Abstract: We Humans, a curious beings developed from the Darwin's principle of natural selection, are accustomed into an inquisition. The question is not 'do we know everything from the triumph of the Higgs boson to the underlying discomfort of ultimate question of life, the universe, and everything?' or it is 'do we know enough?' But how the creative principle resides in mathematics? There's something very mathematical about our gigantic Cosmos, and that the more carefully we look, the more equations are built into nature: From basic arithmetic to the calculation of rocket trajectories, math provides a good understanding of the equations that govern the world around us. Our universe isn't just described by math, but that universe is a "grand book" written in the language of mathematics. We find it very appropriate that mathematics has played a striking role in our growing understanding of the events around us, and of our own existence.
"Our universe may be one of an infinite number of parallel universes, each connected to the others by an infinite series of wormholes."

3 Laws of Universe:

- You cannot get something for nothing because matter and energy are conserved.
- You cannot return to the same energy state because there is always an increase in entropy.
- Absolute zero is unattainable.

Equivalence principle:

The laws of nature in an accelerating frame are equivalent to the laws in a gravitational field.

Matter-energy $\rightarrow$ curvature of space-time

Geometry $\rightarrow$ field theory $\rightarrow$ classical theory $\rightarrow$ quantum theory

"But the creative principle resides in mathematics. In a certain sense, therefore, I hold it true that pure thought can grasp reality, as the ancients dreamed." — Albert Einstein

$\Rightarrow$ Newton's laws of motion tie into almost everything we see in everyday life.
- **Law 1 (law of inertia):** An object at rest stays at rest and an object in motion stays in motion with the same speed and in the same direction unless acted upon by an unbalanced force.

- **Law 2:** Force equals mass times acceleration \((F = ma)\).

- **Law 3:** For every action, there is an equal and opposite reaction.

\[ \text{As a remarkable consequence of the uncertainty principle of quantum mechanics (which implies that certain pairs of quantities, such as the energy and time, cannot both be predicted with complete accuracy), the empty space is filled with what is called vacuum energy.} \]

**Because \(E=MC^2\):**

- Mass is just energy in disguise.
- A small amount of mass can equal a large amount of energy.

**If the mass of the star < 1.4 solar masses**

- Electrons prevent further collapse.
- The core will thus continue to collapse and form a **white dwarf**.

**If the mass of the star > 1.4 solar masses but mass < 3 solar masses**

- Electrons + protons combine to form neutrons.
- Neutrons prevent further collapse.
- The core will thus continue to collapse and form a **neutron star**.

**If the mass of the star > 3 solar masses**

- Gravity wins! Nothing prevents collapse.
- The core will thus continue to collapse and form a black hole.

*Any object with a physical radius smaller than its **Schwarzschild radius** will be a black hole.*
All the laws of physics that we know, breaks down –

- Below this time: (Planck Time)
- Below this length: (Planck Length)
- Above this temperature: (Planck Temperature)

Density parameter ($\Omega$): The ratio of the total amount of matter in the universe divided by the minimum amount of matter needed to cause the big crunch.

- $\Omega < 1$: the Universe will continue to expand forever.
- $\Omega > 1$: the Universe will eventually halt its expansion and recollapse.
- $\Omega = 1$: the Universe contains enough matter to halt the expansion but not enough to recollapse it.

If a black hole has a mass less than the Planck mass, its quantum mechanical size could be outside its event horizon. This wouldn't make sense, Planck mass is the smallest possible black hole.

Absolute zero: (–273°C) – the lowest possible temperature, at which substances contain no heat energy and all vibrations stop — almost.

⇒ If the two quarks would have occupied precisely the same point with the same properties, they would not have stayed in the same position for long. And quarks would have not formed separate, well-defined protons and neutrons. And nor would these, together with electrons have formed separate, well-defined atoms. And the world would have collapsed before it ever reached its present size.

- When 2 similar waves are added, the resultant wave is bigger (constructive interference).
• When 2 dissimilar waves are added, they cancel each other out (destructive interference).

*The different frequencies of light appear as different colors.*

Proton charge + Electron charge = 0

Just what it is if electromagnetism would not dominate over gravity and for the universe to remain electrically neutral.

• It's not their energy; it's their zero rest mass that makes photons to travel at the speed of light.

• Just like a dozen is 12 things, a mole is simply **Avogadro's number of particles**.

Undisturbed space + rigid mass = distorted space.

3 dimensions of space + one dimension of time = **single four-dimensional continuum** (space-time).

Since $h$ – which is one of the most **fundamental numbers in physics**, ranking alongside the speed of light "c" and confined most of these radical departures from life-as-usual to the microscopic realm – is incredibly small (i.e., $6 \times 10^{−34}$ – a decimal point followed by 33 zeros and a 6 – of a joule second), the frequency of the photon is always greater than its energy, so it do not take many quanta to radiate even ten thousand megawatts.

What is GRAVITY?

• **Newtonian view:** Force tells mass how to accelerate. Accelerated mass tells what gravity is.
• **Einsteinian view:** Mass tells space how to curve. Curved space tells what gravity is.

All objects emit **electromagnetic radiation** according to their temperature. Colder objects emit waves with very low frequency (such as radio or microwaves), while hot objects emit visible light or even **ultraviolet** and higher frequencies.

