Integrated Formulas of the Fine-structure Constant and Feigenbaum Constants (viXra:2102.0162v4)

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Dedicated to Prof. Albert Sun-Chi Chan on the occasion of his 70th birthday

Abstract

This paper is a subsequent paper to the previous paper “Formulas of Feigenbaum Constants and Their Physical Meanings” (viXra:2101.0187). In the previous paper, some formulas of Feigenbaum constants in fractional number format were given and the physical meanings of the factors in the formulas were exhibited, especially their relationships with nuclides, the fine-structure constant and $2\pi$. In the previous paper, some integrated formulas of the fine-structure constant, Feigenbaum constants and $2\pi$ were also given, briefly denoted as $\alpha_1\delta(2\pi)\approx1$, and their relationships with nuclides were illustrated. In this paper, some formulas for $\alpha_1\delta(2\pi)\approx1$ are supplemented, some formulas for $\alpha_2(\alpha/\delta)^2\approx1$, $[\alpha_1(2\pi)/(\alpha_2\alpha^2)]\approx1$ and $(2\pi)/\alpha^2\approx1$ are given, some formulas of the fine-structure constant ($\alpha_1$ and $\alpha_2$) based on the key number 103 instead of 112, 173, 137, 83 and 29 are supplemented. In the end, by introducing correction factors $\gamma_1$, $\gamma_2$ and $\gamma$, accurate formulas $\alpha_1(\delta/\gamma_1)^2(2\pi)=1$, $\alpha_2(\delta\alpha/\gamma_2)^2=1$ and $2\pi/(\alpha\gamma)^2=1$ are gained.

Keywords: Formulas; the fine-structure constant; Feigenbaum constants; $2\pi$.

1. Introduction

In our previous papers, we gave or exhibited the following formulas.

$$(2\pi)_{\text{Chen-k}} = e^2 \left(2 \frac{e^2}{1} \left(3 \frac{e^2}{2} \left(5 \frac{e^2}{3} \cdots \left(k+1 \frac{e^2}{k} \right)^{2k+1} \right) \right) \right)$$

$$(2\pi)_{\text{Wallis-k}} = 4 \cdot \left(2 \frac{4}{3} \frac{4}{5} \cdots \frac{2k}{2k+1} \frac{2k+2}{2k+1} \right)$$

$$(2\pi)_{\text{GL-k}} = 8 \cdot \left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots \left(-1\right)^{k+1} \frac{1}{2k+1} \right) \quad (GL \text{ means Gregory-Leibniz})$$

$$(2\pi)_{\text{NC-k}} = 6 + \sum_{n=1}^{k} \frac{(-1)^{n+1}}{n(n+1/2)(n+1)} \quad (NC \text{ means Nilakantha-Chen})$$
\[ \alpha_1 = \frac{\lambda_c}{2\pi a_0} = \frac{36}{7 \cdot (2\pi)_{\text{Chen}-112}} \frac{1}{112} + \frac{1}{75^2} = 1/137.035999037435 \]

\[ \alpha_2 = \frac{2\pi r_c}{\lambda_c} = \frac{13 \cdot (2\pi)_{\text{Chen}-278}}{100} \frac{1}{112 - \frac{1}{64 \cdot 3 \cdot 29}} = 1/137.035999111818 \]

\[ c_{au} \frac{1}{\alpha_c} = \frac{1}{\sqrt{\alpha_c^2}} = \sqrt{1/112 \times (168 - \frac{1}{3} + \frac{1}{12 \cdot 47} - \frac{1}{14 \cdot 112 \cdot (2\cdot 173 + 1)})} = 137.035999074626 \]

\[ \frac{1}{\alpha_1} = 56 + 81 + \frac{1}{28} - \frac{13 \cdot (2\cdot 56 \cdot 11 - 1)}{3 \cdot 5 \cdot (2\cdot 56 \cdot 43 + 1)} = 137.035999073435 \]

\[ \frac{1}{\alpha_2} = 56 + 81 + \frac{1}{28} - \frac{2 \cdot (16 \cdot 27 - 1)}{3 \cdot (16 \cdot 81 + 1)} = 137.035999111818 \]

\[ c_{au} \frac{1}{\alpha_c} = 56 + 81 + \frac{1}{28} - \frac{5 \cdot (4 \cdot 3 \cdot 7 \cdot 17 - 1)}{2 \cdot 5 \cdot (4 \cdot 5 \cdot 7 \cdot 23 + 1) + 1} = 137.035999074626 \]

Note: \( c_{au} \) refers to the speed of light in vacuum in atomic units

Feigenbaum Constants: \( \delta = 4.66920160910299 \)
\[ \alpha = 2.50290787509589 \]

\[ \frac{1}{\delta} = \frac{1}{4.66920160910299} = 0.21416937706236 \]

\[ = \frac{1}{4} \cdot \frac{27}{2} + \frac{1}{2} \cdot \frac{1}{3 \cdot 9} \cdot 23 \cdot (8 \cdot 3 \cdot 17 + 1) + \frac{2 \cdot 23}{4 \cdot 31} = 0.399535280523135 \]

\[ \frac{1}{\alpha} = \frac{1}{2.50290787509589} = 0.399535280523135 \]

\[ = \frac{1}{2} \cdot \frac{1}{9} + \frac{1}{3 \cdot 31} - \frac{1}{23 \cdot (8 \cdot 3 \cdot 17 + 1)} + \frac{1}{17 \cdot 23 \cdot (8 \cdot 3 \cdot 11^4 - 1)} \]

Note: 136=8 \cdot 17, 138=6 \cdot 23

\[ \alpha_i \delta^2 (2\pi) \approx 1 \]

On Feb. 8, 2021, we also noticed that Hieb uploaded a paper in viXra in April of 2017, and gave an approximate formula of the fine-structure constant and Feigenbaum constant as follows, but without any explanations to its physical meanings.

\[ \delta' = (1/(2\pi \alpha))^{1/2} = 4.670114 \approx \delta = 4.669201609 \]

\[ \delta' - \delta = 0.000912 \]

\[ \alpha : \text{the fine-structure constant, } \alpha \approx 1/137.036 \]
2. Integrated Formulas of $\alpha_1, \delta$ and $2\pi$

A Concise Deduction

The Fine-structure Constant:

$$\alpha_1 = \frac{\lambda_c}{2\pi a_0} = \frac{36}{7 \cdot (2\pi)_{\text{Chen-112}}} \cdot \frac{1}{112 + \frac{1}{75^2}} = 1/137.035999037435$$

