A Design for a Quantum Time Machine, Using Weak Measurement and Non-Orthogonal Post-Selection

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Abstract

A design for a time machine, capable of transmitting information backwards in time, is presented. It achieves this by combining weak measurement of photons with a post-selection of non-orthogonal polarisation states. The non-orthogonal outcomes are the result of using a certain photonic metamaterial. As it is based on weak measurement, the accuracy of the time machine is not very high, thus avoiding the occurrence of time paradoxes.

1 Introduction

We propose to use a metamaterial [1] with non-orthogonal eigenpolarisations, coupled with a weak measurement [2, 3], to construct a device capable of transmitting information back in time.

The outline of the approach is as follows: The experiment begins at time t_0 with the emission of a single polarised photon from a source. At a slightly later time t_1, a weak measurement is performed on the polarisation of the photon. At a later time t_2, which may be arbitrarily later, an event occurs, the result of which we wish to communicate back in time. At time t_3, shortly after t_2, the photon enters a block of the metamaterial. With a suitable combination of beam state, weak measurement, metamaterial and its orientation, we will show that the expected value of the weak measurement at t_1 is dependent on the orientation of the metamaterial at t_3. Thus by orienting the metamaterial prior to t_3, in response to the event at t_2, we can communicate the outcome of the event at t_2 back in time to t_1.

We may ensure suitable orientation of the metamaterial at t_3 either by physically rotating it if needed, or we may have two blocks of metamaterial in different orientations, and direct the photon to one or the other with an optical switch depending on what happened at t_2.

Such a device will have a high error rate, due to the nature of weak measurement, and can only communicate one bit of information. The error rate can be reduced by using multiple photons per communication attempt, and the number of bits communicated can be increased by simply using several such devices in parallel.

2 Design details

2.1 Weak measurement strategy

Many photonic metamaterials exist with non-orthogonal polarisation eigenstates. As discussed above, we will send a polarised photon into such a material, performing a weak measurement on it beforehand. It is common to combine weak measurement with post-selection, which means only considering the weak measurement results that are paired with a particular final outcome. The expected value of a weak measurement in this scenario is given by

$$\langle A \rangle_w = \frac{\langle \Psi_{\text{fin}} | A | \Psi_{\text{in}} \rangle}{\langle \Psi_{\text{fin}} | \Psi_{\text{in}} \rangle} \,. \tag{1}$$

Where Ψ_{in} is the initial state, Ψ_{fin} is a post-selected final state and A is the weak measurement operator.

We will post-select both of the non-orthogonal polarisation eigenstates, i.e. everything (also the photon can be absorbed, which we will analyse separately). The contributions of each post-selection to the weak measurement will be added, weighted by their probability to occur. We will then find that the result is dependent on the orientation of the metamaterial, as desired.

It is important to note that although the final photon polarisation states are non-orthogonal, their final position states, when eventually absorbed by something, are orthogonal, so there is no contradiction with the standard quantum mechanical requirement of orthogonal outcomes; only intermediate states are non-orthogonal.

We will mostly discuss the operation of the time machine in terms of individual photons, though the same conclusions hold for beams as well. We will sometimes talk in terms of beams, rather than photons, where it is clearly more convenient to do so.

2.2 The setup

There are many metamaterials, theoretical or actually built, which exhibit non-orthogonal polarisation eigenstates. In every case there is also some absorption of the light. They are built by building up multiple thin layers with certain optical properties. We will consider the case of the metamaterial discussed here [4], of which a single layer has the Jones matrix

$$T = \begin{pmatrix} 0 & P \\ 1 & 0 \end{pmatrix}. \tag{2}$$

This has the property

$$T^2 = PI. \tag{3}$$

So two layers of the material reduce the amplitude of light passing through it by a factor of P, and this is polarisation independent.

The metamaterial has polarisation eigenstates of

$$\Psi_1 = \frac{1}{\sqrt{P+1}} \begin{pmatrix} \sqrt{P} \\ 1 \end{pmatrix}, \tag{4}$$

$$\Psi_2 = \frac{1}{\sqrt{P+1}} \begin{pmatrix} -\sqrt{P} \\ 1 \end{pmatrix}. \tag{5}$$

In general, these eigenpolarisations will experience different refractive indices within the metamaterial, resulting in the splitting of a beam directed into the material.

