Schrödinger Equation of Hydrogen Atom in Atomic Unites,

Theory of Chirality and the Territory of Modern Physics

(viXra:2103.0088v2)

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Dedicated to Prof. Albert Sun-Chi Chan on the occasion of his 70th birthday

Abstract

This paper is a subsequent paper to our previous papers “Chen’s Formulas of the Fine-structure Constant” (viXra:2002.0203) and “Integrated Formulas of the Fine-structure Constant and Feigenbaum Constants” (viXra:2102.0162). In light of the basic concepts illustrated in the above stated previous papers that there are two values of the fine-structure constant ($\alpha_1$ and $\alpha_2$) and there are relationships such as $\alpha_2(\delta\alpha/\gamma_2)^2=1$ between the fine-structure constant $\alpha_2$ and Feigenbaum constants ($\delta$ and $\alpha$), in this paper we deduced Schrödinger equation of hydrogen atom in atomic unites in a concise, reasonable and beautiful form which otherwise couldn’t be gained. In the end of this paper, all theories opened in our previous papers are named to be Theory of Chirality, and a picture of the territory of modern physics including Theory of Relativity, Quantum Theory, Chaos Theory and Theory of Chirality is given.

Keywords: Schrödinger equation; hydrogen atom; the fine-structure constant; Feigenbaum constants; Theory of Chirality, territory of modern physics.

1. Schrödinger Equation of Hydrogen Atom in Atomic Unites

In our previous paper$^1$, we gave the following deduction process.

Stationary Schrodinger Equation: $-\frac{\hbar^2}{2m} \nabla^2 \psi + U \psi = E \psi,$

applied to hydron atom: $\nabla^2 \psi + \frac{2m}{\hbar^2} (E + \frac{e^2}{4\pi\epsilon_0 r}) \psi = 0$, $E = -\frac{m \epsilon^2}{2n^2 (4\pi \epsilon_0)^2 \hbar^2}$
With substitution and simplification:

\[
\frac{2m_e}{\hbar^2} \left( \frac{m_e e^2}{2n^2(4\pi\varepsilon_0)^2\hbar^2} - \frac{e^2}{4\pi\varepsilon_0 r} \right)\psi = \nabla^2 \psi,
\]

\[
\left[ 1 \left( \frac{m_e e^2}{4\pi\varepsilon_0 \hbar^2} \right)^2 - \frac{2m_e e^2}{r 4\pi\varepsilon_0 \hbar} \right] \psi = \nabla^2 \psi,
\]

\[
\left[ 1 \left( \frac{e^2}{4\pi\varepsilon_0 \hbar c^2} \right)^2 - \frac{2 e^2}{r 4\pi\varepsilon_0 \hbar c} \frac{m_e c}{\hbar} \right] \psi = \nabla^2 \psi,
\]

As \( \alpha_1 \alpha_2 = \frac{e}{c} = \frac{e^2}{4\pi\varepsilon_0 \hbar}, \lambda_e = \frac{\hbar}{m_e c} \) and \( \alpha_i = \frac{\lambda_e}{2\pi a_0} ; \)

\[
\left[ 1 \left( \frac{\alpha_1 \alpha_2}{\lambda_e} \right)^2 - \frac{2}{r \sqrt{\alpha_1 \alpha_2}} \right] \psi = \nabla^2 \psi,
\]

\[
\left[ \frac{1}{n^2} \left( \alpha / \lambda_e \right) - \frac{2}{a_0 r \sqrt{\alpha_1 / \alpha_2}} \right] \psi = \nabla^2 \psi,
\]

As \( \alpha_i / \alpha_2 \approx 1 \), simplyfied to:

\[
\left[ \frac{1}{n^2} \alpha_2^2 - \frac{2}{\alpha_2} \right] \psi = \nabla^2 \psi.
\]

factor 2 seems not beautiful

In atomic units (au: \( e = m_e = \hbar = 1 \) and \( \varepsilon_0 = \frac{1}{4\pi} \)),

\[
a_{\text{au}} = \frac{4\pi\varepsilon_0 \hbar^2}{m_e e^2} = 1, \quad \nu_{\text{au}} = \frac{e^2}{4\pi\varepsilon_0 \hbar} = 1, \quad c_{\text{au}} = \frac{v_{\text{au}} c}{\alpha} = \frac{1}{\alpha} \sqrt{\alpha_1 \alpha_2}
\]

\[
\left[ \frac{1}{n^2} \left( \alpha / \lambda_e \right) - \frac{2}{r_{\text{au}} \sqrt{\alpha_1 / \alpha_2}} \right] \psi = \nabla_{\text{au}}^2 \psi.
\]

the above equation could be called Schröedinger equation of hydrogen atom in atomic unites, the later form of the equation shows factor 2 is still reasonable and beautiful.