Feigenbaum Constants: $\delta = 4.66920160910299$

$$\alpha_1 = \frac{36}{7 \cdot (2\pi)_{\text{Chen-112}}} \cdot \frac{1}{112 + \frac{1}{75^2}} \approx \frac{36}{7 \cdot (2\pi)_{\text{Chen-112}}} \cdot \frac{1}{112} \approx \frac{3^2}{14} \cdot \frac{1}{2\pi} \approx \frac{1}{\delta^2 (2\pi)}$$

$$= \frac{4.66920160910299^2 \cdot (2 \cdot 3.14159265358979)}{136.982} \approx 1$$

So it should be reasonable to assume the following approximate formulas:

$$\alpha_1 \delta^2 (2\pi) \approx 1 \text{ or } \frac{1}{\alpha_1 \delta^2 (2\pi)} \approx 1$$

Numerically: $\alpha_1 \delta^2 (2\pi) = \frac{4.6692^2 \times 6.2832}{137.036} = 0.99961 \approx 1$

2021/2/1-3

The above approximate formula $\alpha_1 \delta^2 (2\pi) \approx 1$ is assumed to be the brief form of integrated formulas of $\alpha_1, \delta$ and $2\pi$. There should be some corresponding accurate forms of integrated formulas of $\alpha_1, \delta$ and $2\pi$ as follows.

$$\alpha_1 \delta^2 (2\pi)_{\text{Chen-25-17}} = \frac{4.66920160910299^2 \cdot (e^{\frac{2}{1}} e^{\frac{2}{2}} e^{\frac{2}{3}} \cdots e^{\frac{2}{23}})}{137.035999037435}$$

$$= \frac{4.66920160910299^2 \cdot 6.28564399787948}{137.035999037435} = 1 + \frac{1}{128.9 \cdot (2 \cdot 3.173 + 1) + \frac{7}{10}} = 1.0000008354717 \approx 1$$
\[
\frac{1}{\alpha_0^2(2\pi)^{\text{Chen} - 2517}} = \frac{4.66920160910299^2 \cdot (e^2 \cdot \frac{e^2}{1} \cdot \frac{e^2}{2} \cdot \frac{e^2}{3} \cdot \frac{e^2}{5} \cdots \frac{e^2}{23 \cdot 37})}{137.035999037435} = \frac{4.66920160910299^2 \cdot 6.28564399787948}{137.035999037435}
\]

\[
1 - \frac{1}{2 \cdot 5 \cdot 7 \cdot (2 \cdot 83 \cdot 103 + 1) - \frac{9}{29}} = 0.999999164529037 \approx 1
\]

2021/2/19

\[
\frac{1}{\alpha_0^2(2\pi)^{\text{Wallis} - 9.71}} = \frac{4.66920160910299^2 \cdot 4 \cdot (2 \cdot 4 \cdot 4 \cdot 6 \cdots 1278 \cdot 1280)}{137.035999037435} = \frac{4.66920160910299^2 \cdot 6.28564015562186}{137.035999037435}
\]

\[
1 + \frac{1}{2 \cdot 9 \cdot (8 \cdot 3 \cdot 25 \cdot 7 \cdot 59 - 1) - \frac{5}{17}} = 1.00000022419606 \approx 1
\]

2021/2/1

\[
\frac{1}{\alpha_0^2(2\pi)^{\text{Wallis} - 9.71}} = \frac{4.66920160910299^2 \cdot 4 \cdot (2 \cdot 4 \cdot 4 \cdot 6 \cdots 1278 \cdot 1280)}{137.035999037435} = \frac{4.66920160910299^2 \cdot 6.28564015562186}{137.035999037435}
\]

\[
1 - \frac{1}{2 \cdot 9 \cdot (8 \cdot 3 \cdot 25 \cdot 7 \cdot 59 - 1) + \frac{3.4}{17}} = 0.999999775803991 \approx 1
\]

2021/2/20
\[
\alpha_i \delta^2 (2\pi)_{\text{GL-22.37}} = \frac{4.66920160910299^2 \cdot 8 \cdot (1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots + \frac{1}{2 \cdot 11 \cdot 37 + 1})}{137.035999037435}
\]

\[
= \frac{4.66920160910299^2 \cdot 6.28563929398602}{137.035999037435}
\]

\[
= 1 + \frac{1}{163 \cdot (6 \cdot 11^2 \cdot 97 + 1) + \frac{9}{10}} = 1.00000008711598 \approx 1
\]

\[
\alpha_i \delta^2 (2\pi)_{\text{NC-3}} = \frac{4.66920160910299^2 \cdot 6 \cdot (6 + \sum_{n=1}^{3} \frac{(-1)^n}{n(n+1/2)(n+1)})}{137.035999037435}
\]

\[
= \frac{4.66920160910299^2 \cdot 6.29047619047619}{137.035999037435}
\]

\[
= 1 + \frac{1}{3 \cdot (2 \cdot 7 \cdot 31 - 1)} - \frac{1}{13.89 \cdot (2 \cdot 37 \cdot 53 + 1) - \frac{2 \cdot 9}{25}} = 1.00076960262352 \approx 1
\]

\[
\alpha_i \delta^2 (2\pi)_{\text{NC-3}} = \frac{4.66920160910299^2 \cdot (6 + \sum_{n=1}^{3} \frac{(-1)^n}{n(n+1/2)(n+1)})}{137.035999037435}
\]

\[
= \frac{4.66920160910299^2 \cdot 6.29047619047619}{137.035999037435}
\]

\[
= 1 + \frac{1}{3 \cdot (2 \cdot 7 \cdot 31 - 1)} - \frac{1}{13.89 \cdot (2 \cdot 37 \cdot 53 + 1) - \frac{2 \cdot 9}{25}} = 1.00076960262352 \approx 1
\]
\[ \alpha_2 \delta^2 (2\pi) = \frac{4.66920160910299^2 \cdot (2 \cdot 3.14159265358979)}{137.035999037435} \]

\[ = 0.99960967543223 \approx 1 \]

\[ \alpha_2 = \frac{2 \alpha_2}{\delta^2 (2\pi)} = \frac{2}{0.99960967543223} \approx 2.0004 \]

\[ \alpha_2 \approx \frac{13 \cdot (2\pi)}{100} \approx 0.809 \]

\[ \delta = 4.66920160910299 \]

\[ \alpha = 2.50297087509589 \]

\[ \alpha_2 = \frac{13 \cdot (2\pi)}{100} \approx 0.809 \]

\[ \approx 1 \]

So it should be reasonable to assume the following approximate formulas:

\[ \alpha_2 (\delta \alpha)^2 \approx 1 \text{ or } \frac{1}{\alpha_2 (\delta \alpha)^2} \approx 1 \]