• Longer half-life of nucleus → **Slow Radioactive Decay.**
• Shorter half-life of nucleus → Fast Radioactive Decay.

".. Physics at the atomic and subatomic level ...

… Weird things are possible:

• Energy is quantized \( E = nh\nu \).
• Momentum is quantized \( L = nh \).
• Charge is quantized \( Q = ne \).

Because

\[
E = h\nu, \quad c = \lambda\nu, \quad E = \frac{hc}{\lambda} = pc
\]

\[
\lambda = \frac{h}{p}
\]

*Every particle or quantum entity may be partly described in terms not only of particles, but also of waves.*

The **Thermodynamic Laws think big:** they dictate energy behavior…

• **1 Law:** Energy is conserved; its form can be converted.
• **2 Law:** Energies can flow, equilibrate.
• **3 Law:** "Driving force" for equilibration uniquely defined.
• **0 Law**: Thermal equilibrium is transitive.

**The Life of a Star:**

"More mass

More pressure and temperature

Faster Fusion

**Shorter life"**

"Less mass

Less pressure and temperature

Slower Fusion

**Longer life"**

**String Theory** (A theory that tries to adjust / harmonize / reconcile **General Relativity** and **Quantum mechanics**):

• Different vibrations → Different particles.
• String combinations → Particle interactions.

**The universe is made of**

• 21% Dark Matter.
• 74% Dark Energy.
• 4% Normal Matter.

**MATTER UNDER EXTREME CONDITIONS**
Nuclei + heat + pressure $\rightarrow$ quark-gluon plasma

- **Hydrogen atom**: Diameter about a Billionth of an inch.
- **Electron**: Diameter at least 1000 times smaller than that of proton.
- **Proton**: Diameter about 60,000 times smaller than H atom.
- **Probability distribution** is the only way to locate an electron in an atom.

The Gas laws deal with how gases behave with respect to pressure, volume, temperature ... 

- **Boyle's law**: Volume and pressure are inversely proportional.

- **Charles' law**: Volume is proportional to temperature.

- **Pressure law**: Pressure is proportional to temperature.

**All three combined**:

$$\frac{PV}{T} = \text{constant}$$

n $\rightarrow$ number of neutrons

p $\rightarrow$ number of protons

- **If** $(n/p) \approx 1$ (atomic number between 1 and 20):
  Nucleus has equal number of protons and neutrons to become stable.

- **If** $(n/p) > 1$ (atomic number between 20 and 83):
Number of protons increase and repulsion between them also increases. To balance this force number of neutrons also increases.

- If \( \frac{n}{p} > 1 \) (atomic number > 83):

Nucleus having atomic number higher than 83 has great number of protons and repulsion force between protons. Since the amount of force is too high, number of neutrons cannot balance them and nucleus stays unstable. Thus, we can say that nuclei having atomic number greater than 83 generally undergo transmutation, alpha decay or beta decay.

- Weak nuclear forces + Maxwell equations → Electro weak theory.
- Standard Model of particle physics → explains everything except gravity.

4 NUMBERS describe the characteristics of electrons and their orbitals:

- **Principal quantum number**: a number that describes the average distance of the orbital from the nucleus and the energy of the electron in an atom.
- **Angular momentum quantum number**: a number that describes the shape of the orbital.
- **Magnetic quantum number**: a number that describes how the various orbitals are oriented in space.
- **Spin quantum number**: a number that describes the direction the electron is spinning in a magnetic field — either clockwise or counterclockwise.

The square of the periods of the planets (the times for them to complete one orbit) is proportional to the cubes of their average distance from the Sun. A consequence of this is that the
inner planets move rapidly in their orbits. Venus, Earth and Mars move progressively less rapidly about the Sun. And the outer planets, such as Jupiter and Saturn, move stately and slow.

Wavelength of UV radiation < Wavelength of IR radiation < Wavelength of microwave radiation

- Molecule dissociates (when it absorbs UV radiation).
- Molecule vibrates (when it absorbs IR radiation).
- Molecule rotates (when it absorbs microwave radiation).

⇒ If the expansion of space had overwhelmed the pull of gravity in the beginning – stars, galaxies and humans would never have been able to form. If, on the other hand, gravity had been 5% stronger – stars and galaxies might have formed, but they would have quickly collapsed in on themselves and each other to form a sphere of roughly infinite density.

Neutrons have a mass of 939.56 MeV.

If the mass of a neutron was a seventh of a percent more than it is, stars like most of those we can see would not have existed. If the neutron mass was 0.085% less than it is, the Universe would have been full of neutrons and nothing else.

If we cut the surface of a sphere up into faces, edges and vertices, and let F be the number of faces, E the number of edges and V the number of vertices, we will always get:

\[ V - E + F = 2. \]

Fibonacci numbers – 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144...

- Each number is the sum of the previous two.
• The ratio between the numbers = 1.618034 (golden ratio).
• From pinecones to the Hurricane Sandy, Fibonacci reflects various patterns found in nature.

The paths of anything you throw have the same shape, called an upside-down parabola.

When we observe how objects move around in gravitationally curved trajectories in space, we discover another recurring shape: the ellipse.

