading to an ensemble of $2^{n}$ possible outcomes. To simplify the equations a bit, we
will define $N=2^{n}$. We classify each possible ensemble by how many of the outcomes in it contain event A. Firstly we will calculate some unmodified properties of the ensembles, where a perturbation is just chosen at random without any communication with the future. The probability for us to have an ensemble with $m$ occurrences of event A is given by

$$
\begin{equation*}
P(m)=a_{r}^{m}\left(1-a_{r}\right)^{N-m}\binom{N}{m} . \tag{23}
\end{equation*}
$$

As each perturbation is equally likely to be applied, the probability of event A occurring if we have such an ensemble $(P(A, m))$ is simply

$$
\begin{equation*}
P(A, m)=\frac{m}{N} \tag{24}
\end{equation*}
$$

And thus the overall probability of A occurring, summing over all possible ensembles, is given by

$$
\begin{align*}
P(A) & =\sum_{m=0}^{N} P(m) P(A, m) \\
& =\sum_{m=0}^{N} \frac{m}{N} a_{r}^{m}\left(1-a_{r}\right)^{N-m}\binom{N}{m} \tag{25}
\end{align*}
$$

This of course just sums to $a_{r}$. Now we couple to the improbability generator. If A occurs our history probability gains a factor of $(1-p)^{n}$, and if not then a factor of $p^{n}$. For a given ensemble, the unnormalised probabilities for A to occur $\left(P_{u}(A)\right)$, or for A to not occur $\left(P_{u}(!A)\right)$ are given by

$$
\begin{align*}
P_{u}(A) & =m(1-p)^{n}  \tag{26}\\
P_{u}(!A) & =(N-m) p^{n} \tag{27}
\end{align*}
$$

Thus the correctly normalised probability for A to occur out of an ensemble with $m$ occurrences of A is

$$
\begin{equation*}
P(A, m)=\frac{m(1-p)^{n}}{m(1-p)^{n}+(N-m) p^{n}} . \tag{28}
\end{equation*}
$$

Note that we can only alter $P(A, m)$ with this procedure, not $P(m)$ itself. The overall coupled probability $a_{c}$ for A to occur is given by summing over all ensembles $P(A, m)$ multiplied by $P(m)$, giving

$$
\begin{equation*}
a_{c}=\sum_{m=0}^{N} \frac{m(1-p)^{n}}{m(1-p)^{n}+(N-m) p^{n}} a_{r}^{m}\left(1-a_{r}\right)^{N-m}\binom{N}{m} . \tag{29}
\end{equation*}
$$

If instead of increasing the chance for A to occur, we decrease it, by sending back what we received when A does not occur, we just substitute $p$ with $1-p$ in the above, giving

$$
\begin{equation*}
a_{c}=\sum_{m=0}^{N} \frac{m p^{n}}{m p^{n}+(N-m)(1-p)^{n}} a_{r}^{m}\left(1-a_{r}\right)^{N-m}\binom{N}{m} . \tag{30}
\end{equation*}
$$

### 3.2 Throwing dice example

We now consider some specific examples of classical improbability generation to hopefully make things clearer. First, suppose that we are throwing a pair of dice and would like to roll a pair of sixes. We use a machine to shake the dice, and increase the probability of two sixes by applying perturbations to the shaking, based on the bits sent back from the future. We continue to shake for long enough for the perturbations to have a large effect, and then roll the dice. We will use a generator with 6 bits, with a probability of corruption per bit of $1 \%$, i.e. $p=0.01$. 6 bits gives us 64 possible outcomes from the perturbations. As
the raw chance of rolling a pair of sixes is $\frac{1}{36}$, this should give us a greater than even chance of rolling a 12 . Looking at Eq. (29), we see that the $m=0$ term contributes 0 . The remaining terms may be simplified by using the approximation $p=0$. Due to the factor of $p^{6}$ in the denominator, this will be out by approximately $10^{-12}$, a negligible error. If we set $p=0$, we see that each of the remaining terms just simplifies to $P(m)$, and so

$$
\begin{align*}
a_{c} & \approx \sum_{m=1}^{N} P(m) \\
& =\left[\sum_{m=0}^{N} P(m)\right]-P(0) \\
& =1-P(0) \\
& =1-\left(1-a_{r}\right)^{N} \\
& =1-\left(\frac{35}{36}\right)^{64} \\
& \approx 0.835186 . \tag{31}
\end{align*}
$$

Intuitively this result can be understood as follows: Suppose we have an ensemble of histories, in some of which an event A (rolling a pair of sixes in this case) occurs. Those in which A does not occur will have their probability to be selected suppressed by a factor of $p^{-6}$, or $10^{12}$. Thus one of the histories in which A does happen will almost certainly be selected. The only reasonable circumstance in which A will not occur is if we start with an ensemble of histories in none of which A occurs. The chance of this is $P(0)$, and so $a_{c}=1-P(0)$.

Now we can express this result in $n: 1$ notation to get an improbability. $a_{r}=\frac{1}{36}=35: 1$. For $a_{c}$ we have

$$
\begin{align*}
a_{c} & =0.835186 \\
& =0.197338: 1 \\
& =\frac{35}{177.360}: 1 \tag{32}
\end{align*}
$$

So according to the definition of improbability, we have generated an improbability of around 177:1.