P can take any value between 0 and 1. We can see that the smaller P gets, the greater the non-orthogonality, and also the greater the absorption. So there is a trade off between getting a more non-orthogonal setup, yet retaining enough of the light to determine which of the polarisations a given photon is measured in.

As P gets closer to 0, the polarisations get closer to the y-axis, with the angle from the y-axis given by

$$\tan(\theta) = \sqrt{P}. \tag{6}$$

It will generally be more convenient to work with the eigenpolarisations written in terms of θ rather than P, and in this form they are simply

$$\Psi_1 = \begin{pmatrix} \sin\theta \\ \cos\theta \end{pmatrix}, \tag{7}$$

$$\Psi_2 = \begin{pmatrix} -\sin\theta \\ \cos\theta \end{pmatrix}. \tag{8}$$

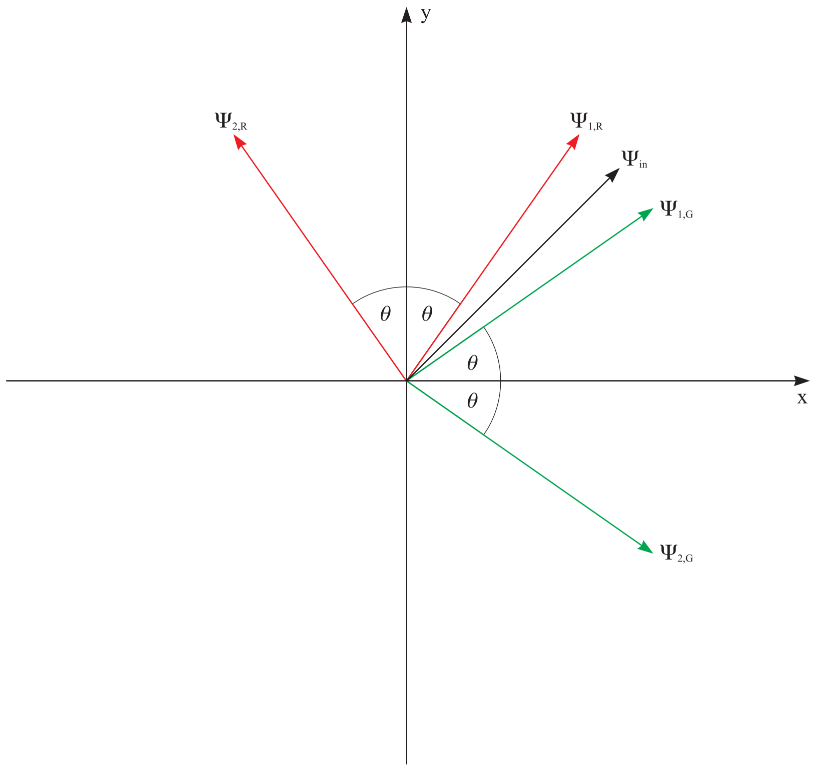

Figure 1: Polarisations of initial and final states for both orientations.

We will consider the effect of this metamaterial on a weak measurement prior to a photon entering it, and also the effect when it is rotated by 90°.

This is shown in Fig. (1), where we are sending the photon along the z-axis, and the eigenpolarisations are in the x-y plane. The red arrows denote the polarisations of the material discussed above, and the green ones that of a 90° rotation of that material in the x-y plane. The beam/photon is initially polarised at 45° to the x-axis as shown. Using the Jones calculus, the initial state is given by

$$\Psi_{\text{in}} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}. \tag{9}$$

We will make a weak measurement of the x-component of the polarisation; thus our weak measurement operator A is given by

$$A = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}. \tag{10}$$

Technically there should be a constant of proportionality here, to indicate by how much a pointer (for example) is moved, in actual centimetres, upon performing the weak measurement. As we're only interested in the relative sizes of various weak measurements we might make, we ignore this complication.

The relative probability (not properly normalised) to obtain a final state Ψ_{fin} is given by

$$P\left(\Psi_{\text{fin}}\right) = \left|\langle \Psi_{\text{fin}} | \Psi_{\text{in}} \rangle\right|^2. \tag{11}$$

There is also a chance of absorption, the effect of which we will analyse separately. We will now calculate the expected value of the weak measurement in the red and green orientations of the metamaterial.