All material particles have properties such as charge and spin.

Space itself has properties such as dimensions.

• These properties are purely mathematical.

Equations aren't the only hints of mathematics that are built into nature: there are also numbers involving not only motion and gravity, but also areas as disparate as classical physics, quantum mechanics, and astronomy.

Strong force → force that is responsible for binding together the fundamental particles of matter to form larger particles.

• If stronger: No hydrogen would have formed; atomic nuclei for most life-essential elements would have been unstable; thus, there would have been no life chemistry.
• If weaker: No elements heavier than hydrogen would have formed: again, no life chemistry.

Weak force → force that is responsible for the radioactive decay of atoms.
• **If stronger:** Too much hydrogen would have been converted to helium in the big bang; hence, stars would have converted too much matter into heavy elements making life chemistry impossible.

• **If weaker:** Too little helium would have been produced from big bang; hence, stars would have converted too little matter into heavy elements making life chemistry impossible.

**Electromagnetic force →** force that is responsible for most of the interactions we see in our environment today.

• **If stronger:** Chemical bonding would have been disrupted; elements more massive than boron would have been unstable to fission.

• **If weaker:** Chemical bonding would have been insufficient for life chemistry.

\(c = 299,792,458 \text{ meters per second}\) – serves as the single limiting velocity in the universe, being an upper bound to the propagation speed of signals and to the speeds of all material particles.

**Ratio of electromagnetic force to gravitational force**

• **If larger:** All stars would have been at least 40% more massive than the sun; hence, stellar burning would have been too brief and too uneven for life support.

• **If smaller:** All stars would have been at least 20% less massive than the sun, thus incapable of producing heavier elements.

**Ratio of electron to proton mass**

• **If larger or smaller:** Chemical bonding would have been insufficient for life chemistry.
Mass of the neutrino

- If smaller: Galaxy clusters, galaxies, and stars would have not formed.
- If greater: Galaxy clusters and galaxies would have been too dense.

Ratio of exotic matter to ordinary matter

- If larger: the universe would have collapsed before the formation of solar-type stars.
- If smaller: no galaxies would have formed.

Number of effective dimensions in the early universe

- If larger or smaller: Quantum mechanics, gravity, and relativity could not have coexisted; thus, life would have been impossible.

Entropy level of the universe

- If larger: Stars would have not formed within proto-galaxies.
- If smaller: No proto-galaxies would have formed.

Polarity of the water molecule

- If greater: Heat of fusion and vaporization would have been too high for life.
- If smaller: Heat of fusion and vaporization would have been too low for life; liquid water would not have worked as a solvent for life chemistry; ice would not have floated, and a runaway freeze-up would have resulted.

\[ F = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2} \]
The force decreases with increasing distance between the charged particles; when the distance is doubled, the force falls by a factor of 4.

Leptons

- electron (stable)
- muon (unstable)
- tau (unstable)
- electron neutrino
- muon neutrino
- tau neutrino

Quarks

- up quark
- down quark
- top quark
- bottom quark
- charmed quark
- strange quark

Not seen as free particles; constituents of protons and neutrons.

Vector bosons

- photon [electromagnetism]
- W-boson [weak force]
- Z-boson [weak force]
- Gluons [strong force]
- Gravitons [gravity]
The greater the distance $d$ to the galaxy, the higher the velocity $v$ with which it receded from us, according to the formula:

$$v = \text{Hubble parameter} \times d$$

The momentum $p$ of something with intrinsic mass $m_0$ moving with velocity $v$ is simply given by

$$p = m_0v$$

as long as $v << c$.

If $v < c$:

$$p = \frac{m_0v}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Energy of the photon = Work Function of the surface + Kinetic energy of the emitted electron

$$h\nu = W + \frac{m_0v^2}{2}$$

If $h\nu < W$:

No photoelectric emission

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$L = L_0 \sqrt{1 - \frac{v^2}{c^2}}$$

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$
If $v = c$:

\[
\begin{align*}
    m & \rightarrow \infty \\
    L & \rightarrow 0 \\
    \Delta t & \rightarrow \infty
\end{align*}
\]

- Neutron $\leftrightarrow$ proton + electron + antineutrino (**beta decay**)
- Proton + electron $\leftrightarrow$ neutrino + neutron (**electron capture**)
- Proton + antineutrino $\leftrightarrow$ positron + neutron (**inverse beta decay**)

- Closed Universe $\rightarrow$ positively curved
- Open Universe $\rightarrow$ negatively curved
- Flat Universe $\rightarrow$ uncurved

\[
\Delta x \cdot \Delta p \geq \frac{\hbar}{2}
\]

The momentum and the position of a particle cannot be simultaneously measured with unlimited precision.
\[
\frac{dA}{dt} = \frac{L}{2m} = \text{constant}
\]

The areal velocity of a planet revolving around the sun in elliptical orbit remains constant which implies one-half its angular momentum divided by its mass remains constant. A consequence of this is that the Planet sweeps out equal areas in equal times.

Curvature of space-time + cosmological constant = density of matter

**Black hole temperature:**

\[
T = \frac{\hbar c^3}{8\pi Gk_B M}
\]

Thus, a smaller black hole is hotter, and consequently radiates more.

