Unruh-DeWitt Detectors in Curved Spacetime

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Abstract

A variant of Semiclassical Gravity is outlined that differs in some interesting ways from the more familiar approach. It extends earlier work in which the renormalization of the stress-energy tensor is replaced by a different protocol. Creation and annihilation operators are assigned their roles based on the behavior of the normal modes making up the solution within an asymptotically simple region in cases where such exist. The question of Rindler particles and Unruh-DeWitt detectors is discussed. Particular attention is paid to the behavior of an Unruh-DeWitt detector that holds stationary outside an eternal black hole. A conjecture is offered to the effect that the positive frequency Wightman function, $D^+(x, x')$, is *always* to be given by $\frac{-1}{4\pi^2}(\text{geodesic distance between } x \text{ and } x')^{-2}$. This assumption allows us to solve the black hole problem numerically. Particles are, in fact, detected and their power spectrum is compared to those seen by rotating and linearly accelerating detectors. This matter is discussed in the context of the Equivalence Principle.

Keywords: Semiclassical Gravity, Rindler Particles, Unruh-DeWitt detectors, Black Holes.

Introduction.

It has been theorized since the seminal work of Fulling (1), DeWitt (2), Unruh (3), and Davies (4) that an accelerating monopole detector will register a thermal bath of particles. In keeping with the Equivalence Principle we would expect a similar result for a detector feeling an acceleration due to gravity.

In an earlier paper this author has argued that the goal of quantizing gravity should be abandoned in favor of Semiclassical Gravity (5). A prescription for doing this was offered that differs in significant ways from much of the previous work in the field. Our world is pictured as a classical, globally hyperbolic, manifold — $\mathcal{M}^4$. We impose upon it a coordinate system and a quantum field theory (QFT) of our choosing. The QFT dealt with here will be real scalar field theory. We try to solve the Klein-Gordon equation defined by $\mathcal{M}^4$ so as to arrive at a set of complete orthonormal modes and their conjugates that we call $u_k(\mathbf{x}, t)$ and $u_k^*(\mathbf{x}, t)$. In a perfect world we can find easy, analytic, expressions for these functions. Unfortunately, this is seldom the case. We write the field as:

1) $\varphi(\mathbf{x}, t) = \frac{1}{\sqrt{V}} \sum_k (u_k(\boldsymbol{x}, t)\, a_k + u_k^*(\boldsymbol{x}, t)\, a_k^\dagger)$ and we imagine the system contained in an enormous periodic box of volume V. This defines a Fock space in which the QFT operates.

We construct the stress-energy tensor — $T_{\mu\nu} = 2\, \frac{\delta \mathcal{L}_{\text{field}}}{\delta g^{\mu\nu}} - g_{\mu\nu}\, \mathcal{L}_{\text{field}}$ — and demand that $G_{\mu\nu} = 8\pi\, (\langle\Psi|T_{\mu\nu}|\Psi\rangle - \langle 0_M|T_{\mu\nu}|0_M\rangle) = 8\pi\, \{\Psi|T_{\mu\nu}|\Psi\}$ where $\{\Psi|\mathcal{O}|\Psi\}$ is defined as $\langle\Psi|\mathcal{O}|\Psi\rangle - \langle 0_M|\mathcal{O}|0_M\rangle$ for any operator $\mathcal{O}$ and $|0_M\rangle$ represents the lowest energy state for which $G_{\mu\nu} = 0$. We call $|0_M\rangle$ the Minkowski vacuum. $|\Psi\rangle$ is the state vector of our reality. We work in the Heisenberg Picture so this vector never changes. If it were $|0_M\rangle$ it is assumed that we automatically default into Minkowski space (MS).

QFT in Curved Spacetime.

I should expect to be asked what a_k and $a_k^\dagger$ are and how they relate to our Fock space. In MS the answer is obvious. In general, we want to associate the former with positive-frequency modes and the latter with the negative-frequency ones. But, in a curved spacetime, this becomes very difficult — negative or positive with respect to what time and where? This has always proved a terrible stumbling block for approaches to quantum and semiclassical gravity. We propose to address it in a rather unusual way.

We will illustrate this by considering an eternal Schwarzschild black hole (BH). We cannot, as a practical matter, find analytic solutions for the $u_k(\boldsymbol{x}, t)$ and $u_k{}^*(\boldsymbol{x}, t)$. But imagine that we can or can work them out numerically somehow. We know that, as r -> infinity, spacetime becomes Minkowskian (6). We know what a_k and $a_k^\dagger$ do there. In particular, the a_k annihilate $|0_M\rangle$. If they do so at infinity it stands to reason that they do so everywhere. Fock space is independent of spacetime and does not "live" in it. We want to identify the $u_k(\mathbf{x}, t)$ that goes to the Minkowski solution (denoted by k) at r = infinity. Depending on how we found our $u_k(\mathbf{x}, t)$ and $u_k{}^*(\mathbf{x}, t)$ we might have to construct various linear combination of these things in order to achieve our goal. But we ought to be able to achieve it (in principle). It would help, of course, if we had analytic solutions for our $u_k(\mathbf{x}, t)$ and $u_k{}^*(\mathbf{x}, t)$. But let us just imagine we did or could find an easy workaround. At least we know what a_k and $a_k^\dagger$ are and what they do. The a_ks *always* annihilate $|0_M\rangle$ and we build $|\Psi\rangle$ for our universe from $|0_M\rangle$ using the $a_k^\dagger$s. We would have to play around a bit to find what $|\Psi\rangle$ is here. It certainly is not any kind of vacuum state since it describes a BH. Otherwise, things are very much like what we know from MS.