2.3 Red case

We will calculate the expected weak value in the red case, $\langle A_w \rangle_R$, by calculating the expected weak values for each of the possible final states, and then summing them weighted by the probability of each outcome. The two possible final states are

$$\Psi_{\text{fin 1,R}} = \begin{pmatrix} \sin\theta \\ \cos\theta \end{pmatrix}, \tag{12}$$

$$\Psi_{\text{fin 2,R}} = \begin{pmatrix} -\sin\theta \\ \cos\theta \end{pmatrix}. \tag{13}$$

Their probabilities to occur are given by

$$P\left(\Psi_{\text{fin 1,R}}\right) = \frac{1}{2}\left(\sin\theta + \cos\theta\right)^2, \tag{14}$$

$$P\left(\Psi_{\text{fin 2,R}}\right) = \frac{1}{2}\left(\cos\theta - \sin\theta\right)^2. \tag{15}$$

Using Eq. (1), we find that the corresponding expected weak values are given by

$$\langle A_w \rangle_{1,R} = \frac{\sin\theta}{\sin\theta + \cos\theta}, \tag{16}$$

$$\langle A_w \rangle_{2,R} = \frac{-\sin\theta}{\cos\theta - \sin\theta}. \tag{17}$$

Adding these, weighted by probabilities, we find the overall expected weak value $\langle A_w \rangle_R$ is given by

$$\begin{aligned} \langle A_w \rangle_R &= \langle A_w \rangle_{1,R} P\left(\Psi_{\text{fin 1,R}}\right) + \langle A_w \rangle_{2,R} P\left(\Psi_{\text{fin 2,R}}\right) \\ &= \frac{1}{2}\frac{\sin\theta}{\sin\theta + \cos\theta}\left(\sin\theta + \cos\theta\right)^2 - \frac{1}{2}\frac{\sin\theta}{\cos\theta - \sin\theta}\left(\cos\theta - \sin\theta\right)^2 \\ &= \sin^2\theta. \end{aligned} \tag{18}$$

2.4 Green case

Similarly for the green case, the two possible final states are

$$\Psi_{\text{fin 1,G}} = \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}, \tag{19}$$

$$\Psi_{\text{fin 2,G}} = \begin{pmatrix} \cos\theta \\ -\sin\theta \end{pmatrix}. \tag{20}$$

Their probabilities to occur are given by

$$P\left(\Psi_{\text{fin 1,G}}\right) = \frac{1}{2}\left(\sin\theta + \cos\theta\right)^2, \tag{21}$$

$$P\left(\Psi_{\text{fin 2,G}}\right) = \frac{1}{2}\left(\cos\theta - \sin\theta\right)^2. \tag{22}$$

Unsurprisingly, these are the same as before, as the angles with Ψ_{in} are the same. The corresponding expected weak values are given by

$$\langle A_w \rangle_{1,G} = \frac{\cos\theta}{\cos\theta + \sin\theta}, \tag{23}$$

$$\langle A_w \rangle_{2,G} = \frac{\cos\theta}{\cos\theta - \sin\theta}. \tag{24}$$