### 3.3 Comparison with quantum improbability

In the quantum case, the improbability generated is an intrinsic property of the generator: the same whatever system it was coupled to. Is this also true in the classical case? We will do a simple check by coupling the same generator as above to a different system; this time we throw only one of the dice and want to increase the probability of rolling a six. Similar to before, we have $a_{r}=\frac{1}{6}=5: 1$, and $a_{c}$ is given by

$$
\begin{align*}
a_{c} & =1-\left(\frac{5}{6}\right)^{64} \\
& =0.99999144145 \\
& =8.55862 \times 10^{-6}: 1 \\
& =\frac{5}{584206}: 1 . \tag{33}
\end{align*}
$$

So using the same generator to increase the chance of rolling a 6 , we get an improbability of approximately $584206: 1$, different to the $177: 1$ improbability when increasing the chance of a pair of sixes. Thus in the classical case the improbability generated is a function of both the generator and the system it is coupled to.

Next we will see if the result from quantum improbability for cooperating generators (Eq. (19)), is also true in the classical case. If we use two generators, identical to the 6 -bit one of the previous examples, to
increase the chance of two sixes being rolled, then if this rule holds we would expect an improbability of $177.36^{2}: 1=31456.7: 1$. Combining two 6 -bit generators is clearly equivalent to a 12 -bit generator, with $2^{12}$ $=4096$ possible histories. We calculate $a_{c}$ as before to give

$$
\begin{align*}
a_{c} & =1-\left(\frac{35}{36}\right)^{4096} \\
& =1-7.72087 \times 10^{-51} \\
& =7.72087 \times 10^{-51}: 1 \\
& =\frac{35}{4.53317 \times 10^{51}}: 1 \tag{34}
\end{align*}
$$

Clearly this is a vastly greater improbability than the quantum cooperating result gives, so in the case of classical improbability we do not have the nice improbability relations of quantum improbability.

Finally we will use the improbability generator to decrease the chance of two sixes being rolled. If we take the limit $p \rightarrow 0$, we can see that only the $m=64$ term in Eq. (30) is nonzero, giving us

$$
\begin{align*}
a_{c} & =a_{r}^{m} \\
& =\left(\frac{1}{36}\right)^{64} \\
& =2.49253 \times 10^{-100} \\
& =1.14628 \times 10^{98} \times 35: 1 \tag{35}
\end{align*}
$$

As this is such a small number, it is perhaps not valid to take the limit $p \rightarrow 0$, and so if we set $p=0.01$, as originally discussed, and evaluate $a_{c}$ using Eq. (30) in full we get

$$
\begin{align*}
a_{c} & =3.08433 \times 10^{-14} \\
& =3.2422 \times 10^{13}: 1 \\
& =9.26342 \times 10^{11} \times 35: 1 \tag{36}
\end{align*}
$$

While this is not as small as before, it is still very, very small. Clearly a classical improbability generator is more effective at suppressing an already unlikely outcome than at making it more likely.

## 4 Getting technology from the future

Suppose we have a time machine that can send large amounts of information many years back in time with high accuracy. We could use such a machine to get the design for, or theory behind, some future technology. We could then make that technology in the present, verify that it actually works, and when the time comes several years later, send the information back in time. Although there is nothing outright paradoxical about this, it is certainly very odd; after all, where did the information actually come from?

Presumably we will need a large number of bits to send something useful back in time, probably thousands, maybe even millions. Let us suppose that we have a time machine capable of sending megabytes of information years back in time, with a very low error rate per bit. With so many bits, it may still be quite likely that some are corrupted even with the low error rate per bit, but we may employ error correction algorithms to limit the effect of this. First, consider that we just send back whatever we receive, regardless of the usefulness of it; it may even be complete gibberish. In this case every possible message (sequence of bits) is equally likely.

Now suppose that we look at what we have received and decide if we like it or not. If we decide the message from the future is useful, then when the time comes we send it back, otherwise we send back the inverse message, inverting the value of every bit. By this procedure we have greatly reduced the probabilities of histories in which useless messages are received, and thus increased the probability of receiving something useful.

This is akin to the proverbial monkeys at keyboards producing the script of Hamlet: normally they wouldn't do this, but it can be made to happen with sufficiently high improbability generation. In getting information from the future like this, we have used very high levels of improbability to spontaneously generate useful ideas. Of course this is not infallible, we might receive nonsense, it always has a chance to happen. Also we might receive an idea that seems convincing but subsequently turns out to be wrong, either due to malice on the part of the sender, or because it is still considered convincing in the future.

## 5 Conclusion

We have seen that time paradoxes are averted by the altering of the probabilities within a time machine. Further, we can use causal loops to alter quantum probabilities of events outside the time machine. It is also possible to use causal loops to alter the probabilities of events in chaotic systems, by utilising the butterfly effect. Finally we see that we can get useful ideas from the future and then send them back to ourselves, and this is just an example of the spontaneous generation of useful information at high improbability.

## References

[1] A.Gray, A design for a quantum time machine, using weak measurement and non-orthogonal postselection, http://viXra.org/abs/2104.0021
[2] A.Gray, A Solution to the Quantum Measurement Problem, arXiv:quant-ph/9712037
[3] B. Tamir and E. Cohen, Introduction to weak measurements and weak values, Quanta, May 2013, p. 7. http://quanta.ws/ojs/index.php/quanta/article/view/14
[4] Y. Aharanov and E. Gruss, Two-time interpretation of quantum mechanics, arXiv:quant-ph/0507269v1