This works for our BH but would not seem to have any general usefulness — $\mathcal{M}^4$ may have no asymptotic MS-like regions. Let us consider a different example. Suppose we live in a universe that expands forever. Cosmology indicates that we do. We assume a Robertson-Walker metric with k = 0. If no new matter is being created we will eventually end up with $\rho = 0$ and the manifold becomes asymptotically Minkowskian. This is most easily seen if we transform to a new coordinate system in which x^i is replaced by $y^i/\sqrt{g_{\text{ii}}}$. Now $g_{\mu\nu}$ -> $\eta_{\mu\nu}$ as t -> infinity. We may write:

$$2)\quad \varphi(\mathbf{y}, \infty) = \frac{1}{\sqrt{V}} \sum_k \left(\frac{e^{-i k\cdot y}}{\sqrt{2\,\omega_k}}\, a_k + \frac{e^{i k\cdot y}}{\sqrt{2\,\omega_k}}\, a_k^\dagger\right).$$

The a_ks always annihilate $|0_M\rangle$ and $[a_k, a_{k'}^\dagger] = \delta_{\text{kk'}}$. We can imagine integrating the above solution backwards in time using the Klein-Gordon equation so as to arrive at expressions for u_k and $u_k{}^*$ during our present epoch. As a practical matter this is easier said than done. We could, in some cases, attack the problem numerically.

In the above we have assumed that there is no cosmological constant. It is illustrative to consider the situation if there is one. In the first place $|0_M\rangle$ and MS disappear from consideration; in the vacuum state we now default into an empty deSitter universe. It is into this spacetime that our world will evolve. Let us call the vacuum $|0_{\text{DS}}\rangle$ (the 'deSitter vacuum'). This is what now figures in the curly brackets that define our Einstein's equation. On the left hand side now also appears - $\Lambda\ g_{\mu\nu}$. An empty deSitter universe possesses a timelike Killing vector. We would try to find sensible positive and negative-frequency modes and integrate these backward in time as above. In this way we know which modes to associate with the a_ks and $a_k^\dagger$s at the present time.

There is an outstanding question that must be addressed. If |Ψ> is in a vacuum state why does $\mathcal{M}^4$ default into MS or an empty deSitter universe instead of one with k = -1, a Taub-NUT space, or any other vacuum solution? We must assume a kind of 'prior geometry' (5). That our universe is characterized by a Robertson-Walker metric (at least at distant future times) is one such assumption. If it is we must also decide whether k = 1, 0, or -1. We must figure out whether there is a cosmological constant and, if so, what it is. These things do not appear to be mandated naturally.

The prescription outlined above allows us to unambiguously define our operators and construct our Fock space. But it only makes sense if $\mathcal{M}^4$ possesses an asymptotic region in which positive and negative frequency modes can be reasonably defined and a timelike Killing vector exists. Most $\mathcal{M}^4$s do not, of course, have this property. The real $\mathcal{M}^4$ we live in apparently does and we speculate that this may be more than a lucky accident. QFT may simply not be possible otherwise.

Particles and Detectors.

Say we are in the Minkowski vacuum state. (Actually, we could not be since we would not even exist. But let us ignore this problem for the moment — we are so small as to not affect the spacetime at all.) A family of Rindler observers (7) defines a coordinate system in which the massless Klein-Gordon equation takes the same form as for the Minkowski observers. Accordingly, they will have a solution of the form 1) but with the a_ks and $a_k^\dagger$s replaced by their operators which we can call b_k and $b_k^\dagger$. They can define "number operators" — $b_k^\dagger b_k$. Since their modes differ from those of the Minkowski observers (and mix the positive and negative-frequency modes of the latter) $b_k^\dagger b_k\ |0_M> \neq 0$. Indeed, they may well conclude that they are living in a thermal bath of particles. But should they? The Minkowskian observers can write $\{\Psi|H|\Psi\} = <\Psi|\ \sum_k \omega_k\, N_k\ |\Psi>$ where $N_k = a_k^\dagger a_k$ and $a_k|0_M> = 0$. Their number operators denominate specific packets of energy. The "number operators" of the Rindler observers do no such thing (as discussed briefly in (5)). We know this since, were it otherwise, there would be a net energy in the world and spacetime could not be Minkowskian. So what, if anything, are the Rindler observers counting? One can imagine attaching a monopole particle detector to a particular Rindler observer and show that it registers a similar thermal bath of particles. If this is not just a very strange coincidence it seems to suggest that we are looking at something real. But real in what sense? This is hard to say. We will return to this question presently. The Rindler observers can define their own "vacuum state," sometimes called a Fulling vacuum, $|0_F>$ such that $b_k\,|\,0_F> = 0$. But $\{0_F|T_{\mu\nu}|0_F\} \neq 0$ so we cannot be in MS. Moreover, if we could be, this state would not even obey the Weak Energy Condition.

Eternal Black Holes.