As \( \alpha_i / \alpha_2 \approx 1 \), simplyfied to:

\[
\left[ \frac{1}{n^2} - \frac{2}{r_{\text{au}}} \right] \psi = \nabla_{\text{au}}^2 \psi.
\]

Discover: 2018/4-6; Revise: 2019/12/13 (add au form)

\[
\alpha / \alpha_2 = \frac{137.035999111818}{137.035999037435} = 1.0000000005428 = 1 + \frac{23 \cdot 59}{25 \cdot 10^{11}} = 1 + \left( 1 + \frac{23 \cdot 59}{50 \cdot 10^{11}} \right)^2
\]

\[
\sqrt{\alpha_i / \alpha_2} = 1 + \frac{23 \cdot 59}{50 \cdot 10^{11}} = 1.0000000002714
\]

Relations to nuclides: \( \begin{array}{cccccccccccc}
\text{Na}^{23}, & \text{Mn}^{55}, & \text{Ru}^{59}, & \text{Pd}^{105}, & \text{Sn}^{119}, & \text{Ba}^{137}, & \text{Pr}^{141}, & \text{Tm}^{169}, & \text{Re}^{110}, & \text{Ra}^{226}, & \text{Pu}^{239}, & \text{A}_{138}
\end{array} \)

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However, in the above deduction, the later form of Schödinger equation of
hydrogen atom in atomic units is not fully correct, and needs correction as follows.

\[
\left( \frac{c_{au}^2}{\alpha_i^2 n^2} - \frac{2c_{au}}{\alpha_i r_{au}} \right) \psi = \nabla \psi
\]

above equation is not fully correct, and is revised to be:

\[
\left( \frac{\alpha_i^2 c_{au}^2}{n^2} - \frac{2\alpha_i c_{au}}{r_{au}} \right) \psi = \nabla \psi
\]

2021/2/27

2. Integrated Formulas of the Fine-structure Constant and Feigenbaum Constants

In our previous papers\textsuperscript{5,6}, some integrated formulas of the fine-structure constant and Feigenbaum constants were given, the following are their summary.

\[
2\pi = 2 \cdot 3.14159265358979
\]

\[
(2\pi)_{\text{Chen-k}} = e^2 \left( \frac{2}{1} \right)^3 \left( \frac{3}{2} \right)^5 \left( \frac{4}{3} \right)^7 \cdots \frac{e^2}{(k + 1)^{2k + 1}}
\]

The Fine-structure Constant:

\[
\alpha_1 = \frac{\lambda_e}{2\pi a_0} = \frac{36}{7 \cdot (2\pi)_{\text{Chen-112}}} \cdot \frac{1}{112 + \frac{1}{75^2}} = 1/137.035999037435
\]

\[
\alpha_2 = \frac{2\pi r_e}{\lambda_e} = \frac{13 \cdot (2\pi)_{\text{Chen-278}}}{100} \cdot \frac{1}{112 - \frac{1}{64 \cdot 3 \cdot 29}} = 1/137.035999111818
\]

Feigenbaum Constants: \( \delta = 4.66920160910299 \), \( \alpha = 2.50290787509589 \)

\[
\alpha_i (\delta / \gamma_i)^2 (2\pi) = 1
\]

\[
\gamma_i = 1 - \frac{1}{47 \cdot 109} + \frac{1}{27 \cdot 7 \cdot (3 \cdot 8 \cdot (3 \cdot 8 \cdot (4 \cdot 137 - 1) - 1) - 1)} = 0.999804818668238
\]

\[
\gamma_i = 1 + \frac{1}{2 \cdot 13 \cdot 197} - \frac{1}{16 \cdot 7 \cdot 17 \cdot (16 \cdot 3 \cdot 23 - 1)} = 1.00019521943495
\]

\[
\gamma_i = 1 + \frac{1}{13 \cdot 197} + \frac{1}{2 \cdot 7 \cdot 41 \cdot (4 \cdot 5 \cdot 19 - 31 - 1)} = 0.99960967543223
\]

\[
\gamma_i = 1 + \frac{1}{512 \cdot 5} - \frac{1}{4 \cdot 9 \cdot 7 \cdot 17 \cdot 19 \cdot 83} = 1.00039047698053
\]