The time that the black hole takes to dissipate is:

\[
t_{ev} = \frac{5120\pi G^2 M^3}{\hbar c^4}
\]

Thus, a black hole of one solar mass \((M_{\odot} = 2.0 \times 10^{30} \text{ kg})\) takes more than \(10^{67}\) years to evaporate — much longer than the current age of the universe at \(14 \times 10^9\) years.
L = \sigma T^4 A

- \text{L = luminosity}
- \text{\sigma = Stefan-Boltzmann constant}
- \text{A = surface area}
- \text{T = temperature in Kelvin}

A consequence of this is that:

- The larger a star is, the more energy it puts out, and the more luminous it is.
- The star with a higher temperature will be more luminous than the star with lower temperatures

If A stands for the surface area of a black hole (area of the event horizon), then the black hole entropy is given by:

\[ S_B = \frac{k_B A}{4L_{\text{Planck}}^2} \]

- The larger a black hole is, the more entropy it possess.

\textbf{Wien's Law:}

The wavelength of peak emission is inversely proportional to the temperature of the emitting object.
\[ \lambda_{\text{max}} = \frac{b}{T} \]

b is a constant of proportionality called Wien's displacement constant, equal to \( 2.897771955... \times 10^{-3} \text{ mK} \)

Thus, hotter objects emit most of their radiation at shorter wavelengths; hence they will appear to be bluer.

**Stellar Radiation Pressure:**

\[ P_{\text{radiation}} = \frac{4\sigma T^4}{3c} \]

Thus, a doubling of temperature means an increase of radiation pressure by a factor of 16.

The nuclear radius \( R \) can be approximated by the following formula:

\[ R = r_0 \sqrt[3]{A} \]

\( A = \) Atomic mass number (the number of protons \( Z \), plus the number of neutrons \( N \)) and \( r_0 = 1.25 \text{ fm} = 1.25 \times 10^{-15} \text{ m} \).

Thus, size of nucleus depends on the mass number of nucleus.
If electrons were bosons, rather than fermions, then they would not obey the **Pauli Exclusion Principle**. There would be no life chemistry.

$$F = \frac{G m_1 m_2}{r^2}$$

$G$ represents the gravitational constant, which has a value of $6.674 \times 10^{-11}$ N (m/kg)$^2$. Because $G$ is small, gravitational force is very small unless large masses are involved.

**Eddington limit:**

$$L_{\text{Edd}} = \frac{4\pi GMm_p c}{\sigma_T}$$

$\sigma_T$ is the Thomson scattering cross-section for the electron and $m_p$ is the mass of a proton.

If a star exceeds this limit, its luminosity would be so high that it would blow off the outer layers of the star.
Viral Theorem for star:

Thermal energy + gravitational potential energy = \( \frac{1}{2} \) gravitational potential energy

Thermal energy = \( -\frac{1}{2} \) gravitational potential energy

\[ K = -\frac{1}{2} U \]

As a consequence of this is that:

The thermal energy increases if the gravitational potential energy becomes more negative.

- Wavelength of light \( \ll \) size of particle: Geometrical scattering
- Wavelength of light \( \sim \) size of particle: Mie scattering
- Wavelength of light \( \gg \) size of particle: Rayleigh scattering

\[ k_B T \ll KE_{\text{Fermi}} : \text{the electron gas is fully degenerate} \]
\[ k_B T \sim KE_{\text{Fermi}} : \text{the electron gas is partially degenerate} \]
\[ k_B T \gg KE_{\text{Fermi}} : \text{the electron gas is non-degenerate} \]
The spin of the neutron, proton and electron are all $\frac{1}{2}$. If beta decay involves just a neutron becoming a
proton and an electron, spin is not conserved.

Neutron $\rightarrow$ proton + electron

Half integral $\rightarrow$ integral

$$\frac{1}{2} \rightarrow \frac{1}{2} + \frac{1}{2}$$

Hence, the above reaction cannot take place if spin is to be conserved.

The electrostatic repulsion between two protons is $\frac{e^2}{4\pi\varepsilon_0 r^2}$ while the gravitational attraction between them is $\frac{Gm_p^2}{r^2}$. The ratio of these two forces is $\frac{e^2}{4\pi\varepsilon_0 Gm_p^2}$. This expression is independent of distance between them, so the relative strength of the forces is the same throughout all space.

If

$$\frac{mv^2}{2} > \frac{GMm}{r}$$

Object of mass m will escape the gravitational field of mass M.
In classical physics, it is possible to exactly specify both position and momentum simultaneously.