We would like to know about the particles that would be observed outside a BH by an Unruh-DeWitt monopole detector (a UD) were it sitting stationary outside the event horizon. Assuming the particles are massless we would need to compute the power spectrum given by:

3) $$\mathrm{P}(E) = \int_{-\infty}^{\infty} \mathrm{Exp}[-\,i\,E\,\Delta\tau\,]\,D^+(x(\tau),\,x(\tau'))\,d\Delta\tau.$$

We need to find $D^+(x(\tau),\,x(\tau')) = <\Psi_{\mathrm{BH}}|\ \varphi(\mathrm{x})\ \varphi(\mathrm{x}')\ |\Psi_{\mathrm{BH}}>$ where $\varphi(\mathrm{x})$ is the real scalar field around the BH and $|\Psi_{\mathrm{BH}}>$ is the state in our Fock space that corresponds to the existence of the BH. We know it is not a vacuum state of any kind since it describes a BH. We are not altogether certain what it is, however. Nor do we have a useful, analytic, expression for φ(x). The problem seems quite hopeless.

We might find a path towards its solution by noting that, in MS, $D^{+}(x,\ x') = -\frac{1}{4\pi^2}[\frac{1}{(t-t'-i\epsilon)^2-(\mathbf{x}-\mathbf{x}')^2}]$ for a massless scalar field which is proportional to (ignoring the tiny - $i\,\epsilon$ term) $\frac{1}{(\text{Straight Line Distance between } x \text{ and } x')^2}$ = $\frac{1}{\mathfrak{G}(x,x')^2}$ where $\mathfrak{G}$(x, x') denotes the 'geodesic distance' between the two points (i.e. the distance travelled by the UD if it move from x to x' along a geodesic path). This has every appearance of being a rather fundamental geometrodynamical relationship and, indeed, we will speculate that it is.

Conjecture:

$D^{+}(x(\tau), x(\tau'))$ = <Ψ | φ(x) φ(x') |Ψ > for a massless field is always to be given by $-\frac{1}{4\pi^2}\frac{1}{\mathfrak{G}(x,x')^2}$ where we understand t' to be adjusted with small - $i\,\epsilon$ term. We suggest that this is true in any $\mathcal{M}^4$.

This nomenclature may cause a bit of confusion since we are used to seeing |Ψ> as some sort of vacuum state for D^{+}. But, in our scheme of things, this cannot be the case if a BH is present.

This conjecture has some happy consequences. For one thing, if the UD is freely falling $\frac{1}{\mathfrak{G}(x,x')^2}$ will be $\frac{1}{(\tau-\tau'-i\,\epsilon)^2}$ where τ designates the UD's proper time. P(E) therefore vanishes and no particles are detected. This is very good news for the Equivalence Principle. Of course the UD we are interested in does not move along a geodesic. But our conjecture allows us to calculate P(E) numerically.

To do this we suppose the UD is sitting still at R_0. We set up a large table of interpolating functions that solve for the geodesic path followed by a test particle that starts at R_0 with an outward radial velocity. After a while it will fall back to R_0. Call τ the proper time along the test particle's geodesic path. Given our choice of radial velocity we can figure out the τ when it returns to R_0. We also know t(τ) — the time at infinity — when it does this. We ask the computer to find numerically the τ and radial velocity that satisfy r(τ) = R_0 and t(τ) = s/$\sqrt{1-2M/R_0}$ where s is the UD's proper time. We invert these solutions to obtain an interpolating function that gives us τ as a function of s. We then calculate (see Supplementary Material):

$$\int_{-\infty}^{\infty}\frac{\text{Exp}[-i\,E\,s]}{(\tau(s)-i\,\epsilon)^2}\,ds.$$

The simulation was performed for R_0 = 3 and M = 1. The result is shown below (fig. 1). The Rindler result for a UD experiencing the same proper acceleration is shown for comparison (solid line).

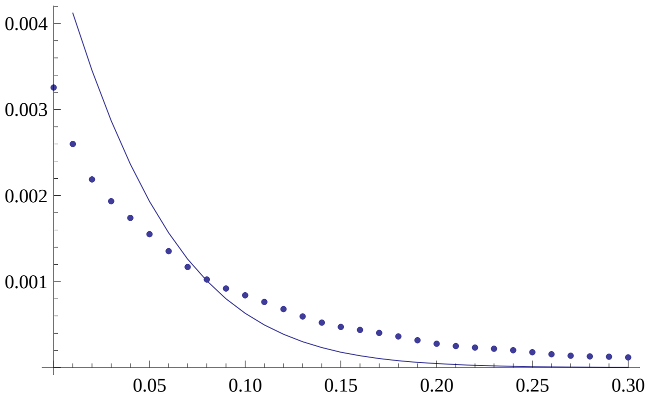

fig. 1

In keeping with the Equivalence Principle the two curves are rather similar. That they are not identical is doubtless due to geometrical factors. A detector that rotates in MS at a radius of 3 and the same proper acceleration gives us a curve that matches the BH result even more closely. (Fig. 2. The solid line is that of the rotating detector (8).)

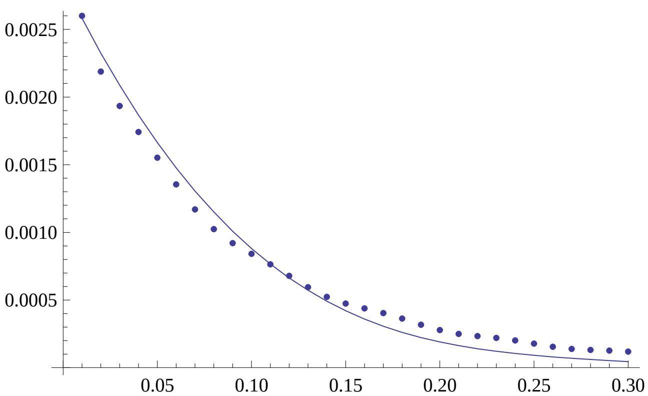

fig. 2

We also note that the existence of an event horizon is of no consequence here. The UD could just as well be sitting on a large planet. A similar exercise could be carried out for a UD in rotation around a BH. (Here we should better work in a coordinate system that rotated with it.) We already know the result we would obtain if it were in a stable (geodesic) orbit — no particles would be detected.