Numerically:

\[ \alpha_2 (\delta \alpha)^2 = \frac{(4.6692 \times 2.5029)^2}{137.036} \approx 0.99664 \approx 1 \]
The above approximate formula $\alpha_2(\delta \alpha)^2 \approx 1$ is assumed to be the brief form of integrated formulas of $\alpha_2$, $\delta$ and $\alpha$. There should be some corresponding accurate forms of integrated formulas of $\alpha_2$, $\delta$ and $\alpha$ as follows.

\[
\alpha_2(\delta \alpha)^2 = \frac{(4.66920160910299 \cdot 2.50290787509589)^2}{137.03599911181} = 0.996644586263908 \approx 1
\]

\[
= 1 - \frac{1}{2 \cdot 1.49} + \frac{1}{29 \cdot 31 \cdot (2 \cdot 3 \cdot 49 \cdot 13 + 1) - 16} = 0.996644586263908 \approx 1
\]

\[
= 1 - \frac{1}{2 \cdot 3 \cdot 25 \cdot 11 \cdot (4 \cdot 11 \cdot 47 + 1) - \frac{2}{5}} = 1.00036671044256 \approx 1
\]

\[
= 1 - \frac{1}{2 \cdot 3 \cdot 25 \cdot 11 \cdot (2 \cdot 9 \cdot 5 \cdot 23 - 1) - \frac{2}{5}} = 1.00036671044256 \approx 1
\]

4. Integrated Formulas of $\alpha_1$, $\alpha_2$, $\alpha$ and $2\pi$

\[
\alpha_1 \delta^2 (2\pi) = 0.999661 \approx 1
\]

\[
\alpha_2 (\delta \alpha)^2 = 0.996644 \approx 1
\]

\[
\frac{\alpha_1 \delta^2 (2\pi)}{\alpha_2 (\delta \alpha)^2} = \frac{\alpha_1 (2\pi)}{\alpha_2 \alpha^2} \approx \frac{2\pi}{\alpha_2} = 1.002975 \approx 1
\]

2021/2/11
\[
\frac{\alpha(2\pi)}{\alpha^2} = \frac{137.035999111818 \cdot (2 \cdot 3.14159265358979)}{137.035999037435 - 2.50297875095892} = 1 + \frac{1}{16 \cdot 3.7} - \frac{1}{2 \cdot 3.7 \cdot (4 \cdot 17 - 2 \cdot 157)} - \frac{1}{2} \approx 1
\]

\[
\frac{\alpha^2}{\alpha(2\pi)} = \frac{137.035999037435 - 2.50297875095892}{137.035999111818 \cdot (2 \cdot 3.14159265358979)} = 1 - \frac{1}{16 \cdot 3.7 + 1} + \frac{1}{7 \cdot 19 \cdot (2 \cdot 3.7^2 \cdot 23 - 1)} + \frac{16}{3.13} = 0.997033753032614 \approx 1
\]

\[
\alpha(2\pi) = \frac{2 \cdot 3.14159265358979}{2.50290787509589} = 1 + \frac{1}{16 \cdot 3.7 + 1} - \frac{1}{3 \cdot 13} \cdot (2 \cdot 9 \cdot 41 + 1) - \frac{1}{22} \approx 1
\]

\[
\alpha^2 = \frac{2.50290787509589}{2 \cdot 3.14159265358979} = 1 - \frac{1}{16 \cdot 3.7 + 1} + \frac{1}{8 \cdot 81 - 19 \cdot 73} + \frac{1}{3 \cdot 13} \approx 1
\]

5. Marvelous Coincidences

There are some marvelous coincidences of factors with nuclides in the above formulas. One typical example of these coincidences is listed as follows, which indicates the methodology and the formulas in this paper should be correct.
\[ \alpha_1 \delta^2 (2\pi)_{\text{Chen-2517}} = 1 + \frac{1}{128 \cdot 9 \cdot (2 \cdot 3 \cdot 173 + 1) + \frac{7}{10}} \approx 1 \]

\[ \frac{1}{\alpha_1 \delta^2 (2\pi)_{\text{Chen-2517}}} = 1 - \frac{1}{2 \cdot 5 \cdot 7 \cdot (2 \cdot 83 \cdot 103 + 1) - \frac{9}{29}} \approx 0.999999164529037 \]

\[ (2\pi)_{\text{Chen-2517}} = e^2 \left( \frac{e^2}{(2^1)^3} \right) \left( \frac{e^2}{(2 \cdot 3)^5} \right) \left( \frac{e^2}{(4 \cdot 3)^7} \right) \cdots \left( \frac{e^2}{(2 \cdot 3 \cdot 71)^{2337}} \right) \]

6. Formulas of the Fine-structure Constant based on 103

In our previous paper\(^1\)\(^,\)\(^2\)\(^,\)\(^4\), many formulas of the fine-structure constant based on the key numbers 112, 173, 137, 83 and 29 were given. As shown in the above two formulas in Section 5, it seems 103 is another key number comparable to the above stated key numbers, so some formulas of the fine-structure constant based on the key number 103 instead of them are constructed as follows.

\[ \alpha_1 = \frac{137}{29 \cdot e^2 \left( \frac{e^2}{(2^1)^3} \right) \left( \frac{e^2}{(2 \cdot 3)^5} \right) \left( \frac{e^2}{(4 \cdot 3)^7} \right) \cdots \left( \frac{e^2}{(2 \cdot 3 \cdot 71)^{1033}} \right) 103} + \frac{1}{32 \cdot (32 \cdot 29 + 1) - \frac{3}{2 \cdot 17}} \]

\[ = \frac{137}{29 \cdot e^2 \left( \frac{e^2}{(2^1)^3} \right) \left( \frac{e^2}{(2 \cdot 3)^5} \right) \left( \frac{e^2}{(4 \cdot 3)^7} \right) \cdots \left( \frac{e^2}{(2 \cdot 3 \cdot 71)^{1033}} \right) 103} + \frac{1}{81 \cdot (2 \cdot 3 \cdot 61 + 1) + \frac{31}{2 \cdot 17}} \]

\[ = \frac{137}{29 \cdot e^2 \left( \frac{e^2}{(2^1)^3} \right) \left( \frac{e^2}{(2 \cdot 3)^5} \right) \left( \frac{e^2}{(4 \cdot 3)^7} \right) \cdots \left( \frac{e^2}{(2 \cdot 3 \cdot 71)^{1033}} \right) 103} + \frac{1}{81 \cdot (16 \cdot 23 + 1) - \frac{31}{2 \cdot 17}} \]

\[ = 1/137.035999037435 \]

