Abstract: In this note we will consider an idea of Fermat for the stop in connection with the division by zero calculus. Here, in particular, we will see some mysterious logic on the stop in connection with the concepts of differential and differential coefficient.

David Hilbert:

_The art of doing mathematics consists in finding that special case which contains all the germs of generality._

Oliver Heaviside:

_Mathematics is an experimental science, and definitions do not come first, but later on._

Key Words: Division by zero, division by zero calculus, isolated singular point, analytic function, Laurent expansion, $1/0 = 0/0 = z/0 = \tan(\pi/2) = \log 0 = 0$, $[(z^n)/n]_{n=0} = \log z$, $[e^{(1/z)}]_{z=0} = 1$.

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1 Introduction

For a fixed $\ell > 0$, we will consider the area $S(a)$ by

$$S(a) = a(\ell - a)$$
and the maximum of $S(a)$ for $0 \leq a \leq \ell$. Of course, this problem is very simple with elementary calculus.

However, Fermat (1629) considered this problem in the following way:

Assume that

$$(a + \epsilon)(\ell - a - \epsilon) = a(\ell - a). \quad (1.1)$$

Then, formally we have the identity

$$\epsilon^2 + \epsilon(2a - \ell) = 0. \quad (1.2)$$

See [2], 358-359.

From this logic and identity, could we obtain the desired result

$$a = \frac{\ell}{2} \quad (1.3)$$

? 

Note that

$$S'(a) = \ell - 2a.$$

Firstly, note that the identity (1.1) is not valid except $\epsilon = 0$ and $\epsilon = \ell - 2a$, as we from the representation of $S(a)$; that is the identity is nonsense when we consider a small variation. Of course, (1.2) is valid for $\epsilon = 0$ and $\epsilon = \ell - 2a$. So, we wonder the above logic is nonsense.

The logic is not on any variation of the area $S(a)$ essentially that may be related to differential and differential coefficient.

For this question, we will be able to give our interpretation with the concept of the division by zero calculus.

### 2 Division by zero calculus

We will simply introduce the division by zero calculus.

For any Laurent expansion around $z = a$,

$$f(z) = \sum_{n=-\infty}^{-1} C_n(z - a)^n + C_0 + \sum_{n=1}^{\infty} C_n(z - a)^n, \quad (2.1)$$

we will define

$$f(a) = C_0. \quad (2.2)$$
We define the value of the function \( f(z) \) at the singular point \( z = a \) by ignoring the singular parts of the Laurent expansion.

For the correspondence (2.2) for the function \( f(z) \), we will call it the **division by zero calculus**. By considering derivatives in (2.1), we can define any order derivatives of the function \( f \) at the singular point \( a \); that is,

\[
 f^{(n)}(a) = n!C_n.
\]

With this assumption (definition), we can obtain many new results and new concepts. See [20] and the references in this paper.

### 3 Interpretation by the division by zero calculus

In the formal formula (1.2), from the identity

\[
\frac{\epsilon^2}{\epsilon} + (2a - \ell) = 0,
\]

we obtain the desired result (1.3) at \( \epsilon = 0 \)

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**References**


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