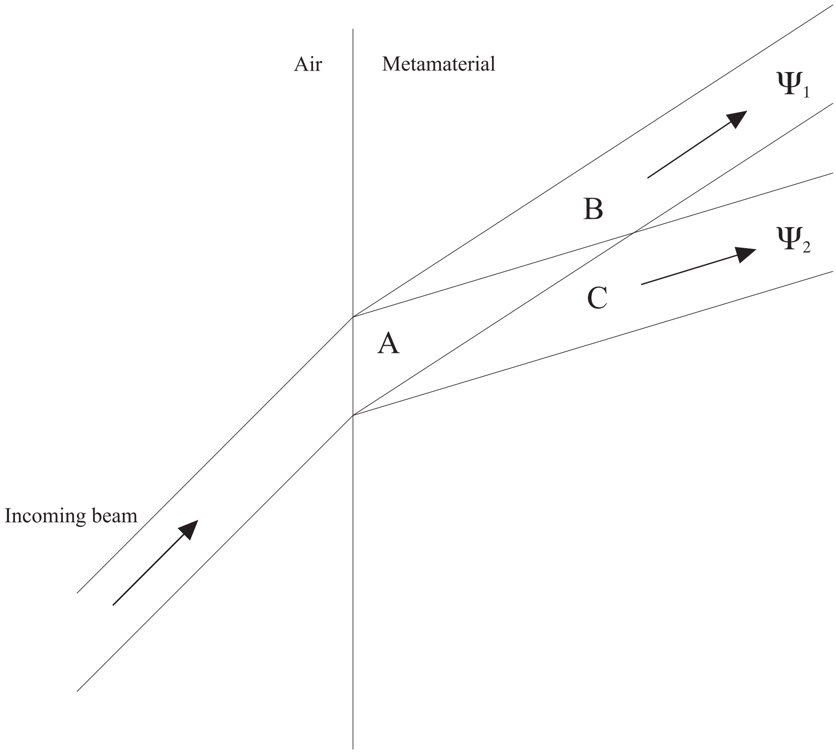

Figure 2: Beam entering the metamaterial and splitting into two polarised beams.

Adding these, weighted by probabilities, we find the overall expected weak value $\langle A_w \rangle_G$ is given by

$$\begin{aligned}
\langle A_w \rangle_G &= \langle A_w \rangle_{1,G} \, P\left(\Psi_{\text{fin 1,G}}\right) + \langle A_w \rangle_{2,G} \, P\left(\Psi_{\text{fin 2,G}}\right) \\
&= \frac{1}{2}\frac{\cos\theta}{\cos\theta + \sin\theta}\left(\sin\theta + \cos\theta\right)^2 + \frac{1}{2}\frac{\cos\theta}{\cos\theta - \sin\theta}\left(\cos\theta - \sin\theta\right)^2 \\
&= \cos^2\theta. \qquad (25)
\end{aligned}$$

2.5 Comparison

The difference between the two weak values is

$$\cos^2\theta - \sin^2\theta = \cos 2\theta. \qquad (26)$$

We can see that at $\theta = 45^\circ$ there is no difference, as this is the orthogonal eigenpolarisations limit. The closer θ is to 0 the greater the difference, so one might think that we would want the polarisations to be as non-orthogonal as possible. Unfortunately, as discussed above, the greater the non-orthogonality the greater the chance of absorption, in which case we might not end up with Ψ_{fin} in either of the eigenpolarisations. We will now analyse the impact of absorption on the situation.

2.6 Effect of absorption

In Fig. (2) we show what happens when a beam of light enters the metamaterial. It splits into two polarised beams. In region B it is polarised as Ψ_1, and in C as Ψ_2. In region A the beams overlap and the light here is not yet in either of the two polarised states, but still in the initial state Ψ_{in}. If absorbed in region B, then it must have entered the material in state Ψ_1, so $\Psi_{\text{fin}} = \Psi_1$, and our analysis above is unaffected. Similarly for absorption in C and Ψ_2. If you believe that a photon has to exit the metamaterial and hit a detector for us to declare what Ψ_{fin} is, then suppose that the metamaterial only occupies the A volume, with B and C being volumes of air; thus absorption doesn't happen in them, achieving the same result.

It is absorption in the A region that makes a difference. In this case the beam is still in the Ψ_{in} state; thus $\Psi_{\text{fin}} = \Psi_{\text{in}}$. Using Eqs. (1, 9, 10) we find that the contribution to $\langle A_w \rangle$ if absorbed in the A region is $\frac{1}{2}$.

We denote the probability to be absorbed in the A region by p. Clearly p will depend on the properties of the beam as well as of the metamaterial, so we won't be calculating it from first principles here. It will however be the same in both the red and green cases, as the eigen-refractive indices are the same, and assuming the beam properties are also the same, the three regions will be the same. The only difference is the ratio of the beam intensities in the B and C regions, which does not affect p. Thus, taking absorption into account, we find that the expected weak values in the red and green cases are

$$\langle A_w \rangle_R = (1-p)\sin^2\theta + \frac{p}{2}\,, \tag{27}$$

$$\langle A_w \rangle_G = (1-p)\cos^2\theta + \frac{p}{2}\,. \tag{28}$$

The difference between the two weak values is now

$$(1-p)\left(\cos^2\theta - \sin^2\theta\right) = (1-p)\cos 2\theta. \tag{29}$$

We will refer to this as the signal strength and label it Δ.