2021/2/25
\[
\alpha_1 = \frac{137}{29 \cdot 4 \cdot (\frac{2}{3} \cdot 4 \cdot 6 \cdots \frac{1548 \cdot 2 - 25 \cdot 31}{\frac{2}{3} \cdot 5 \cdot 5 \cdots \frac{1549 \cdot 2 - 2 \cdot 9 \cdot 43 + 1}{2 \cdot 3 \cdot 5 \cdot (2 \cdot 3 \cdot 5 \cdot 7 \cdot 13 - 1) - \frac{3}{17}})} + 1 \]

\[
= 1 / 137.03599037435
\]

\[
\alpha_2 = \frac{137}{29 \cdot 8 \cdot (1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots + \frac{1}{2 \cdot 2 \cdot 17 \cdot 29 + 1})} + \frac{1}{103} + \frac{1}{7 \cdot (4 \cdot 7 \cdot 199 + 1) + \frac{4}{7}}
\]

\[
= 1 / 137.03599037435
\]

\[
\alpha_1 = \frac{137}{29 \cdot (6 + \sum_{n=1}^{7} \frac{-1)^{n+1}}{n(n+1/2)(n+1)}} + \frac{1}{103} + \frac{1}{3 \cdot 19} - \frac{1}{125 \cdot (8 \cdot 7 \cdot 11 + 1) + \frac{1}{4}}
\]

\[
= 1 / 137.03599037435
\]
\[
\alpha_2 = \frac{25 \cdot 8 \cdot (1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots - \frac{1}{5 \cdot 23 \cdot 83})}{11 \cdot 19} \frac{1}{103 - \frac{1}{2 \cdot 3 \cdot 5 \cdot 13 \cdot 23 \cdot (2 \cdot 11 \cdot 17 - 1)}} = \frac{1}{137.0359991111818}
\]

\[
\alpha_2 = \frac{25 \cdot (6 + \sum_{n=1}^{11} n(n+1/2)(n+1))}{11 \cdot 19} \frac{1}{103 - \frac{1}{4 \cdot 5 \cdot 23} + \frac{1}{2 \cdot (2 \cdot 3 \cdot 5 \cdot 7 + 1) \cdot (2 \cdot 9 \cdot 17 + 1) + \frac{2}{3}}} = \frac{1}{137.0359991111818}
\]

7. Integrated Formulas of \(\alpha_1, \delta, 2\pi\) and \(\gamma_1\)

By introducing a correction factor \(\gamma_1\), some integrated formulas of \(\alpha_1, \delta, 2\pi\) and \(\gamma_1\) in the format of \(\alpha_1(\delta \gamma_1)^2(2\pi) = 1\) could be obtained as follows.

\[\alpha_1(\delta \gamma_1)^2(2\pi) = 1\]

\[\gamma_1 = \sqrt{2 \pi \alpha_1 \delta} = \sqrt{\frac{6.28564399787948 \cdot 4.66920160910299}{137.035999037435}} = 0.999804818668238\]

\[\gamma_1 = \frac{1}{\sqrt{2 \pi \alpha_1 \delta}} = \frac{1}{\sqrt{\frac{137.035999037435}{2 \cdot 3.14159265358979}} \cdot 4.66920160910299} = 1.00019521943495 \approx 1\]

2021/2/28
\[ \alpha_1(\delta / \gamma_1) = 1 \]

\[ \gamma_1(\delta / \gamma_1) = \sqrt{(2\pi) \cdot \alpha_1} \]

\[ \sqrt{\frac{e^2 + e^2 + e^2 + e^2 + e^2}{2 - 3 \cdot 71}} = \frac{4.66920160910299}{25 \cdot 17} \]

\[ = \sqrt{\frac{6.28564399787948 \cdot 4.66920160910299}{137.035999037435}} \]

\[ = 1 + \frac{1}{2 \cdot 79 \cdot 109 \cdot 139 - \frac{2}{17} \text{or} \frac{3}{25}} = 1.00000041773574 \]

\[ = 1 + \frac{1}{2 \cdot 79 \cdot 109 \cdot 139 - \frac{15}{17} \text{or} \frac{22}{25}} = 0.99999958226432 \]

\[ \gamma_1(\delta / \gamma_1) = \sqrt{(2\pi) \cdot \alpha_1} \]

\[ = \sqrt{\frac{4 \cdot 2 \cdot 4 \cdot 6 \cdot 1278}{3 \cdot 3 \cdot 5 \cdot 5 \cdot 1279 \cdot 2 \cdot 9 \cdot 71 + 1}} = \frac{4.66920160910299}{137.035999037435} \]

\[ = \frac{6.28564399787948 \cdot 4.66920160910299}{137.035999037435} \]

\[ = 1 + \frac{1}{4 \cdot 9 \cdot (8 \cdot 3 \cdot 25 \cdot 7 \cdot 59 - 1)} = 1.00000011209802 \]

\[ \gamma_1(\delta / \gamma_1) = \sqrt{(2\pi) \cdot \alpha_1} \]

\[ = \sqrt{\frac{4 \cdot 2 \cdot 4 \cdot 6 \cdot 1278}{3 \cdot 3 \cdot 5 \cdot 5 \cdot 1279 \cdot 2 \cdot 9 \cdot 71 + 1}} = \frac{4.66920160910299}{137.035999037435} \]

\[ = 1 + \frac{1}{4 \cdot 9 \cdot (8 \cdot 3 \cdot 25 \cdot 7 \cdot 59 - 1)} = 1.00000011209802 \]

\[ \gamma_1(\delta / \gamma_1) = \sqrt{(2\pi) \cdot \alpha_1} \]

\[ = \sqrt{\frac{4 \cdot 2 \cdot 4 \cdot 6 \cdot 1278}{3 \cdot 3 \cdot 5 \cdot 5 \cdot 1279 \cdot 2 \cdot 9 \cdot 71 + 1}} = \frac{4.66920160910299}{137.035999037435} \]

\[ = 1 + \frac{1}{4 \cdot 9 \cdot (8 \cdot 3 \cdot 25 \cdot 7 \cdot 59 - 1)} = 1.00000011209802 \]
\[
\frac{1}{\gamma_{\text{Wallis}}-9.71} = \frac{1}{\sqrt{2\pi} \gamma_{\text{Wallis}}-9.71} \\
= 1 - \frac{1}{5 \cdot 7 \cdot 32 - 27 - 5 \cdot 59 - 1} = 0.99999987901990
\]

\[
\begin{align*}
\alpha_1 (\delta / \gamma_{\text{GL-22.37}})^2 (2\pi)_{\text{GL-22.37}} &= 1 \\
\gamma_{\text{GL-22.37}} &= \sqrt{2\pi} \alpha_1 \delta \\
&= \sqrt{8 \cdot (1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots + \frac{1}{2 \cdot 11 \cdot 37 + 1}) \cdot 4.66920160910299} \\
&= \sqrt{6.28563929398602 \cdot 4.66920160910299} \\
&= \frac{1}{\sqrt{137.035999037435}} \\
&= 1 + \frac{6}{4.25 \cdot 7 \cdot (2 \cdot 23^3 \cdot 31 - 1)} \\
&= 1.00000004355799
\end{align*}
\]