\[
\alpha_i (\delta / \gamma_{i-\text{Chen-2517}})^2 (2\pi)_{\text{Chen-2517}} = 1
\]

\[
(2\pi)_{\text{Chen-2517}} = e^2 \left( \frac{2}{1} \right)^3 \left( \frac{3}{2} \right)^5 \left( \frac{4}{3} \right)^7 \cdots \frac{e^2}{(25 \cdot 17)^{23 \cdot 37}} = 6.28564399787948
\]
\[ \gamma_{\text{Chen}-2517} = 1 + \frac{1}{2 \cdot 79 \cdot 109 \cdot 139 - \frac{3}{25}} = 1.00000041773574 \]

\[ \frac{1}{\gamma_{\text{Chen}-2517}} = 1 - \frac{1}{2 \cdot 79 \cdot 109 \cdot 139 + \frac{22}{25}} = 0.999999582264432 \]

\[ \gamma_{\text{Chen}-2517}^2 = 1 + \frac{1}{128 \cdot 9 \cdot (2 \cdot 3 \cdot 173 + 1) + \frac{7}{10}} = 1.0000008354717 \]

\[ \frac{1}{\gamma_{\text{Chen}-2517}^2} = 1 - \frac{1}{2 \cdot 5 \cdot 7 \cdot (2 \cdot 83 \cdot 103 + 1) - \frac{9}{29}} = 0.999999164529037 \]

\[ \alpha_2(\delta \alpha / \gamma_2)^2 = 1 \]

\[ \gamma_2 = 1 - \frac{1}{5 \cdot 7 \cdot 17} + \frac{1}{4 \cdot 3 \cdot 17 \cdot 23 \cdot 137 - \frac{11}{59}} = 0.998320883415699 \]

\[ \frac{1}{\gamma_2} = 1 + \frac{1}{2 \cdot 27 \cdot 11} - \frac{1}{5 \cdot 17 \cdot (16 \cdot 3 \cdot 157 + 1) + \frac{16}{17}} = 1.00168194075892 \]

\[ \gamma_2^2 = 1 - \frac{1}{2 \cdot 149} + \frac{1}{29 \cdot 31 \cdot (2 \cdot 3 \cdot 49 \cdot 13 + 1) - \frac{16}{19}} = 0.996644586263908 \]

\[ \frac{1}{\gamma_2^2} = 1 + \frac{1}{27 \cdot 11} - \frac{1}{2 \cdot 3 \cdot 25 \cdot 11 \cdot (4 \cdot 11 \cdot 47 + 1) - \frac{2}{5}} = 1.00336671044256 \]

\[ \frac{2\pi}{(\alpha \gamma)^2} = 1 \]

\[ \gamma = 1 + \frac{1}{32 \cdot 3 \cdot 7} - \frac{1}{23 \cdot 151 \cdot 173 + \frac{9}{4 \cdot 7}} = 1.00148643087192 \]

\[ \frac{1}{\gamma} = 1 - \frac{1}{32 \cdot 3 \cdot 7 + 1} + \frac{1}{2 \cdot 41 \cdot (2 \cdot 3 \cdot 25 \cdot 49 - 1) - \frac{1}{6}} = 0.99851577532546 \]

\[ \gamma^2 = 1 + \frac{1}{16 \cdot 3 \cdot 7} - \frac{1}{3 \cdot 13 \cdot 31 \cdot (2 \cdot 9 \cdot 41 + 1) - \frac{1}{22}} = 1.0029750712205 \]

\[ \frac{1}{\gamma^2} = 1 - \frac{1}{16 \cdot 3 \cdot 7 + 1} + \frac{1}{8 \cdot 81 \cdot 19 \cdot 73 + \frac{1}{3 \cdot 13}} = 0.997033753573803 \]
3. The Final Form of Schrödinger Equation of Hydrogen Atom in Atomic Unites

As we have:
\[
\left( \frac{\alpha_1^2 \alpha_2^2}{n^2} - \frac{2\alpha_1 \alpha_2 \alpha_{au}}{r_{au}} \right) \psi = \nabla_{au}^2 \psi
\]
\[
\alpha_1 (\alpha / \gamma_1)^3 (2\pi) = 1, \quad \alpha_2 (\alpha / \gamma_2)^3 = 1
\]
So:
\[
\left( \frac{\epsilon_{au}^2}{(\alpha / \gamma_1)^3 n^2} - \frac{2\epsilon_{au}}{(\alpha / \gamma_2)^3} r_{au} \right) \psi = \nabla_{au}^2 \psi
\]
2021/2/27