2.7 How to perform the weak measurement

We will briefly discuss how to actually perform a weak measurement of the x-polarisation of a photon. There are presumably many ways to do this apart from the one shown here, which is meant as an example only.

First, we pass the initial beam through a down converter, splitting it into two beams, each of half frequency. One of these beams we will call the main beam, and it will continue on to meet the metamaterial at some point in the future. The other beam we will call the weak measurement (WM) beam, and we will perform a weak measurement on the x-polarisation of the photons in it. As each photon in the WM beam is entangled with one in the main beam, this is equivalent to performing a weak measurement on the photons in the main beam.

To weakly measure the polarisation of the WM beam photons, we pass the beam through a thin birefringent crystal, with eigenpolarisations in the x and y directions. This will cause the x and y polarisation components of the beam to separate into two further beams. If the separation is arranged to be very small, and much less than the beam width, the beams will still largely overlap after passing through the birefringent crystal. Thus measuring the position of any photon in the overlapping beams constitutes a weak measurement of its polarisation.

As this position measurement will absorb the photon, we cannot perform such a measurement directly on the photons of the main beam, hence the need to create a beam of entangled photons.

3 Signal strength

In practice most photonic metamaterials are constructed from alternating layers with different optical properties. For example, in the case considered here, each layer of the material is constructed from two sub-layers: a partial polariser and a half-wavelength retarder. For the material to work it is necessary that the layers be thin enough that the material, to a good approximation, appears homogeneous to a beam passing through it. Thus the layers must be of comparable size to the beam wavelength, or smaller. As a beam will typically be hundreds or thousands of wavelengths in width, we can expect to need hundreds or thousands of layers of the metamaterial to achieve beam separation. Unfortunately each layer absorbs some of the light, which reduces the signal strength. We can reduce the absorption per layer, but only at the expense of reducing the non-orthogonality, thus reducing the difference between the two weak measurements, making the signal harder to detect. Obviously we will want to make the beam as narrow as possible and angle it so that beam separation is achieved as soon as possible.

Suppose we have optimised the beam as best we can, and have determined that we need N layers of the metamaterial to achieve beam separation. How does the signal strength then depend on the properties of the metamaterial?

We see from Eq. (3) that passing through two layers of the metamaterial multiplies a beam's amplitude by P, or its intensity by P^2. Thus each layer multiplies the intensity by P. We will consider the case where P is nearly 1, i.e.

$$P = 1 - \delta. \tag{30}$$

After passing through n layers the intensity I_n of the beam will be

$$\begin{aligned} I_n &= P^n \\ &= (1-\delta)^n \\ &\approx e^{-\delta n}. \end{aligned} \tag{31}$$

As discussed above, we denote the number of layers required to separate the beams, i.e. the number of layers in the A region of Fig. (2), by N. If there is no absorption, then per layer a fraction $\frac{1}{N}$ of the original photons enters region B or C. In the presence of absorption, we multiply this by the beam intensity at the layer in question. Thus the fraction of photons that end up in either of the eigenpolarisations, the $1-p$ of Eq. (29) is approximately given by

$$\begin{aligned} 1-p &= \int_0^N dn \frac{e^{-\delta n}}{N} \\ &= \frac{1}{\delta N}\left(1 - e^{-\delta N}\right). \end{aligned} \tag{32}$$

To find the overall signal strength Δ, we must multiply this by $\cos 2\theta$. We can see that 2θ is just the angle between the two eigenpolarisations. Using the polarisations given in Eqs. (7, 8), we can see that

$$\begin{aligned} |\Psi_1|\,|\Psi_2| \cos 2\theta &= \Psi_1.\Psi_2 \\ &= \frac{1-P}{1+P}. \end{aligned} \tag{33}$$

And so

$$\begin{aligned} \cos 2\theta &= \frac{1-P}{1+P} \\ &\approx \frac{\delta}{2}. \end{aligned} \tag{34}$$

Thus the overall signal strength Δ is given by

$$\begin{aligned} \Delta &= (1-p)\cos 2\theta \\ &= \frac{1}{2N}\left(1 - e^{-\delta N}\right). \end{aligned} \tag{35}$$

We can see that this takes the value 0 at $\delta = 0$, the orthogonal case, and as δ increases converges to $\frac{1}{2N}$, being close to this value once $\delta N > 1$. Thus it may seem that greater non-orthogonality is optimal, however we must bear in mind that N will depend on δ, complicating the analysis. This is because the refractive indices of the two polarisations will vary with δ, and the ease with which the beam can be split will depend on the difference between those refractive indices. Nevertheless, whatever the dependence of N on δ, N will be fairly large, and at best $\Delta = \frac{1}{2N}$, so absorption is always going to cause substantial signal loss.