\[
\begin{align*}
\gamma_{\text{GL-22.37}} &= \frac{1}{\alpha_1 \delta} \\
&= 1 - \frac{1}{2.9 \cdot 11 \cdot 47 \cdot (2.9 \cdot 137 + 1) - \frac{5}{11}} \\
&= 0.999999956442012
\end{align*}
\]

\[
\begin{align*}
\alpha_1 (\delta / \gamma_{\text{NC-3}})^2 (2\pi)_{\text{NC-3}} &= 1 \\
\gamma_{\text{NC-3}} &= \sqrt{2\pi} \alpha_1 \delta \\
&= \sqrt{\left(6 + \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n(n+1/2)(n+1)} \right) \cdot 4.66920160910299} \\
&= \frac{6.29047619047619 \cdot 4.66920160910299}{\sqrt{137.035999037435}} \\
&= 2021/3/2
\end{align*}
\]
\[
= 1 + \frac{1}{23 \cdot 113} - \frac{1}{2 \cdot 3 \cdot 257 \cdot (4 \cdot 5 \cdot 29 \cdot 31 + 1)} = 1.0003847273 \times 10^{-4}
\]

2021/3/3

\[
\gamma_1 = \frac{1}{\sqrt{(2\pi)}_{NC-3} \alpha_1 \delta} = \frac{1}{179 - (8 \cdot 9 \cdot (2 \cdot 3 \cdot (2 \cdot 179 + 1) - 1) + 1)} - \frac{3}{7} = 0.999615420653963
\]

8. Integrated Formulas of \(\alpha_2, \alpha, \delta \) and \(\gamma_2\)

By introducing a correction factor \(\gamma_2\), some integrated formulas of \(\alpha_2, \alpha, \delta \) and \(\gamma_2\) in the format of \(\alpha_2 (\delta \alpha \gamma_2)^2 = 1\) could be obtained as follows.

\[
\begin{align*}
\alpha_2 (\delta \alpha \gamma_2)^2 & = 1 \\
\gamma_2 & = \sqrt{\alpha_2 (\alpha \delta)} = \frac{4.66920160910299 \cdot 2.50290787509589}{\sqrt{137.035999111818}} \\
& = 1 - \frac{1}{5 \cdot 7 \cdot 17} + \frac{1}{4 \cdot 3 \cdot 17 \cdot 23 \cdot 137 - \frac{11}{59}} = 0.998320883415699
\end{align*}
\]

2021/2/28

\[
\begin{align*}
\gamma_2 & = \frac{1}{\sqrt{\alpha_2 \delta \alpha}} = \frac{4.66920160910299 \cdot 2.50290787509589}{\sqrt{137.035999111818}} \\
& = 1 + \frac{1}{2 \cdot 27 \cdot 11} - \frac{1}{5 \cdot 17 \cdot (16 \cdot 3 \cdot 157 + 1) + 16} = 1.00168194075892
\end{align*}
\]

2021/2/27

\[
\begin{align*}
\gamma_2 & = \frac{1}{\sqrt{\alpha_2 \delta \alpha}} = \frac{4.66920160910299 \cdot 2.50290787509589}{\sqrt{137.035999111818}} \\
& = 1 + \frac{1}{2 \cdot 27 \cdot 11} - \frac{1}{5 \cdot 17 \cdot (16 \cdot 3 \cdot 157 + 1) + 16} = 1.00168194075892
\end{align*}
\]
9. Integrated Formulas of $\alpha_1$, $\alpha_2$, $\alpha$, $2\pi$, $\gamma_1$ and $\gamma_2$

\[
\frac{\alpha_1(2\pi)}{\alpha_2(\gamma_1 / \gamma_2)} = 1
\]

\[
\gamma_1 = \sqrt{\frac{137.035999111818 \cdot (2 \cdot 3.14159265358979)}{137.035999037435 \cdot 2.50290787509589}}
\]

\[
\gamma_2 = 1 + \frac{1}{32 \cdot 3 \cdot 7} - \frac{1}{32 \cdot 89 \cdot (4 \cdot 53 - 1) - \frac{25}{2 \cdot 23}} = 1.00148643114372
\]

\[
\gamma_1 = \frac{2.50290787509589}{137.035999037435} = 1.00148643114372
\]

\[
\gamma_2 = 1 + \frac{1}{32 \cdot 3 \cdot 7} + \frac{1}{4 \cdot 53 \cdot (2 \cdot 49 - 29 + 1) + \frac{2 \cdot 3}{23}} = 0.99851577505446
\]

10. Integrated Formulas $a$, $2\pi$ and $\gamma$

By introducing a correction factor $\gamma$, some integrated formulas of $2\pi$, $\alpha$ and $\gamma$ in the format of $2\pi/(\alpha \gamma)^2 = 1$ could be obtained as follows.

\[
\frac{2\pi}{(\alpha \gamma)^2} = 1
\]

\[
\gamma = \sqrt{\frac{2 \cdot 3.14159265358979}{2.50290787509589}}
\]

\[
= 1 + \frac{1}{32 \cdot 3 \cdot 7} - \frac{1}{23 \cdot 151 \cdot 173 + \frac{9}{4 \cdot 7}} = 1.00148643087192
\]

\[
\gamma = 1 + \frac{1}{32 \cdot 3 \cdot 7 + 1} + \frac{1}{2 \cdot 41 \cdot (2 \cdot 3 \cdot 25 \cdot 49 - 1) - \frac{1}{6}} = 0.99851577532546
\]

2021/3/3
11. Summary

The above integrated formulas of the fine-structure constant and Feigenbaum constants are summarized as follows.