**Quantum mechanics:**
If we try to localize a particle spatially, we lose information about its momentum.

A light year is the distance traveled by light in a year:

\[
1 \text{ light year} = (\text{speed of light}) \times (1 \text{ year}) = 3 \times 10^{10} \text{ cm s}^{-1} \times 3 \times 10^7 \text{ s} = 9 \times 10^{17} \text{ cm}.
\]

- Water freezes at 273 K (≡ 0°C)
- Water boils at 373 K (≡ 100°C)

**Intrinsic energy of proton = KE of quarks + PE of quarks + intrinsic energy of quarks**

Any reaction that can take place in nature must follow this rule, no matter what force is responsible for the reaction:

- In any reaction the total charge of all the particles entering the reaction must be the same as the total charge of all the particles after the reaction.
Because \( E = mc^2 \) (the equation that represents the correlation of energy to matter: essentially, energy and matter are but two different forms of the same thing) and due to the fuzziness of quantum theory (that implies: photon carries mass proportional to its frequency i.e., \( h\nu = mc^2 \)), some of the most incredible mysteries of the quantum realm (a jitter in the amorphous haze of the subatomic world) get far less attention than Schrödinger’s famous cat. Virtual particle-antiparticle pairs of mass \( \Delta m = \frac{h\Delta \nu}{c^2} \) are continually created out of energy \( \Delta E \) of the empty space consistent with the Heisenberg’s uncertainty principle of quantum mechanics (which implies):

\[
\Delta mc^2 \times \Delta t \geq \frac{\hbar}{2}
\]

where: \( \Delta t \) stands for time during which virtual particle-antiparticle pairs of energy \( \Delta mc^2 = h\Delta \nu \) appear together, move apart, then come together and annihilate each other giving energy back to the space without violating the law of energy conservation (which states that energy can neither be created nor destroyed; rather, it can only be transformed from one form to another).

Spontaneous births and deaths of roiling frenzy of particles so called virtual matter – antimatter pairs momentarily occur everywhere, all the time – violate the Energy-momentum relationship: \( E = \sqrt{m_0^2c^4 + p^2c^2} \) – is the conclusion that mass and energy are interconvertible; they are two different forms of the same thing. The word virtual particles literally mean that these particles are not observed directly, but their indirect effects are measured to a remarkable degree of accuracy. Their properties and consequences are well established and well understood consequences of quantum mechanics.

---

**Hubble's law**

Consequence of the expansion of the space through which light is travelling.
For particles moving at speeds very close to that of light:

\[
E = \sqrt{m_0^2c^4 + p^2c^2}
\]

\[
pc \gg m_0c^2:
\]

\[
E = pc
\]

Hence the matter behaves similarly to the radiation.

- The angles in a triangle when added together sum up to 180°.
- The circumference of a circle divided by its diameter is a fixed number called \(\pi\).
- In a right angled triangle the lengths of the sides are related by \(c = \sqrt{a^2 + b^2}\) where \(c\) is the length of the side opposite to the right angle.

- 1 eV = 1.6 × 10⁻¹⁹ J
- 1 keV is a thousand eV
- 1 MeV is a million eV
- 1 GeV is a thousand million eV
- 1 TeV is a million million eV
Particles can only spin at a rate that is a multiple of \( \frac{h}{2\pi} \).

- Fermions (quarks and leptons) spin at \( \frac{1}{2} \times \frac{h}{2\pi} \).

- Bosons (photons and gluons) spin at \( 1 \times \frac{h}{2\pi} \) or \( 2 \times \frac{h}{2\pi} \).

**Euler's formula:**

\[
e^{\pi i} + 1 = 0
\]

Connects the five fundamental constants of mathematics (\( e, \pi, i, 0, 1 \)).

[Imaginary number \( i = \sqrt{-1} \)].

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Noether's Theorem (1918):

"For every continuous symmetry there is a corresponding conserved quantity [such as electric charge] and vice versa."

"Even if there is only one possible unified theory, it is just a set of rules and equations…

What is it that breathes fire into the equations and makes a universe for them to describe?"

"Why does the universe go to the bother of existing?"

− Stephen Hawking

\[(i \gamma_\mu \frac{d}{d^\mu} - m) \psi = 0\]

The Dirac Equation that predicts the existence of antimatter

where:

- \(i\) = imaginary number
- \(\gamma_\mu\) = Pauli matrices
- \(d^{\mu}\) = derivative in 4 dimensions
- \(m\) = fermion mass
- \(\psi\) = wave function
Bayes' Theorem:

\[
P(H \mid E) = \frac{P(E \mid H) \cdot P(H)}{P(E)}
\]

H represents a hypothesis and E the evidence.

- \( P(H \mid E) \) – the probability of H given E is true
- \( P(E \mid H) \) – the probability of E given H is true
- \( P(E) \) – the probability of E
- \( P(H) \) – the probability of H

The number 0 is the neutral element of addition:

\[
\begin{align*}
1 + 0 &= 1 \\
23 + 0 &= 23
\end{align*}
\]

\( F = G \times \frac{m_1 m_2}{r^2} \)

Constant that controls the strength of gravity

Consisting of one oxygen atom and two hydrogen atoms, water molecule plays a special role in the chemistry of life.
The $\Delta^{++}$ is more massive than the sum of the masses of the proton and the pion ($\pi^+$). This means that it is energetically possible for the $\Delta^{++}$ to decay into a proton and a pion.

The Planck mass $\sqrt{\frac{\hbar c}{G}}$ is roughly $24,000,000,000,000,000,000,000 (2.4 \times 10^{22})$ times the mass of the electron.

Observations
  ↓
Hypothesis
  ↓
Experiment
  ↓
Laws
  ↓
Theory
- Planck's law is accurate at all wavelengths.
- Wien's Law is a good approximation at short wavelengths.
- The Rayleigh-Jeans Law is a good approximation at large wavelength.