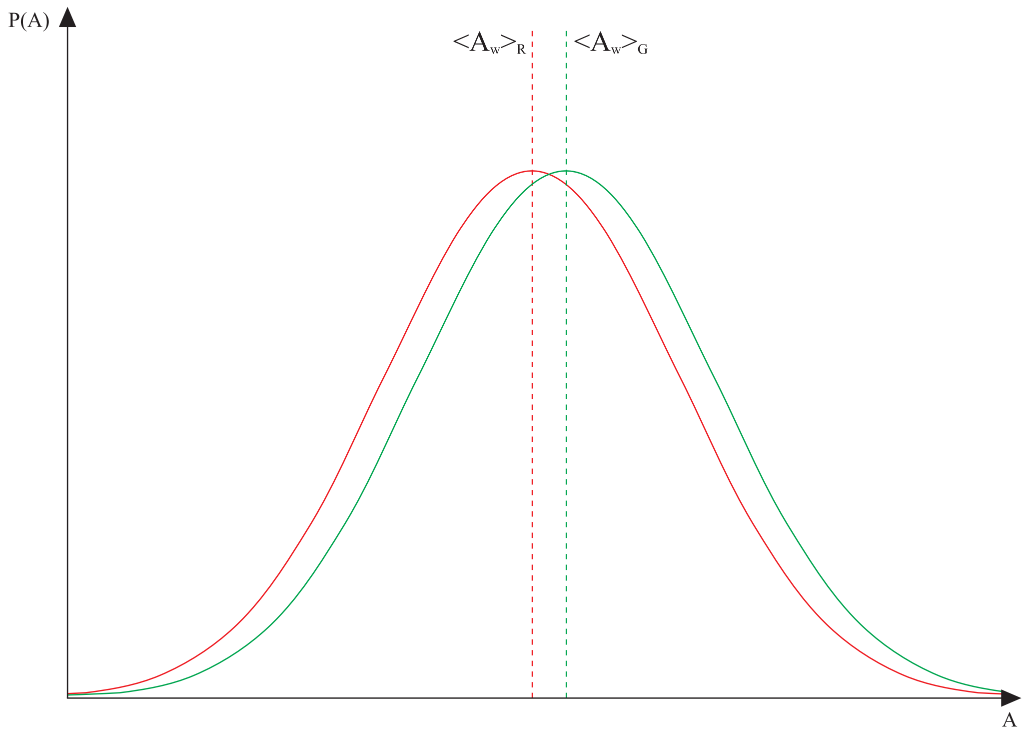

Figure 3: Distribution of weak measurement A in both cases.

3.1 Avoidance of absorption signal loss

As we have seen that absorption leads to significant signal loss, it is natural to seek a way to avoid it. Fortunately this can be achieved by placing a layer of material of high refractive index (e.g. diamond) on top of the metamaterial, and then angling the beam such that one of the eigenpolarisations is reflected due to total internal reflection, while the other enters the material. As their refractive indices are different, it should be possible to arrange this. If a photon is reflected, we know which polarisation it has, and if it is absorbed, or passes through the metamaterial, we know it must have the other polarisation. Thus absorption no longer causes any loss of information, and thus no signal degradation.

4 Paradoxes

A common objection to the possibility of time travel is that it results in various paradoxes. For example, suppose we use this time machine to predict if a photon will end up in the red or green orientation of the metamaterial, and then ensure that the metamaterial is actually in the opposite orientation when the photon arrives there. Surely this is impossible if the time machine actually works?