The Fine-structure Constant:

\[
\alpha_1 = \frac{\frac{\lambda_e}{2\pi a_0}}{7 \cdot (2\pi)_{Chen-112}} = 1/137.035999037435127 \quad (2) \\
\alpha_2 = \frac{2\pi r_e}{\lambda_e} = \frac{13 \cdot (2\pi)_{Chen-278}}{100 - \frac{1}{112 - \frac{1}{64 \cdot 3 \cdot 29}}} = 1/137.035999111818
\]

Feigenbaum Constants: \( \delta = 4.66920160910299 \)
\( \alpha = 2.50290787509589 \)

\( 2\pi = 2 \cdot 3.14159265358979 \)

\[
\alpha_1 (\delta / \gamma_1)^2 (2\pi) = 1
\]

\[
\gamma_1 = 1 + \frac{1}{47 \cdot 109} - \frac{1}{27 \cdot 7 \cdot (3 \cdot 8 \cdot (3 \cdot 8 \cdot (4 \cdot 137 - 1) - 1) - 1)} = 0.999804818668238
\]

\[
\frac{1}{\gamma_1} = 1 + \frac{1}{16 \cdot 7 \cdot 17 \cdot (16 \cdot 3 \cdot 23 - 1)} = 1.00019521943495
\]

\[
\gamma_1^2 = 1 + \frac{1}{13 \cdot 197} - \frac{1}{2 \cdot 7 \cdot 41 \cdot (4 \cdot 5 \cdot 19 - 31 - 1)} = 0.9996097543223
\]

\[
\frac{1}{\gamma_1^2} = 1 + \frac{1}{512 - \frac{1}{4 \cdot 9 \cdot 7 \cdot 17 \cdot 19 \cdot 83}} = 1.00039047698053
\]

\[
\alpha_1 (\delta / \gamma_{1-Chen-2517})^2 (2\pi)_{Chen-2517} = 1
\]

\[
(2\pi)_{Chen-2517} = \frac{e^2}{(2 \cdot 3)} \quad \frac{e^2}{(2 \cdot 3)} \quad \frac{e^2}{(2 \cdot 3)} \quad \frac{e^2}{(2 \cdot 3 \cdot 71)} = 6.28564399787948
\]

\[
\gamma_{1-Chen-2517} = 1 + \frac{1}{2 \cdot 79 \cdot 109 \cdot 139 - \frac{3}{25}} = 1.00000041773574
\]

\[
\frac{1}{\gamma_{1-Chen-2517}} = 1 - \frac{1}{2 \cdot 79 \cdot 109 \cdot 139 + \frac{22}{25}} = 0.999999582264432
\]

\[
\gamma_{1-Chen-2517}^2 = 1 + \frac{1}{128 \cdot 9 \cdot (2 \cdot 3 \cdot 173 + 1) + \frac{7}{10}} = 1.0000008354717
\]

\[
\frac{1}{\gamma_{1-Chen-2517}^2} = 1 - \frac{1}{2 \cdot 5 \cdot 7 \cdot (2 \cdot 83 \cdot 103 + 1) - \frac{9}{29}} = 0.999999164529037 \approx 1
\]
\[ \alpha_1 \left( \frac{\delta}{\gamma_{\text{Wallis}-971}} \right)^2 \left( 2\pi \right)_{\text{Wallis-971}} = 1 \]

\[ (2\pi)_{\text{Wallis-971}} = 4 \cdot \left( \frac{2 \cdot 4 \cdot 6 \cdots 1278 \cdot 1280}{3 \cdot 5 \cdot 7 \cdot 1279 \cdot 2 \cdot 9 \cdot 71 + 1} \right) = 6.28564015562186 \]

\[ \gamma_{\text{Wallis-971}} = 1 + \frac{1}{4 \cdot 9 \cdot (8 \cdot 3 \cdot 25 \cdot 7 \cdot 59 - 1)} = 1.00000011209802 \]

\[ \frac{1}{\gamma_{\text{Wallis-971}}} = 1 - \frac{1}{5 \cdot 7 \cdot (32 \cdot 27 \cdot 5 \cdot 59 - 1)} = 0.99999887901990 \]

\[ \gamma_{\text{Wallis-971}}^2 = 1 + \frac{1}{2 \cdot 9 \cdot (8 \cdot 3 \cdot 25 \cdot 7 \cdot 59 - 1) - \frac{5}{17}} = 1.00000022419606 \]

\[ \frac{1}{\gamma_{\text{Wallis-971}}}^2 = 1 - \frac{1}{2 \cdot 9 \cdot (8 \cdot 3 \cdot 25 \cdot 7 \cdot 59 - 1) + \frac{3 \cdot 4}{17}} = 0.9999977580399 \]

\[ \alpha_1 \delta^2 \left( \frac{2\pi}{\gamma_{\text{GL-2237}}} \right)_{\text{GL-2237}} = 1 \]

\[ (2\pi)_{\text{GL-2237}} = 8 \cdot \left( 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots + \frac{1}{2 \cdot 2 \cdot 11 \cdot 37 + 1} \right) = 6.28563929398602 \]

\[ \gamma_{\text{GL-2237}} = 1 + \frac{1}{4 \cdot 25 \cdot 7 \cdot (2 \cdot 23^2 \cdot 31 - 1) + \frac{6}{11}} = 1.0000004355799 \]

\[ \frac{1}{\gamma_{\text{GL-2237}}} = 1 - \frac{1}{2 \cdot 9 \cdot 11 \cdot 47 \cdot (2 \cdot 9 \cdot 137 + 1) + \frac{5}{11}} = 0.99999956442012 \]

\[ \gamma_{\text{GL-2237}}^2 = 1 + \frac{1}{163 \cdot (6 \cdot 11^2 \cdot 97 + 1) + \frac{9}{10}} = 1.0000008711598 \]

\[ \frac{1}{\gamma_{\text{GL-2237}}}^2 = 1 - \frac{1}{9 \cdot 11 \cdot 47 \cdot (2 \cdot 9 \cdot 137 + 1) - \frac{2}{25}} = 0.99999912884025 \]

\[ \alpha_1 \left( \frac{\delta}{\gamma_{\text{NC-3}}} \right)^2 \left( 2\pi \right)_{\text{NC-3}} = 1 \]

\[ (2\pi)_{\text{NC-3}} = (6 + \sum_{n=1}^{\infty} \frac{(-1)^n+1}{n(n+1/2)(n+1)}) = 6.29047619047619 \]

\[ \gamma_{\text{NC-3}} = 1 + \frac{1}{23 \cdot 113 - 2 \cdot 3 \cdot 257 \cdot (4 \cdot 5 \cdot 29 \cdot 31 + 1)} = 1.00038472730421 \]

\[ \frac{1}{\gamma_{\text{NC-3}}} = 1 - \frac{1}{8 \cdot 25 \cdot 13 + 179 \cdot (8 \cdot 9 \cdot (2 \cdot 3 \cdot (2 \cdot 179 + 1) - 1) + 1) - \frac{3}{7}} = 0.999615420653963 \]
\[ \gamma_{1-NC}^2 = 1 + \frac{1}{3 \cdot (2 \cdot 7 \cdot 31 - 1)} - \frac{1}{13 \cdot 89 \cdot (2 \cdot 37 \cdot 53 + 1) - \frac{2 \cdot 9}{25}} = 1.00076960262352 \]

\[ \frac{1}{\gamma_{1-NC}^2} = 1 - \frac{1}{4 \cdot 25 \cdot 13} + \frac{1}{4 \cdot 9 \cdot 25 \cdot (2 \cdot 25 \cdot (4 \cdot 25 + 1) + 1} + \frac{2}{7} = 0.999230989209198 \]