Five Equations That Changed the World:

- \( F = \frac{GMm}{d^2} \) (Newton's Law of Universal Gravitation)
- \( P + \rho \times \frac{1}{2} v^2 = \text{constant} \) (Bernoulli's Law of Hydrodynamic Pressure)
- \( \nabla \times E = -\frac{\partial B}{\partial t} \) (Faraday's Law of Induction)
- \( E = mc^2 \) (Albert Einstein's mass–energy equivalence)
- \( S_{\text{universe}} > 0 \) (Clausius's Law of Thermodynamics)

**Fine structure constant**

Constant characterizing the strength of interaction between charged particles.
Titius–Bode law:

The distance from the Sun to the \( n \)\(^{th} \) planet is \( 0.075 \times 2^n + 0.4 \) astronomical units.

\[
c = \frac{1}{\sqrt{\text{Vacuum permeability} \times \text{Vacuum permittivity}}}
\]

\[
c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}
\]

Determined by the electromagnetic properties of free space – \( \mu_0 \) and \( \varepsilon_0 \)

Quantum mechanics + general theory of relativity \( \rightarrow \) quantum theory of gravity
If the density perturbations were much weaker, then galaxies may never have coalesced. Without galaxies there would be no buildup of heavy elements, and it is unlikely that planets, and life, would have emerged.

In the presence of gravity, time slows down — the stronger the effect of gravity the more that time slows down.

\[
\text{Einstein curvature tensor} = \frac{8\pi G}{c^4} \times \text{Energy-momentum stress tensor}
\]

- Entropy of Universe = entropy of visible Universe + entropy of dark matter + entropy of black holes
- The total energy of the star = internal energy due to thermal motion and radiation + gravitational potential energy

- Stars with mass > 0.08 \(M_{\text{sun}}\) burn hydrogen.
- Stars with mass > 0.5 \(M_{\text{sun}}\) burn hydrogen and helium.
- Stars with mass in the range of 1–8 \(M_{\text{sun}}\) continue nucleosynthesis up till the production of carbon.
- Stars with mass > 10 \(M_{\text{sun}}\) synthesize all the elements up to iron and nickel.
Rate of energy production in the **pp-process** of hydrogen burning $\propto$ (Temperature)$^4$

Rate of energy production in the **CNO-process** of hydrogen burning $\propto$ (Temperature)$^{18}$

---

Supersymmetry

Quantum Gravity

Grand Unified Theories

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- Size of our universe $\sim 10^{26}$ m
- The distance Earth–Sun is $\sim 1.5 \times 10^{11}$ m
- The radius of the Sun is $\sim 7 \times 10^8$ m
- The radius of the Earth is $\sim 6.4 \times 10^6$ m
- Rocks, Humans, . . . $\sim 1$ m
- Grains of sand $\sim 10^{-3}$ m
- Viruses $\sim 10^{-7}$ m
- Simple molecules $\sim 10^{-9}$ m
- Atoms $\sim 10^{-10}$ m

---

Redshift = \[
\frac{\lambda_{\text{observed}}}{\lambda_{\text{emitted}}} - 1
\]

always positive, i.e. observed radiation is redder than the emitted one.

Implies:

Universe is in expansion
Temperature of relativistic particles $\propto \frac{1}{\text{scale factor of the universe}}$

Temperature of non-relativistic particles $\propto \frac{1}{(\text{scale factor of the universe})^2}$

<table>
<thead>
<tr>
<th>Electromagnetic wave</th>
<th>Wavelength</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radio</td>
<td>$\sim 10\text{cm or larger}$</td>
</tr>
<tr>
<td>Microwave</td>
<td>$\sim 1\text{cm}$</td>
</tr>
<tr>
<td>Infrared</td>
<td>$\sim 10^{-3}\text{ cm}$</td>
</tr>
<tr>
<td>Visible</td>
<td>$\sim 10^{-5}\text{ cm}$</td>
</tr>
<tr>
<td>Ultraviolet</td>
<td>$\sim 10^{-6}\text{ cm}$</td>
</tr>
<tr>
<td>X-rays</td>
<td>$\sim 10^{-8}\text{ cm}$</td>
</tr>
<tr>
<td>Gamma-rays</td>
<td>$\sim 10^{-9}\text{ cm or smaller}$</td>
</tr>
</tbody>
</table>

**Principle of equivalence:**

$m$ in $(m \times a)$ and the $m$ in \(\frac{GMm}{r^2}\) are identical.

Inertial mass and gravitational mass are identical.
Gravity will affect anything carrying energy

Root of the construction of Einstein's equations which describe gravity

Light intensity drops as \( \frac{1}{(\text{distance})^2} \)

- In an open universe (negative curvature): the angles in a triangle add up to less than 180°.
- In a closed universe (positive curvature): the angles in a triangle add up to more than 180°.
- In a flat universe (zero curvature): the angles in a triangle add up to 180°.

The energy of the universe is constant.
The entropy of the universe tends to a maximum.