Conclusion.

We have tried to lay out a practical form of Semiclassical Gravity. It differs in some salient respects from what is usually encountered in the literature. For one thing, we dismiss the problem of stress-energy tensor renomalization opting instead for $G_{\mu\nu} = 8\,\pi\,\{\Psi|T_{\mu\nu}|\Psi\}$. We work in the Heisenberg Picture so $|\Psi>$ never changes. To address the question of defining the creation and annihilation operators we require the spacetime manifold we live in — $\mathcal{M}^4$ — to be consistent, at least asymptotically, with well-defined positive and negative-frequency modes. It must, in this limit, possess a timelike Killing vector field. For example, a spacetime that expands away into a Minkowkian 'heat death' would satisfy the requirement. Indeed, we may speculate that only $\mathcal{M}^4$s that conform to this requirement can be inhabited by workable QFTs. The problem of UDs has also to be addressed in this scheme of things. With no analytic solutions available for our BH problem for we offer a conjecture we know works in MS. A numerical simulation is performed that provides results that are in general agreement with the Equivalence Principle.

Behind all of this lies the question of the ontological status of these 'Rindler particles.' The question has been examined somewhat by Unruh and Wald (9). We can say that these particles do not represent actual mass-energy since they can be detected in MS which, by definition, is devoid of such. They may be detected but we know they cannot be being continuously created — our universe would be full of them. They represent an enigma, or perhaps even unphysical nonsense. We are dissuaded from the latter opinion by the remarkable appearance of the Plank factor. In one of Feynman's Red Books (10) it says 'Same equations, same solutions.' But it occurs to us that very little looks the same between the mathematics that Plank uses to derive his black body spectrum and the mathematics that allows for Rindler particles. Something seems to be eluding us here.

References.

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SUPPLEMENTARY MATERIAL

Below we solve for a detector sitting still outide a BH.

```
S[V_, T_] := S[V, T] = NDSolve[{x[1]′[τ]^2/(2 x[1][τ] - x[1][τ]^2) + x[1]″[τ] == (2 - x[1][τ]) x[4]′[τ]^2/x[1][τ]^3,
    x[4]″[τ] == 2 x[1]′[τ] x[4]′[τ]/(2 x[1][τ] - x[1][τ]^2), x[1][0] == 3, x[1]′[0] == V, x[4][0] == 0,
    x[4]′[0] == Sqrt[3 (1 + V^2 3)]}, {x[1], x[4]}, {τ, 0, T}][[1]]
```