\[ \alpha_2 (\delta \alpha / \gamma_2)^2 = 1 \]

\[ \gamma_2 = 1 + \frac{1}{5 \cdot 7 \cdot 17} + \frac{1}{4 \cdot 3 \cdot 17 \cdot 23 \cdot 137 - \frac{11}{59}} = 0.998320883415699 \]

\[ \frac{1}{\gamma_2} = 1 + \frac{1}{2 \cdot 27 \cdot 11} - \frac{1}{5 \cdot 17 \cdot (16 \cdot 3 \cdot 157 + 1) + \frac{16}{17}} = 1.00168194075892 \]

\[ \gamma_2^2 = 1 + \frac{1}{2 \cdot 149} + \frac{1}{29 \cdot 31 \cdot (2 \cdot 3 \cdot 49 + 1) - \frac{16}{19}} = 0.996644586263908 \]

\[ \frac{1}{\gamma_2^2} = 1 + \frac{1}{2 \cdot 27 \cdot 11} - \frac{1}{2 \cdot 3 \cdot 25 \cdot 11 \cdot (4 \cdot 11 \cdot 47 + 1) - \frac{2}{5}} = 1.00336671044256 \]

\[ \frac{\alpha_3 (2\pi)}{\alpha_2 (\alpha \gamma_1 / \gamma_2)^2} = 1 \]

\[ \gamma_1 = 1 + \frac{1}{32 \cdot 3 \cdot 7} - \frac{1}{32 \cdot 89 \cdot (4 \cdot 53 - 1) - \frac{25}{23}} = 1.00148643114372 \]

\[ \frac{\gamma_2}{\gamma_1} = 1 - \frac{1}{32 \cdot 3 \cdot 7 + 1} + \frac{1}{4 \cdot 53 \cdot (2 \cdot 49 + 1) + \frac{2 \cdot 3}{23}} = 0.99851577505446 \]

\[ \left( \frac{\gamma_2}{\gamma_1} \right)^2 = 1 + \frac{1}{16 \cdot 3 \cdot 7} - \frac{1}{2 \cdot 3 \cdot 7 \cdot (4 \cdot 17 \cdot (2 \cdot 157 - 1)) - \frac{23}{4}} = 1.00297507176499 \]

\[ \left( \frac{\gamma_2}{\gamma_1} \right)^2 = 1 + \frac{1}{16 \cdot 3 \cdot 7 + 1} + \frac{1}{7 \cdot 19 \cdot (2 \cdot 3 \cdot 7 \cdot 23 - 1) + \frac{16}{3 \cdot 13}} = 0.997033753032614 \]
\[
\frac{2\pi}{(\alpha\gamma)} = 1
\]
\[
\gamma = 1 + \frac{1}{32 \cdot 3.7} - \frac{1}{23 \cdot 151} + \frac{9}{4 \cdot 7} = 1.00148643087192
\]
\[
\frac{1}{\gamma} = 1 - \frac{1}{32 \cdot 3.7 + 1} + \frac{1}{2 \cdot 13 \cdot 31 \cdot (2 \cdot 9 \cdot 41 + 1) - \frac{1}{16} = 0.99851577532546
\]
\[
\gamma^2 = 1 + \frac{1}{16 \cdot 3.7} - \frac{1}{3 \cdot 13 \cdot 31 \cdot (2 \cdot 9 \cdot 41 + 1) - \frac{1}{16} = 1.0029750712205
\]
\[
\frac{1}{\gamma^2} = 1 - \frac{1}{16 \cdot 3.7 + 1} + \frac{1}{8 \cdot 81 \cdot 19 \cdot 73 + \frac{1}{3 \cdot 13} = 0.997033753573803
\]

12. Some Supplement Formulas of Feigenbaum Constants

\[
(\delta\alpha)^2 = (4.66920160910299 \cdot 2.50290787509589)^2 = 136.576186638059
\]
\[
100 \cdot (112 + \frac{1}{3 \cdot 5 \cdot 73} - \frac{1}{2 \cdot 5 \cdot 73 \cdot (32 \cdot 3 \cdot 23 - 1)}) = 13 \cdot e^2 \left( 2 \alpha \right)^2 \left( \frac{2}{3} \right)^2 \left( \frac{4}{3} \right)^2 \cdots \left( \frac{42}{41} \right)^2
\]

\[
\begin{align*}
24.25, 26 & \quad Mg \quad Al \quad K \quad Ti \quad V \quad Cr \quad Mn \quad Fe \\
7.32 & \quad Ge \quad As \quad Sb \quad Bi \\
47.109 & \quad Ag \quad Cd \quad Hg \\
157.16 & \quad Gd \quad Tb \quad Dy \\
235.23 & \quad U \quad Th \quad Pa \\
(\delta\alpha)^2 = (4.66920160910299 \cdot 2.50290787509589)^2 = 136.57618663 \cdot 1518059
\end{align*}
\]
\[
100 \cdot (112 - \frac{1}{2 \cdot 53} + \frac{1}{2 \cdot 5 \cdot 19 \cdot 181}) = 13 \cdot e^2 \left( 2 \alpha \right)^2 \left( \frac{2}{3} \right)^2 \left( \frac{4}{3} \right)^2 \cdots \left( \frac{43}{42} \right)^2
\]

\[
\begin{align*}
19 \quad F \quad 27 & \quad Al \quad 14 \quad Si \quad 39, 40, 41 \\
5.10 & \quad Mg \quad 13, 14 \quad K \quad 19 \quad Ca \\
4, 14, 15, 16 & \quad Al \quad 14 \quad K \quad 19 \quad V \\
63, 65 & \quad Sc \quad 14, 15, 16 \quad Ca \quad 50, 51 \quad Sc \\
14, 15, 16 & \quad Si \quad 39, 40, 41 \quad V \quad 50, 51 \quad Sc \\
33, 34 & \quad Ti \quad 24, 25, 26 \quad Cr \quad 52, 53 \quad Ti \\
145 & \quad V \quad 50, 51 \quad Sc \\
60, 62, 64 & \quad Cr \quad 52, 53 \quad Ti \\
92, 94 & \quad Sc \quad 33, 34 \quad Ti \quad 24, 25, 26 \quad Cr \\
181 & \quad V \quad 50, 51 \quad Sc \\
68, 70, 72 & \quad Cr \quad 52, 53 \quad Ti \\
110 & \quad V \quad 50, 51 \quad Sc \\
108 & \quad Cr \quad 52, 53 \quad Ti \\
125 & \quad V \quad 50, 51 \quad Sc \\
110 & \quad Cr \quad 52, 53 \quad Ti \\
108 & \quad V \quad 50, 51 \quad Sc \\
125 & \quad Cr \quad 52, 53 \quad Ti \\
110 & \quad V \quad 50, 51 \quad Sc \\
108 & \quad Cr \quad 52, 53 \quad Ti \\
125 & \quad V \quad 50, 51 \quad Sc \\
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108 & \quad V \quad 50, 51 \quad Sc \\
125 & \quad Cr \quad 52, 53 \quad Ti \\
110 & \quad V \quad 50, 51 \quad Sc \\
108 & \quad Cr \quad 52, 53 \quad Ti \\
125 & \quad V \quad 50, 51 \quad Sc \\
110 & \quad Cr \quad 52, 53 \quad Ti \\
108 & \quad V \quad 50, 51 \quad Sc \\
125 & \quad Cr \quad 52, 53 \quad Ti \\
110 & \quad V \quad 50, 51 \quad Sc \\2021/3/23
\end{align*}
\]
\[(\delta\alpha)^2 = (4.66920160910299 \cdot 2.50290787509589)^2 = 136.576186638059\]