Theory

External Reality Theory (ERT)
External reality exists completely independent of human beings

Mathematical Universe Theory (MUT)
External physical reality is a mathematical structure
When associated with c and with the reduced Planck's constant $\hbar$, it leads to the definition of the Planck's time:

$$t_{\text{Planck}} = \sqrt{\frac{\hbar G}{c^5}} = 5.4 \times 10^{-44} \text{ seconds}$$

The shortest possible time interval that can be measured.

**Faraday constant:**

Average charge carried by the Avogadro number of electrons

$$F = e \ N_A$$

Average number of carbon atoms needed to form 12 gram of a carbon material.

The nuclear charge Q can be approximated by the following formula:

$$Q = Ze$$

$Z = \text{Atomic number (the number of protons)}.$

Thus, charge of nucleus depends on the number of protons.
\[ Q = \frac{\text{number of protons}}{N_A} \times F \]

\[ Q = \text{number of moles of proton} \times F \]

**The strong coupling constant** defines the strength of the force that holds protons and neutrons together.

The Universe is made up of three things:

- Vacuum
- Matter
- Photons

**Total energy density of the universe:**

\[ \rho = \rho_{\text{vacuum}} + \rho_{\text{matter}} + \rho_{\text{radiation}} \]

\[ \rho_{\text{vacuum}} = \frac{\Lambda c^2}{8\pi G} \]

is constant and independent of time. The cosmological constant "\( \Lambda \)" has negative pressure equal and opposite to its energy density and so causes the expansion of the universe to accelerate.
Matter:

\[ E = mc^2 \]

For N particles:

\[ E = Nmc^2 \]

\[ \rho_{\text{matter}} \propto \frac{Nmc^2}{R^3} \]

Radiation:

\[ E = \frac{hc}{\lambda} \text{ and } \lambda \propto R \]

For N photons:

\[ E \propto \frac{Nhc}{R} \]

\[ \rho_{\text{radiation}} \propto \frac{Nh\text{c}}{R^4} \]

- \( \rho_{\text{vacuum}} \propto R^0 \)
- \( \rho_{\text{matter}} \propto R^{-3} \)
- \( \rho_{\text{radiation}} \propto R^{-4} \)
Changing their values changes the physical phenomena

When the Universe was at the Planck temperature
\[ \sqrt{\frac{\hbar c^5}{G k_B^2}} \]
and the mean energy of photons was close to the Planck energy \[ \sqrt{\frac{\hbar c^5}{G}}. \]

If the thermal kinetic energy of stellar gas cloud wins over gravity:

\[ \frac{3}{2} k_B T > \frac{3GM^2}{5R} \rightarrow \text{Expansion} \]

If

\[ \frac{3GM^2}{5R} > \frac{3}{2} k_B T \rightarrow \text{gravity wins: collapse!} \]

Solar mass = \( 2 \times 10^{30} \text{ kg} \) – of which about 70% is hydrogen, 28% helium, and 2% consists of other elements. Only about a seventh part of that hydrogen mass is available at any time for hydrogen fusion in the core of the Sun.
Because relativistic mass = \( \frac{\text{rest mass}}{\sqrt{1 - \frac{v^2}{c^2}}} \): the faster an particle moves, the more kinetic energy it possess. But according to \( E = mc^2 \), kinetic energy adds to an particle's mass, so the faster an particle moves, the harder it is to further increase the particle's speed. This effect is really significant only for particles moving at speeds close to the speed of light. As a particle approaches the speed of light, its mass increases ever more quickly to infinite, so it take infinite amount of energy to speed it up further. This is the reason that any material particle is forever confined by relativity to move at speeds slower than the speed of light. Only photons that have no intrinsic mass move at the speed of light.

\[
E^2 = p^2c^2 + m_0^2c^4
\]

\( m_0 = 0 \):

\[
E = pc
\]

The **Compton wavelength of a particle** characterizes the length scale at which the wave property of a given particle starts to show up. In an interaction that is characterized by a length scale larger than the Compton wavelength, particle behaves classically (i.e., no observation of wave nature). For interactions that occur at a length scale comparable than the Compton wavelength, the wave nature of the particle begins to take over from classical physics.

At Planck length (\( \sqrt{\frac{\hbar G}{c^3}} \)), the gravitational force is as strong as the other forces and space-time is "foamy" – filled with tiny bubbles and wormholes appearing and disappearing into the vacuum.
**Rayleigh scattering law:** The amount of scattering of light is inversely proportional to the fourth power of the wavelength.

\[ I \propto \frac{1}{\lambda^4} \]

Thus, Rayleigh scattering is more intense at shorter wavelengths.

**Supersymmetry**

The positive zero point energy of the boson field exactly cancels the negative zero point energy of the fermion field.

\[ h \rightarrow 6.62607004 \times 10^{-34} \text{ m}^2 \text{ kg/s} \]

Because \( h \) is too small: **Quantum mechanics is for little things.**

**Einstein's Relativity:**

Motion and Gravity make Clocks tick slowly.
Gravity pulls everything in, but a mysterious force called dark energy tries to push it all back together again. Our fate relies on which force will win the desire to succeed.

Because of

**CP violation** (violation of charge conjugation parity symmetry)

there was more matter than antimatter right after the Big Bang.

\[
m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}
\]

Tachyons (if they exist) have \( v > c \). This means that \( m \) is imaginary!