Define:
\[
c_{au} = \frac{c}{v_e} = \frac{1}{\sqrt{\alpha_1 \alpha_2}} = \frac{1}{\sqrt{\frac{\lambda_e}{2\pi r_e}}} \sqrt{\frac{\alpha_0}{\lambda_e}} = \sqrt{\frac{2\pi (\alpha / \gamma_1)^3 (\alpha / \gamma_2)^2}{\gamma_1 \gamma_2}} = \frac{\delta^2 \alpha}{\sqrt{2\pi}}
\]
\[
c_{au-Ch} = \sqrt{\frac{\alpha_2}{\alpha_1}} = \alpha_2 c_{au} = \frac{c_{au}}{(\alpha / \gamma_2)^2} = \sqrt{\frac{2\pi}{\alpha \gamma_1 / \gamma_2}} = 2\pi \sqrt{\frac{r_e \alpha_0}{\lambda_e}}
\]
\[
= 1 - \frac{23.59}{50 \times 10^{11}} = 0.9999999994572
\]
Relationships with nuclides: \( ^{50,51}_{21}Y \), \(^{105}_{46}Pd \), \(^{119}_{50}Sn \), \(^{169}_{69}Tm \)

c_{au-Ch} could be called chaotic speed of light in atomic unites or Chen speed

So:
\[
\left( \frac{c_{au-Ch}^2}{n^2} - \frac{2c_{au-Ch}}{r_{au}} \right) \psi = \nabla_{au}^2 \psi
\]
the above last formula could be called:
the final form of Schrödinger equation of hydrogen atom in atomic unites
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Note that if \( \alpha_1 = \alpha_2 \), the above equation will be simplified to:
\[
\left( \frac{1}{n^2} - \frac{2}{r_{au}} \right) \psi = \nabla_{au}^2 \psi
\]

But this simplest equation is not reasonable and beautiful because the factor 2 is very strange and in lack of beauty and reasonability. On the other hand, the consistence of \( c_{au-Ch}^2 \) and \( 2c_{au-Ch} \) is very good, reasonable and beautiful. So our basic concept that there are two values of the fine-structure constant, i.e., \( \alpha_1 \) and \( \alpha_2 \), is critical for Schrödinger equation of hydrogen atom to be a reasonable and beautiful equation. This also demonstrates all theories in our previous papers should be correct.
4. Theory of Chirality

All theories opened in our previous papers \(^1\)–\(^6\), registered as copyrights \(^7\)–\(^9\) and other unopened as manuscripts (such as “New Theory of Chemical Bonds” written by us in 2009) are hereby named to be Theory of Chirality. The main principles and conclusions of Theory of Chirality are listed as follows.

**Chirality** = \(\pm 2\pi\)

\[
(2\pi)_{\text{Chirality}} = e^2 - \frac{e^2}{(\frac{2}{3})^2} + \frac{e^2}{(\frac{4}{3})^3} - \frac{e^2}{(\frac{6}{3})^4} + \cdots = 1/137.035999074626
\]

\[
\alpha = \frac{\lambda}{2\pi a_0} = \frac{28}{7\cdot(2\pi)_{\text{Chirality}}} = 1/137.035999037435
\]

\[
1/\alpha = 56 + 81 + \frac{1}{28} - \frac{13\cdot(2.56 \cdot 11 - 1)}{3.5\cdot(2.56 \cdot 43 + 1)} = 137.035999037435
\]

\[
\alpha_2 = \frac{2\pi\alpha}{\alpha} = \frac{13\cdot(2\pi)_{\text{Chirality}}} {100} = 1/137.035999111818
\]

\[
1/\alpha_2 = 56 + 81 + \frac{1}{28} - \frac{2\cdot(16 \cdot 27 - 1)}{3\cdot(16 \cdot 81 + 1)} = 137.035999111818
\]

\[
c_{\text{aw}} = \frac{c}{v} = \frac{1}{\alpha} = \frac{1}{\sqrt{\alpha_1\alpha_2}} = \sqrt{112\times(168 - \frac{1}{3} + \frac{1}{12\cdot47} - \frac{1}{14\cdot112\cdot(2\cdot173 + 1)})}
\]

\[
= 137.035999074626
\]

\[
c_{\text{aw}} = 56 + 81 + \frac{1}{28} - \frac{5\cdot(4\cdot3\cdot7 \cdot 17 - 1)}{2\cdot5\cdot(4\cdot5\cdot7 \cdot 23 + 1) + 1} = 137.035999074626
\]