S solves the geodesic equation at r = 3 outside a BH of M = 1. Below we define functions that we will need.

```
Rad[V_, L_, Inc_] := Table[{{ i, V}, x[1][ i] /. S[V, L]}, {i, 0, L, Inc}]

F[{{a_, b_}, c_}] := {{-a, b}, c}

H[{{a_, b_}, c_}] := {{-a, b}, -c}

RR[V_, L_, Inc_] := Union[Rad[V, L, Inc], Map[F, Rad[V, L, Inc]]]

Time[V_, L_, Inc_] := Table[{{ i, V}, x[4][ i] /. S[V, L]}, {i, 0, L, Inc}]

TT[V_, L_, Inc_] := Union[Time[V, L, Inc], Map[H, Time[V, L, Inc]]]

IntR00 = Interpolation[Union[RR[0, 1, .01], RR[.01, 1, .01],
   RR[.02, 1, .01], RR[.03, 1, .01], RR[.04, 1, .01], RR[.05, 1, .01]]]

InterpolatingFunction[{{-1., 1.}, {0., 0.05}}, <>]

IntT00 = Interpolation[Union[TT[0, 1, .01], TT[.01, 1, .01],
   TT[.02, 1, .01], TT[.03, 1, .01], TT[.04, 1, .01], TT[.05, 1, .01]]]

InterpolatingFunction[{{-1., 1.}, {0., 0.05}}, <>]

IntR05 = Interpolation[Union[RR[.05, 2, .02], RR[.06, 2, .02],
   RR[.07, 2, .02], RR[.08, 2, .02], RR[.09, 2, .02], RR[.1, 2, .02]]]

InterpolatingFunction[{{-2., 2.}, {0.05, 0.1}}, <>]

IntT05 = Interpolation[Union[TT[.05, 2, .02], TT[.06, 2, .02],
   TT[.07, 2, .02], TT[.08, 2, .02], TT[.09, 2, .02], TT[.1, 2, .02]]]

InterpolatingFunction[{{-2., 2.}, {0.05, 0.1}}, <>]

IntR10 = Interpolation[Union[RR[.1, 4, .04], RR[.11, 4, .04],
   RR[.12, 4, .04], RR[.13, 4, .04], RR[.14, 4, .04], RR[.15, 4, .04]]]

InterpolatingFunction[{{-4., 4.}, {0.1, 0.15}}, <>]

IntT10 = Interpolation[Union[TT[.1, 4, .04], TT[.11, 4, .04],
   TT[.12, 4, .04], TT[.13, 4, .04], TT[.14, 4, .04], TT[.15, 4, .04]]]

InterpolatingFunction[{{-4., 4.}, {0.1, 0.15}}, <>]

IntR15 = Interpolation[Union[RR[.15, 5, .05], RR[.16, 5, .05],
   RR[.17, 5, .05], RR[.18, 5, .05], RR[.19, 5, .05], RR[.2, 5, .05]]]

InterpolatingFunction[{{-5., 5.}, {0.15, 0.2}}, <>]

IntT15 = Interpolation[Union[TT[.15, 5, .05], TT[.16, 5, .05],
   TT[.17, 5, .05], TT[.18, 5, .05], TT[.19, 5, .05], TT[.2, 5, .05]]]

InterpolatingFunction[{{-5., 5.}, {0.15, 0.2}}, <>]

IntR20 = Interpolation[Union[RR[.2, 6, .06], RR[.21, 6, .06],
   RR[.22, 6, .06], RR[.23, 6, .06], RR[.24, 6, .06], RR[.25, 6, .06]]]

InterpolatingFunction[{{-6., 6.}, {0.2, 0.25}}, <>]
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IntT20 = Interpolation[Union[TT[.2, 6, .06], TT[.21, 6, .06],
   TT[.22, 6, .06], TT[.23, 6, .06], TT[.24, 6, .06], TT[.25, 6, .06]]]

InterpolatingFunction[{{-6., 6.}, {0.2, 0.25}}, <>]

IntR25 = Interpolation[Union[RR[.25, 7, .07], RR[.26, 7, .07],
   RR[.27, 7, .07], RR[.28, 7, .07], RR[.29, 7, .07], RR[.3, 7, .07]]]

InterpolatingFunction[{{-7., 7.}, {0.25, 0.3}}, <>]

IntT25 = Interpolation[Union[TT[.25, 7, .07], TT[.26, 7, .07],
   TT[.27, 7, .07], TT[.28, 7, .07], TT[.29, 7, .07], TT[.3, 7, .07]]]

InterpolatingFunction[{{-7., 7.}, {0.25, 0.3}}, <>]

IntR30 = Interpolation[Union[RR[.3, 8, .08], RR[.31, 8, .08],
   RR[.32, 8, .08], RR[.33, 8, .08], RR[.34, 8, .08], RR[.35, 8, .08]]]

InterpolatingFunction[{{-8., 8.}, {0.3, 0.35}}, <>]

IntT30 = Interpolation[Union[TT[.3, 8, .08], TT[.31, 8, .08],
   TT[.32, 8, .08], TT[.33, 8, .08], TT[.34, 8, .08], TT[.35, 8, .08]]]

InterpolatingFunction[{{-8., 8.}, {0.3, 0.35}}, <>]

IntR35 = Interpolation[Union[RR[.35, 10, .1], RR[.36, 10, .1],
   RR[.37, 10, .1], RR[.38, 10, .1], RR[.39, 10, .1], RR[.4, 10, .1]]]

InterpolatingFunction[{{-10., 10.}, {0.35, 0.4}}, <>]

IntT35 = Interpolation[Union[TT[.35, 10, .1], TT[.36, 10, .1],
   TT[.37, 10, .1], TT[.38, 10, .1], TT[.39, 10, .1], TT[.4, 10, .1]]]

InterpolatingFunction[{{-10., 10.}, {0.35, 0.4}}, <>]

IntR40 = Interpolation[Union[RR[.4, 12, .12], RR[.41, 12, .12],
   RR[.42, 12, .12], RR[.43, 12, .12], RR[.44, 12, .12], RR[.45, 12, .12]]]

InterpolatingFunction[{{-12., 12.}, {0.4, 0.45}}, <>]

IntT40 = Interpolation[Union[TT[.4, 12, .12], TT[.41, 12, .12],
   TT[.42, 12, .12], TT[.43, 12, .12], TT[.44, 12, .12], TT[.45, 12, .12]]]

InterpolatingFunction[{{-12., 12.}, {0.4, 0.45}}, <>]

IntR45 = Interpolation[Union[RR[.45, 14, .14], RR[.46, 14, .14],
   RR[.47, 14, .14], RR[.48, 14, .14], RR[.49, 14, .14], RR[.5, 14, .14]]]

InterpolatingFunction[{{-14., 14.}, {0.45, 0.5}}, <>]

IntT45 = Interpolation[Union[TT[.45, 14, .14], TT[.46, 14, .14],
   TT[.47, 14, .14], TT[.48, 14, .14], TT[.49, 14, .14], TT[.5, 14, .14]]]

InterpolatingFunction[{{-14., 14.}, {0.45, 0.5}}, <>]

IntR50 = Interpolation[Union[RR[.5, 18, .18], RR[.51, 18, .18],
   RR[.52, 18, .18], RR[.53, 18, .18], RR[.54, 18, .18], RR[.55, 18, .18]]]

InterpolatingFunction[{{-18., 18.}, {0.5, 0.55}}, <>]
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IntT50 = Interpolation[Union[TT[.5, 18, .18], TT[.51, 18, .18],
   TT[.52, 18, .18], TT[.53, 18, .18], TT[.54, 18, .18], TT[.55, 18, .18]]]

InterpolatingFunction[{{-18., 18.}, {0.5, 0.55}}, <>]

IntR55 = Interpolation[Union[RR[.55, 24, .24], RR[.56, 24, .24],
   RR[.57, 24, .24], RR[.58, 24, .24], RR[.59, 24, .24], RR[.6, 24, .24]]]

InterpolatingFunction[{{-24., 24.}, {0.55, 0.6}}, <>]

IntT55 = Interpolation[Union[TT[.55, 24, .24], TT[.56, 24, .24],
   TT[.57, 24, .24], TT[.58, 24, .24], TT[.59, 24, .24], TT[.6, 24, .24]]]

InterpolatingFunction[{{-24., 24.}, {0.55, 0.6}}, <>]

IntR60 = Interpolation[Union[RR[.6, 32, .32], RR[.61, 32, .32],
   RR[.62, 32, .32], RR[.63, 32, .32], RR[.64, 32, .32], RR[.65, 32, .32]]]

InterpolatingFunction[{{-32., 32.}, {0.6, 0.65}}, <>]

IntT60 = Interpolation[Union[TT[.6, 32, .32], TT[.61, 32, .32],
   TT[.62, 32, .32], TT[.63, 32, .32], TT[.64, 32, .32], TT[.65, 32, .32]]]

InterpolatingFunction[{{-32., 32.}, {0.6, 0.65}}, <>]

IntR65 = Interpolation[Union[RR[.65, 47, .47], RR[.66, 47, .47],
   RR[.67, 47, .47], RR[.68, 47, .47], RR[.69, 47, .47], RR[.7, 47, .47]]]

InterpolatingFunction[{{-47., 47.}, {0.65, 0.7}}, <>]

IntT65 = Interpolation[Union[TT[.65, 47, .47], TT[.66, 47, .47],
   TT[.67, 47, .47], TT[.68, 47, .47], TT[.69, 47, .47], TT[.7, 47, .47]]]

InterpolatingFunction[{{-47., 47.}, {0.65, 0.7}}, <>]

IntR70 = Interpolation[Union[RR[.7, 80, .8], RR[.71, 80, .8],
   RR[.72, 80, .8], RR[.73, 80, .8], RR[.74, 80, .8], RR[.75, 80, .8]]]

InterpolatingFunction[{{-80., 80.}, {0.7, 0.75}}, <>]

IntT70 = Interpolation[Union[TT[.7, 80, .8], TT[.71, 80, .8],
   TT[.72, 80, .8], TT[.73, 80, .8], TT[.74, 80, .8], TT[.75, 80, .8]]]

InterpolatingFunction[{{-80., 80.}, {0.7, 0.75}}, <>]

IntR75 = Interpolation[Union[RR[.75, 182, 1.82], RR[.76, 182, 1.82],
   RR[.77, 182, 1.82], RR[.78, 182, 1.82], RR[.79, 182, 1.82], RR[.8, 182, 1.82]]]

InterpolatingFunction[{{-182., 182.}, {0.75, 0.8}}, <>]

IntT75 = Interpolation[Union[TT[.75, 182, 1.82], TT[.76, 182, 1.82],
   TT[.77, 182, 1.82], TT[.78, 182, 1.82], TT[.79, 182, 1.82], TT[.8, 182, 1.82]]]

InterpolatingFunction[{{-182., 182.}, {0.75, 0.8}}, <>]

IntR80 = Interpolation[Union[RR[.8, 1430, 14.3], RR[.81, 1430, 14.3],
   RR[.82, 1430, 14.3], RR[.83, 1430, 14.3], RR[.84, 1430, 14.3], RR[.85, 1430, 14.3]]]

InterpolatingFunction[{{-1430., 1430.}, {0.8, 0.85}}, <>]
```

```
IntT80 = Interpolation[Union[TT[.8, 1430, 14.3], TT[.81, 1430, 14.3],
   TT[.82, 1430, 14.3], TT[.83, 1430, 14.3], TT[.84, 1430, 14.3], TT[.85, 1430, 14.3]]]
```

```
InterpolatingFunction[{{-1430., 1430.}, {0.8, 0.85}}, <>]
```