There is in fact no problem here, as the time machine, when performing a weak measurement on a single photon, is not very accurate. This is because weak measurements themselves are not very accurate. While we may know with some precision the average value $\langle A_w \rangle$ of a given weak measurement, a set of these weak measurements will be distributed around that average, with a large standard deviation. For example, the distribution of weak measurements in the red and green orientations may look like that in Fig. (3), with a large amount of overlap between the two.

To make a prediction with a single photon, we would make a weak measurement on it, and conclude that the red orientation will be encountered later if the value is closer to $\langle A_w \rangle_R$ than $\langle A_w \rangle_G$, otherwise we predict the green orientation to be encountered. Due to the substantial overlap between the two distributions, such a prediction will often be wrong. Suppose both orientations are equally likely to occur, then the machine may be right, for example, 51% of the time, slightly better than the 50% chance when making a random guess. Essentially there is no paradox here as the time machine has an error rate. One can reduce it, of course, by using more photons, but never to zero.

5 Necessity of non-orthogonality

Given that metamaterials with non-orthogonal eigenpolarisations are both difficult to make, and have the unwanted feature of absorption, one might wonder if we can achieve a similar result by post-selecting everything entering a birefringent crystal with orthogonal eigenpolarisations, and varying the orientation of that crystal. We will show here that, with orthogonal outcomes, no information about the future can be obtained.

As before, our beam will travel in the z-direction and we will perform a weak measurement on the x-component of the polarisation. We will have Ψ_{in} polarised at an angle ϕ in the x-y plane. The eigenpolarisations of the crystal will be at angles θ and $\theta + \frac{\pi}{2}$; these polarisation states are denoted by Ψ_1 and Ψ_2. So the relevant polarisation states are given by

$$\begin{aligned}
\Psi_{\text{in}} &= \begin{pmatrix} \cos\phi \\ \sin\phi \end{pmatrix}, && (36)\\
\Psi_1 &= \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}, && (37)\\
\Psi_2 &= \begin{pmatrix} -\sin\theta \\ \cos\theta \end{pmatrix}. && (38)
\end{aligned}$$

The probabilities for Ψ_{fin} to be Ψ_1 or Ψ_2 are given by

$$\begin{aligned}
P(\Psi_1) &= \cos^2(\phi-\theta), && (39)\\
P(\Psi_2) &= \sin^2(\phi-\theta). && (40)
\end{aligned}$$

As before we use the weak measurement operator A:

$$A = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}. \tag{41}$$

The weak measurement expected values are calculated to be

$$\begin{aligned}
\langle A_w\rangle_1 &= \frac{\cos\theta\cos\phi}{\cos(\phi-\theta)}, && (42)\\
\langle A_w\rangle_2 &= \frac{-\sin\theta\cos\phi}{\sin(\phi-\theta)}. && (43)
\end{aligned}$$

Weighting by probabilities to get the overall expected weak value, we find

$$\begin{aligned}
\langle A_w\rangle &= \langle A_w\rangle_1 P(\Psi_1) + \langle A_w\rangle_2 P(\Psi_2)\\
&= \frac{\cos\theta\cos\phi}{\cos(\phi-\theta)}\cos^2(\phi-\theta) - \frac{\sin\theta\cos\phi}{\sin(\phi-\theta)}\sin^2(\phi-\theta)\\
&= \cos\theta\cos\phi(\cos\theta\cos\phi+\sin\theta\sin\phi) - \sin\theta\cos\phi(\sin\phi\cos\theta-\cos\phi\sin\theta)\\
&= \left(\cos^2\theta+\sin^2\theta\right)\cos^2\phi\\
&= \cos^2\phi. && (44)
\end{aligned}$$

So the weak measurement outcome is independent of the angle θ, and thus can give us no information about it.

Also, it is necessary for the non-orthogonality to occur at the moment the beam splits. Whatever condition causes the beam to split, in this case the polarisation state, is the thing that we are measuring and thus post-selecting on. Therefore, splitting the beam into a pair with orthogonal polarisation, one of which we then rotate afterwards to make non-orthogonal, will not work.

6 Conclusion

Using weak measurement theory combined with non-orthogonal final states, it appears to be possible to make a time machine capable of sending information from the future to the past. As weak measurement theory is derived from standard quantum mechanics, and has been found to be true in experiments to date, then either time travel of this nature is indeed possible, or quantum theory needs to be modified in some way to prevent it. Either possibility is very interesting.

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