\[
\delta = \frac{1}{4.66920169010299} = \frac{1}{4} - \frac{1}{27} + \frac{1}{4\cdot9\cdot23} - \frac{1}{2\cdot3\cdot7\cdot23\cdot(2\cdot5\cdot3\cdot11 - 1) + 1} + \frac{2\cdot23}{3\cdot19}
\]

\[
\alpha = \frac{1}{2.50290787509589} = \frac{1}{2} - \frac{1}{9} + \frac{1}{3\cdot31} - \frac{1}{23\cdot(8\cdot3\cdot17 + 1) + 1} + \frac{17\cdot23\cdot(8\cdot3\cdot11^4 - 1)}
\]

\[
\alpha_1(\delta/\gamma_1)^2(2\pi) = 1
\]

\[
\gamma_1 = 1 - \frac{1}{47.109} + \frac{1}{27\cdot7\cdot(3\cdot8\cdot(3\cdot8\cdot(4\cdot137 - 1) - 1) - 1)} = 0.999804818668238
\]

\[
\alpha_2(\alpha\delta/\gamma_2)^2 = 1, \quad \gamma_2 = 1 - \frac{1}{5\cdot7\cdot17} + \frac{1}{4\cdot3\cdot17\cdot23\cdot137 - 11\cdot59} = 0.998320883415699
\]
\[
\frac{2\pi}{(\alpha\gamma)^2} = 1, \quad \gamma = 1 + \frac{1}{32 \cdot 3.7} = \frac{1}{23.151 - 173 + \frac{9}{4.7}} = 1.00148643087192
\]
\[
(\frac{a_2 c_{\alpha\omega}}{n^2} - \frac{2a_2 c_{\omega\alpha}}{n^2})\psi = \nabla_{\alpha\omega}^2 \psi, \quad (\frac{c_{\omega\alpha-Ch}}{n^2} - \frac{2c_{\alpha\omega-Ch}}{n^2})\psi = \nabla_{\alpha\omega}^2 \psi
\]
\[
c_{\alpha\omega} = \frac{c}{v_c} = \frac{1}{\sqrt{\alpha_2\alpha_1}}, \quad r_c = \frac{\delta^2 \alpha}{\gamma_1\gamma_2} \sqrt{2\pi}
\]
\[
c_{\alpha\omega-Ch} = \sqrt{\frac{\alpha_2}{\alpha_1}} = \alpha_2 c_{\alpha\omega} = \frac{c_{\alpha\omega}}{(\alpha\delta / \gamma_2)^2} = \frac{\sqrt{2\pi}}{\alpha_\gamma_1 / \gamma_2} = \frac{2\pi \sqrt{r_0a_0}}{\lambda_c} = 1 - \frac{23.59}{50 \cdot 10^{11}}
\]

5. The Territory of Modern Physics

The territory of modern physics including Theory of Relativity, Quantum Theory, Chaos Theory and Theory of Chirality is shown as follows (Fig. 1).

\[
T = \frac{\hbar c^3}{8\pi MGk}
\]
\[
S = \frac{A\hbar c^3}{4Gh}
\]

\[
c_{\alpha\omega-Ch} = \sqrt{\frac{\alpha_2}{\alpha_1}} = \frac{2\pi \sqrt{r_0a_0}}{\lambda_c} = \frac{\sqrt{2\pi}}{\alpha_\gamma_1 / \gamma_2}
\]

\[
\alpha_1 = \frac{\lambda_c}{2\pi a_0} = \frac{1}{137.03599037435}
\]
\[
\alpha_2 = \frac{2\pi r_c}{\lambda_c} = \frac{1}{137.0359911818}
\]

The Territory of Modern Physics
Gang Chen, 2021/3/11-16
References:

7. G. Chen and T-M. Chen, Copyright Registration, Chen’s Periodic Table of Elements and Natural Group Theory, GuoZuoDengZi-2018-L-00472808.

Acknowledgements

Yichang Huifu Silicon Material Co., Ltd., Guangzhou Huifu Research Institute Co., Ltd. and Yichang Huifu Nanometer Material Co., Ltd. have been giving Dr. Gang Chen a part-time employment since Dec. 2018. Thank these companies for their financial support. Specially thank Dr. Yuelin Wang and other colleagues of these companies for their appreciation, support and help.

Thank Prof. Wenhao Hu, the dean of School of Pharmaceutical Sciences, Sun Yet-Sen University, for providing us an apartment in Shanghai since January of 2021 and hence facilitating the process of writing this paper.
## Appendix I: Research History

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Note: Date was recorded according to Beijing Time.