Below are the radial distance and time at infinity as functions of t (the UDs proper time if it had moved on the geodesic) and v (the initial radial velocity.

```
Radius[t_, v_] := Which[0 ≤ v < .05, IntR00[t, v], .05 ≤ v < .1, IntR05[t, v], .1 ≤ v < .15,
  IntR10[t, v], .15 ≤ v < .2, IntR15[t, v], .2 ≤ v < .25, IntR20[t, v], .25 ≤ v < .3,
  IntR25[t, v], .3 ≤ v < .35, IntR30[t, v], .35 ≤ v < .4, IntR35[t, v], .4 ≤ v < .45,
  IntR40[t, v], .45 ≤ v < .5, IntR45[t, v], .5 ≤ v < .55, IntR50[t, v], .55 ≤ v < .6,
  IntR55[t, v], .6 ≤ v < .65, IntR60[t, v], .65 ≤ v < .7, IntR65[t, v], .7 ≤ v < .75,
  IntR70[t, v], .75 ≤ v < .8, IntR75[t, v], .8 ≤ v < .85, IntR80[t, v], True, 10^5]
```

```
FarTime[t_, v_] := Which[0 ≤ v < .05, IntT00[t, v], .05 ≤ v < .1, IntT05[t, v], .1 ≤ v < .15,
  IntT10[t, v], .15 ≤ v < .2, IntT15[t, v], .2 ≤ v < .25, IntT20[t, v], .25 ≤ v < .3,
  IntT25[t, v], .3 ≤ v < .35, IntT30[t, v], .35 ≤ v < .4, IntT35[t, v], .4 ≤ v < .45,
  IntT40[t, v], .45 ≤ v < .5, IntT45[t, v], .5 ≤ v < .55, IntT50[t, v], .55 ≤ v < .6,
  IntT55[t, v], .6 ≤ v < .65, IntT60[t, v], .65 ≤ v < .7, IntT65[t, v], .7 ≤ v < .75,
  IntT70[t, v], .75 ≤ v < .8, IntT75[t, v], .8 ≤ v < .85, IntT80[t, v], True, 10^5]
```