\[
11.19 \cdot (103 + \frac{1}{2 \cdot 109} - \frac{1}{4 \cdot 3.11 \cdot (4 \cdot 13^2 + 1) + \frac{19}{30}})
\]

\[
= 25 \cdot e^2 \left( \frac{e^2}{2} \left( \frac{\varepsilon^2}{3} \left( \frac{4^2}{3^5} \left( \frac{48}{47} \right)^{19} \right) \right) \right)
\]

\[
11.19 \cdot (103 + \frac{1}{2 \cdot 109} - \frac{1}{4 \cdot 3.11 \cdot (6 \cdot 113 - 1) + \frac{19}{30}})
\]

\[
= 25 \cdot e^2 \left( \frac{e^2}{2} \left( \frac{\varepsilon^2}{3} \left( \frac{4^2}{3^5} \left( \frac{48}{47} \right)^{19} \right) \right) \right)
\]

2021/3/24

\[(\delta\alpha)^2 = (4.66920160910299 \cdot 2.50290787509589)^2 = 136.576186638059\]

\[
= 137 - \frac{1}{2} + \frac{1}{13} - \frac{1}{23 \cdot 59} + \frac{1}{8 \cdot 3 \cdot 19 \cdot (16 \cdot 3 \cdot 5 \cdot 19 + 1)}
\]

\[
= 137 - \frac{1}{2} + \frac{1}{13} - \frac{1}{23 \cdot 59} + \frac{1}{8 \cdot 3 \cdot 19 \cdot (16 \cdot 3 \cdot 5 \cdot 19 + 1)}
\]

2021/3/24

\[(\delta\alpha)^2 = (4.66920160910299 \cdot 2.50290787509589)^2 = 136.576186638059\]

\[
= 137 - \frac{1}{2} + \frac{1}{13} - \frac{1}{23 \cdot 59} + \frac{1}{8 \cdot 3 \cdot 19 \cdot (16 \cdot 3 \cdot 5 \cdot 19 + 1)}
\]

2021/3/24

\[(\delta\alpha)^2 = (4.66920160910299 \cdot 2.50290787509589)^2 = 136.576186638059\]
\[
\delta^2 = (4.66920160910299)^2 = 21.8014436664500
\]

\[
7 \times (112 + \frac{1}{8} - \frac{1}{3 \cdot 101} + \frac{1}{3 \cdot 31 - (4 \cdot 11 - 23 + 1) + \frac{4}{7}})
= 21.8014436664499
\]

\[
\delta^2 = (4.66920160910299)^2 = 21.8014436664500
\]

\[
29 \times (103 - \frac{1}{11 - 13} + \frac{1}{2 \cdot 131 - 191 + 9 \cdot \frac{5}{11}})
= \frac{137}{137}
\]

\[
\delta^2 = (4.66920160910299)^2 = 21.8014436664500
\]

\[
22 \times \frac{1}{5} + \frac{1}{4 \cdot 137} - \frac{1}{11^2 \cdot 23^2 - \frac{4}{19}}
\]

\[
\delta^2 = (4.66920160910299)^2 = 21.8014436664500
\]

\[\text{2021/4/5}\]

\[\text{2021/3/23}\]

\[\text{2021/3/24}\]

\[\text{2021/4/3}\]
\[ \delta^2 = (4.66920160910299)^2 = 21.80144366664500 \]

\[ = 22 - \frac{1}{5 + \frac{1}{27 + \frac{1}{1 + \frac{1}{34 + \frac{1}{13 + \frac{1}{47 + \frac{1}{2 \cdot 17 \cdot 19}}}}}}} \]

\[ = 22 - \frac{1}{5 + \frac{1}{27 + \frac{1}{1 + \frac{1}{34 + \frac{1}{47}}}}} \]

\[ = 22 - \frac{1}{5 + \frac{1}{139 \cdot 163}} \]

\[ = 22 - \frac{1}{5 + \frac{139 \cdot 163}{4 \cdot 13 \cdot (2 \cdot 3 \cdot 11 \cdot 13 + 1)}} \]

\[ = 22 - \frac{1}{5 + \frac{139 \cdot 163}{22}} \]

\[ = 22 - \frac{1}{5 + \frac{139 \cdot 163}{22}} \]

\[ = 22 - \frac{1}{5 + \frac{139 \cdot 163}{22}} \]

\[ = 22 - \frac{1}{5 + \frac{139 \cdot 163}{22}} \]

\[ = 22 - \frac{1}{5 + \frac{139 \cdot 163}{22}} \]

\[ = 22 - \frac{1}{5 + \frac{139 \cdot 163}{22}} \]
References:

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## Appendix I: Research History

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
<th>Date</th>
<th>Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>Abstract</td>
<td>1</td>
<td>2021/2/20</td>
<td>Hanyuan</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>2021/2/20-21</td>
<td></td>
</tr>
<tr>
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<td></td>
<td>2021/1/31</td>
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<tr>
<td>2</td>
<td>3-6</td>
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<tr>
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<td></td>
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<tr>
<td>3</td>
<td>6-7</td>
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<td>Chengdu</td>
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<td>7</td>
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<tr>
<td>5</td>
<td>8-9</td>
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<td>6</td>
<td>9-11</td>
<td>2021/2/25-26</td>
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</tr>
<tr>
<td>7</td>
<td>11-14</td>
<td>2021/2/28-3/3</td>
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<td>15</td>
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<td>15</td>
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<tr>
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<td>19-22</td>
<td>2021/3/23-4/5</td>
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<td>22-23</td>
<td>2021/4/4-5</td>
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<td>1-24</td>
<td>2021/1/31-4/5</td>
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Note: Time was recorded according to Beijing Time.