Einstein's field equations– the unimodular defect
(A summary)

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Albert Einstein first presented his gravitational field equations in unimodular coordinates. In these coordinates, the field equations can be written explicitly in terms of the Einstein pseudotensor for the energy-momentum of the gravitational field. Since this pseudotensor produces, by contraction, a first-order intrinsic differential invariant, it violates the rules of pure mathematics. This is sufficient to prove that Einstein's unimodular field equations are invalid. Since the unimodular form must hold in the General Theory of Relativity, it follows that the latter is also unsound, lacking a proper mathematical foundation.

1. INTRODUCTION

In his original derivation of this field equations for the gravitational field, Albert Einstein utilised the so-called 'unimodular coordinates' [1, 2]. This set of coordinates is characterised by $\sqrt{-g} = 1$, where g is the determinate of the metric tensor.

Using the unimodular coordinates Einstein's field equations can be written explicitly in terms of his 'pseudotensor'. Since his pseudotensor is a meaningless concoction of mathematical symbols it follows immediately that his field equations are also a meaningless concoction of mathematical symbols. This proves that his General Theory of Relativity is false.

II. THE UNIMODULAR FIELD EQUATIONS

Einstein [1, 2] advanced his field equations in the *absence* of matter in this form:

$$\frac{\partial \Gamma^{\alpha}_{\mu\nu}}{\partial x^{\alpha}} + \Gamma^{\alpha}_{\mu\beta}\Gamma^{\beta}_{\nu\alpha} = 0$$
$$\sqrt{-g} = 1 \tag{1}$$

Einstein [2] advanced his field equations in the *presence* of matter in this form:

$$\frac{\partial}{\partial x^{\alpha}}\left(g^{\sigma\beta}\Gamma^{\alpha}_{\mu\beta}\right) = -\kappa\left[\left(t^{\sigma}_{\mu} + T^{\sigma}_{\mu}\right) - \frac{1}{2}\delta^{\sigma}_{\mu}(t + T)\right]$$
$$\sqrt{-g} = 1 \tag{2}$$

Setting $T^{\sigma}_{\mu} = 0$ the field equations in the *absence* of matter become:

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$$\frac{\partial}{\partial x^\alpha}\left(g^{\sigma\beta}\Gamma^\alpha_{\mu\beta}\right) = -\kappa\left(t^\sigma_\mu - \frac{1}{2}\delta^\sigma_\mu t\right)$$
$$\sqrt{-g} = 1$$

(3)

Equations (2) and (3) contain Einstein's pseudotensor t^σ_μ, defined by [2]:

$$\kappa t^\alpha_\sigma = \frac{1}{2}\delta^\alpha_\sigma g^{\mu\nu}\Gamma^\lambda_{\mu\beta}\Gamma^\beta_{\nu\lambda} - g^{\mu\nu}_\sigma\Gamma^\alpha_{\mu\beta}\Gamma^\beta_{\nu\sigma}$$

(4)

But it has already been proven that Einstein's pseudotensor is a meaningless concoction of mathematical symbols because it violates the rules of pure mathematics [3, 4]. Consequently, the field equations (2) and (3) are also meaningless concoctions of mathematical symbols. Now after some tedious mathematical manipulations, Eqs. (3) can be written as:

$$\frac{\partial \Gamma^\alpha_{\mu\nu}}{\partial x^\alpha} + \Gamma^\alpha_{\mu\beta}\Gamma^\beta_{\nu\alpha} = 0$$
$$\sqrt{-g} = 1$$

(5)

which is just the field equations (1). Hence equations (1) are also meaningless concoctions of mathematical symbols. Consequently, Einstein's General Theory of Relativity is false.

This result cannot be overcome by alternative constructions employing the Landau-Lifshitz pseudotensor instead because it too is a meaningless concoction of mathematical symbols for the very same reason that Einstein's pseudotensor is invalid [4, 5].

III. CONCLUSION

The General Theory of Relativity is false because it violates the rules of pure mathematics.

REFERENCES

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