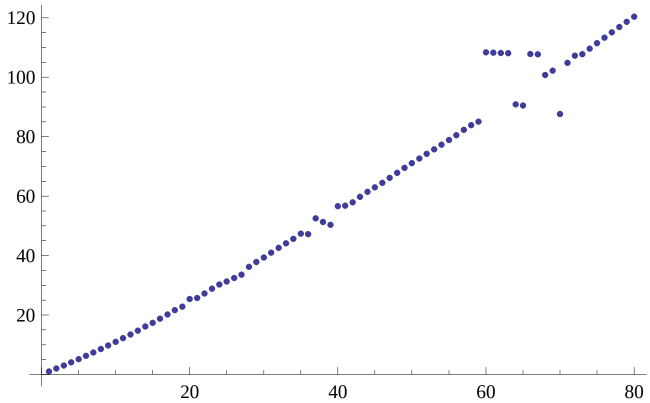
Below we find the ' geodesic time ' as a function of s (the real proper time registered by the detector).

```
B[s_] := FindRoot[{Radius[t, v] == 3, FarTime[t, v] == Sqrt[3] s}, {{t, 11}, {v, .4}}]
```

```
ProperTime[s_] := t /. B[s]
```

Below we plot the geodesic time against s. We see that it has major inaccuracies around s = 40 and s = 65. This is because FindRoot fails to find the root due to poor selection of the starting values for t and v. We can change the starting values but this only moves the problems from one s to another. An easier way is to construct a solution out of pieces that work. Essentially, we excise the parts that do not work and join the working parts by interpolation.

```
ListPlot[Table[ProperTime[s], {s, 1, 80, 1}]]
```




```
T1 = Table[{{s, ProperTime[s]}, Radius[t, v] /. B[s]}, {s, 0, 20, .1}];
```

Below we throw away solutions that do not work and drop the Radius part which no longer care about.

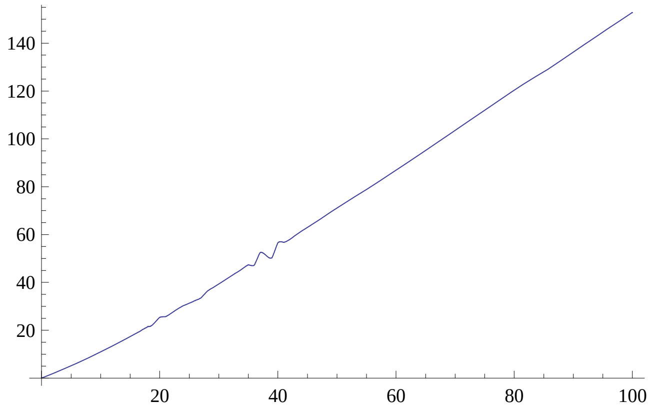
```
crit[{{a_, b_}, c_}] := If [c == 3, True, False]

M[{{a_, b_}, c_}] := {a, b}

Map[M, Select[T1, crit]] // Chop;

I1 = Interpolation[%]

InterpolatingFunction[{{0., 20.}}, <>]
```

Below we do the same things for 18 < s < 55 and 80 < s < 100.

```
T2 = Table[{{s, ProperTime[s]}, Radius[t, v] /. B[s]}, {s, 18, 55, 1}];

T3 = Table[{{s, ProperTime[s]}, Radius[t, v] /. B[s]}, {s, 80, 100, 1}];

U = Union[T2, T3];

crit2[{{a_, b_}, c_}] := If [2.9999 < c < 3.0001, True, False]

Select[U, crit2];

Map[M, U];

I2 = Interpolation[%]

InterpolatingFunction[{{18., 100.}}, <>]

PP[s_] := If[s ≤ 18, I1[s], I2[s]]
```

The result is now much better.

```
Plot[PP[s], {s, 0, 100}]
```



Below de define the ' geodesic distance ' (PD).

```
PD[s_] := If[s ≥ 0, PP[s], -PP[-s]]
```

Below we find the power spectrum we want.

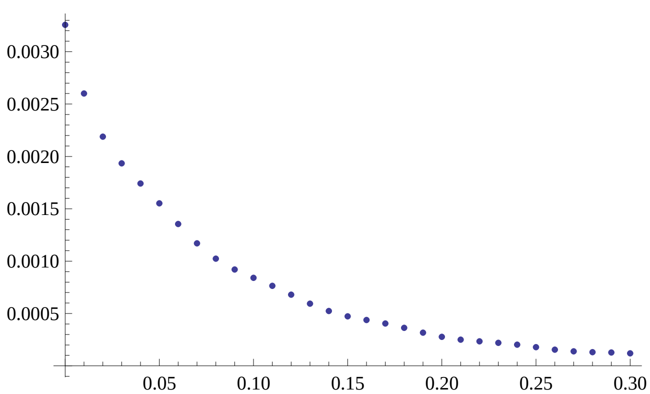
```
Int[En_, ϵ_, L_] := NIntegrate[Exp[-I En τ] / ((PD[τ] - I ϵ)^2), {τ, -L, L}] / (-4 Pi^2)
```

We adjust L and ϵ until the result is stable (minimal artifacts).

```
Table[{i, Int[i, .001, 100]}, {i, 0, .3, .01}] // Chop;
```