**General theory of relativity** describes gravity, ignoring quantum mechanics.

Math in Nature:
Hexagon | Bee Hive
---|---
Concentric Circles | Ripples of a pond when a stone hits the surface of the water.

**Doppler Effect:**
- When a wave source moves toward an observer, its waves appear to have a shorter wavelength.
- When a wave source moves away from an observer, its waves appear to have a longer wavelength.

In more than three space dimensions, planetary orbits would be unstable and planets would either fall into the sun or escape its attraction altogether.

What goes up must get down

Newton's law of gravity

What goes up need not descend— if it is shot upward faster than the escape velocity \( \sqrt{\frac{2GM}{R}} \).

Since gravity weakens with distance, the earth pulls on your head with less force than it pulls on your feet, which are a meter or two closer to the earth's center. The difference is so tiny we cannot feel it, but an astronaut near the surface of a black hole would be literally torn apart.
Because:

\[ 2\pi r = n\lambda \]

Only orbits with circumferences corresponding to a whole number of electron wavelengths could survive without destructive interference.

Because:

\[ r = \frac{3GM}{c^2} \]

The photon spheres can only exist in the space surrounding an extremely compact object (a black hole or possibly an "ultracompact" neutron star).

- **In phase** → wave crests and troughs reinforce each other.
- **Out of phase** → wave crests and troughs cancel out.

- The energy above which (Grand unification energy \( \approx 10^{16}\text{GeV} \)), the electro-magnetic force, weak force, and strong force become indistinguishable from each other.
- Since the graviton has no mass of its own, the **gravitational force of attraction** between the sun and every planet is long range.
- The proton and neutron masses are so similar; they differ only by the replacement of an up quark with a down quark.
Because:

\[
\frac{E}{B} = c
\]

Electric and magnetic fields turn into each other in a wavelike motion, creating an electromagnetic field that travels at the speed of light.

Inside the nucleus of an atom, a proton is never permanently a proton and a neutron is never permanently a neutron. They keep on changing into each other. A neutron emits a pi meson and become proton and a proton absorbs a pi meson and become a neutron.

\[
\begin{align*}
\text{Neutron} &\rightarrow \text{proton} + \pi^- \\
\text{Proton} + \pi^- &\rightarrow \text{neutron}
\end{align*}
\]

When two black holes collide, they merge, and the area of the final black hole is greater than the sum of the areas of the original holes.

There is no escape from a black hole in classical theory, but quantum theory enables energy and information to escape.
The **inherent goal of unification** is to show that all of these forces are, in fact, manifestations of a single force. We can't perceive this unity at the low energies of our everyday lives, or even in our most powerful accelerators at CERN. But close to the Big Bang temperatures, at inconceivably high energies… If the forces unify, the proton can be unstable, and eventually decay …

\[
\text{Proton} \rightarrow \text{positron} + \text{neutral pion}
\]

\[
\text{Neutral pion} \rightarrow 2 \text{gamma ray photons}
\]

- Accelerated massive bodies give off gravitational waves just as bound electrons in an atom emit electromagnetic radiation
- A rotating neutron star (a tiny, burnt out star) generates regular pulses of radio waves.
- Quantum mechanics says that the position of a particle is uncertain, and therefore that there is some possibility that a particle will be within an energy barrier rather than outside of it. The process of moving from outside to inside without traversing the distance between is known as **quantum tunneling**, and it is very important for the fusion reactions in stars like the Sun.

According to string theory, our universe is made up of small vibrating strings whose size is $10^{23}$ times smaller than protons. A **String** can do something aside from moving – it can vibrate in different ways. One mode of vibration makes the string appear as a photon, another as a neutron, and so on...
Because

\[ dM = \frac{k}{\delta \pi} \, dA + \Omega dJ + \Phi dQ \]

M stands for mass, k for surface gravity, A for area of the event Horizon, J for angular momentum, \( \Omega \) for angular velocity, Q for charge and \( \Phi \) for the electrostatic potential.

The size and shape of the black hole depends only on its mass, charge and rate of rotation, and not on the nature of the star that had collapsed to form it.

The electromagnetic and weak interactions lose their symmetry below 100 GeV.

Entropy (a thermodynamic measure of untidiness in a system and a measure of how much information a system contains) is defined as:

\[ S = k_B \ln \{\text{number of states}\} \]

which, for N particles of the same type, is

\[ S = k_B \ln \{\text{(no of one-particle states)} \, N\} \]
\[ S = k_B \ln \{\text{a not-too-big number}\} \]

\[ S = k_B N \]

This means: the more particles, the more disorder.

From the properties of subatomic particles and the realm of quantum physics to the formation of a giant mathematical object (universe), math proves unquestionably effective in describing and predicting their physical reality. However, a question that lies at the intersection of philosophy and science arises: Is Math the Language of the Universe?

**References:**

- Light – The Physics of the Photon, by Ole Keller.
- University Physics with Modern Physics by Hugh D. Young.
- Isaac Newton and the Laws of Motion by Andrea Gianopoulos.
- Our Mathematical Universe: My Quest for the Ultimate Nature of Reality by Max Tegmark.

**Source of information:**

- [https://www.wikipedia.org/](https://www.wikipedia.org/)