Below is our result. Here acc = .19245

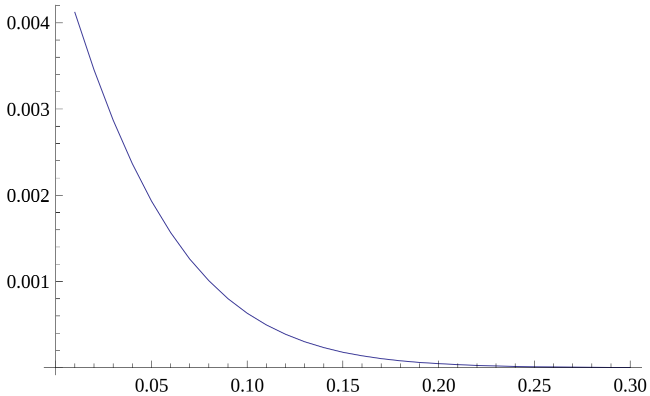
```
ListPlot[%]
```



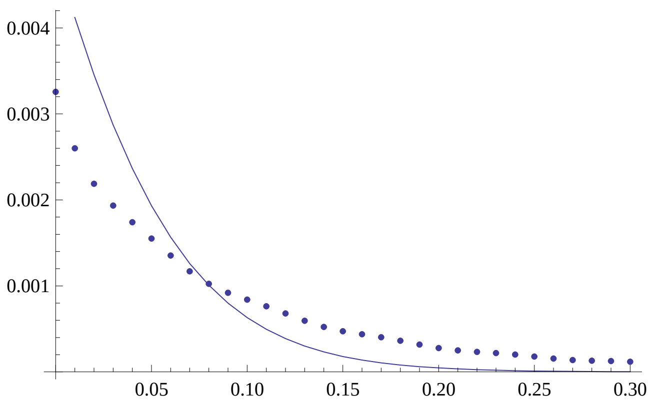
Below is the Rindler result.

```
Table[{En, (En / (2 Pi)) / (Exp[2 Pi En / .19245] - 1)}, {En, 0, .3, .01}];
```

```
ListPlot[%, Joined → True]
```




```
Show[%, %70]
```



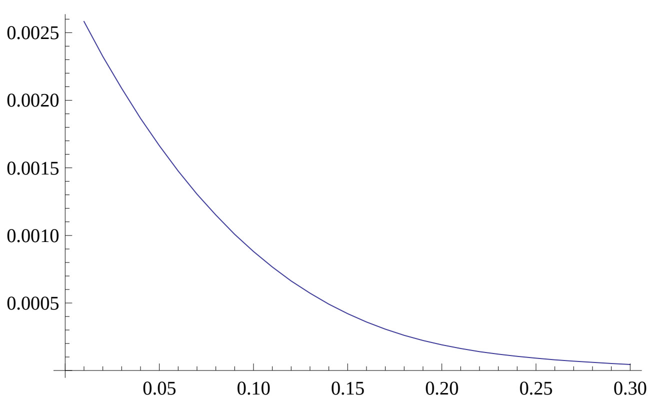
Below we do a rotating detector in Flat spacetime. R = 3 and its proper acceleration is .19245.

```
Int2[En_, R_, w_, ϵ_, L_] := NIntegrate[
   Exp[-I En τ] / ((τ / Sqrt[1 - R^2 w^2] - I ϵ)^2 - 2 R^2 (1 - Cos[w τ / Sqrt[1 - R^2 w^2]])),
   {τ, -L, L}] / (-4 Pi^2)
```

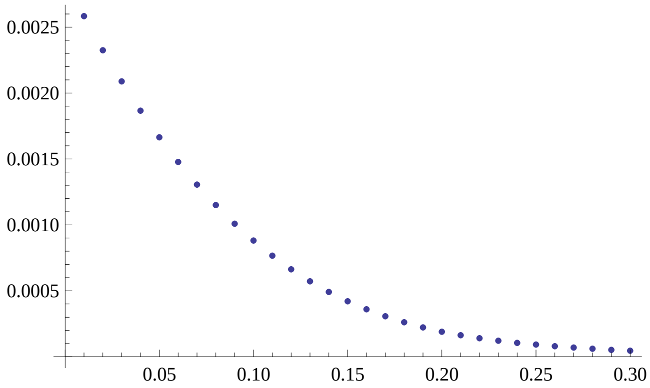
Here is the result.

```
Table[{En, Int2[En, 3, .2, .01, 1000]}, {En, 0, .3, .01}] // Chop;
```

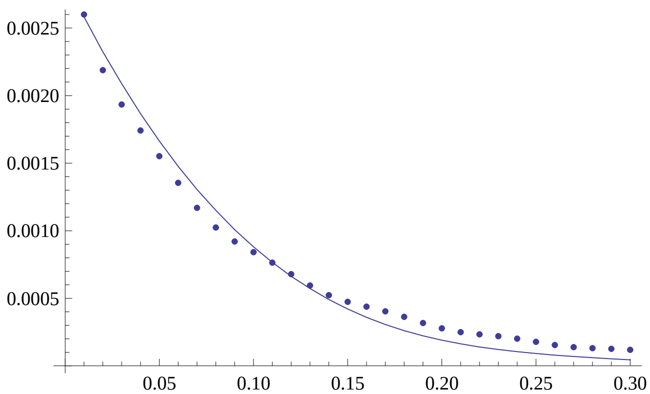
```
ListPlot[%, Joined → True]
```



```
ListPlot[%]
```




```
Show[%, %70]
```



BH result looks similar.