Elementary Equations

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ABSTRACT

In this note we give some series for the expression:

$$S=\frac{\pi}{4}\sqrt{\frac{\sqrt{13}-1}{78}}+\sqrt{\frac{\sqrt{13}-1}{78}}\tan^{-1}\left(\frac{\sqrt{5+2\sqrt{13}}-3}{\sqrt{5+2\sqrt{13}}+3}\right)+\sqrt{\frac{\sqrt{13}+1}{78}}\ln\left(\frac{1+\sqrt{13}+\sqrt{2\sqrt{13}-2}}{4}\right)$$

Series

$$S=\frac{4}{13}\sum_{n=0}^{\infty}(-1)^n\left(\frac{2}{13}\right)^n\sum_{k=0}^{n}\binom{n}{k}\frac{2^{-k}}{2n+2k+1} \tag{1}$$

$$S=\frac{4}{13}\sum_{n=0}^{\infty}\frac{(-1)^n}{2n+1}\left(\frac{2}{13}\right)^n\sum_{k=0}^{n}(-1)^k\binom{n-k}{k}\left(\frac{13}{4}\right)^k \tag{2}$$

$$S=\frac{1}{3}\sum_{n=0}^{\infty}(-1)^n\left(\frac{1}{12}\right)^n\sum_{k=0}^{2n}\binom{2n}{k}\frac{1}{2k+1} \tag{3}$$

$$S=\frac{8}{13}\sum_{n=0}^{\infty}\binom{2n}{n}\frac{13^{-n}}{n+1}\sum_{k=0}^{n}\frac{(-3)^k\,4^{n-k}\binom{n}{k}}{\binom{4n-2k+2}{2n-k+1}\binom{2n-k+1}{n-k}} \tag{4}$$

$$S=\frac{8}{13}\sum_{n=0}^{\infty}\left(-\frac{8}{13}\right)^n\frac{1}{(2n+1)!\binom{4n+2}{2n+1}}\sum_{k=0}^{n}\left(\frac{3}{8}\right)^k\binom{n}{k}\binom{2n+2k}{n+k}(n+k)!(n-k)! \tag{5}$$

$$S=\frac{1}{4}+\frac{16}{13^2}\sum_{n=0}^{\infty}\left(-\frac{2}{13}\right)^n(n+1)\sum_{k=0}^{n}\binom{n}{k}2^{-k}\left(\frac{1}{2n+2k+3}+\frac{1}{2n+2k+5}\right) \tag{6}$$

$$S=\frac{4}{13}\sum_{n=0}^{\infty}\frac{(-1)^n}{4n+1}\left(\frac{1}{13}\right)^n F\left(n+1,2n+\frac{1}{2},2n+\frac{3}{2},-\frac{2}{13}\right) \quad (7)$$

$$S=\frac{4}{13}\sum_{n=0}^{\infty}\frac{(-1)^n}{2n+1}\left(\frac{2}{13}\right)^n F\left(n+1,\frac{2n+1}{4},\frac{2n+5}{4},-\frac{1}{13}\right) \quad (8)$$

$$S=\frac{4}{13}\sum_{n=0}^{\infty}\frac{1}{2n+1}\left(\frac{2\sqrt{13}-2}{13}\right)^n F\left(2n+2,n+\frac{1}{2},n+\frac{3}{2},-\frac{1}{\sqrt{13}}\right) \quad (9)$$

$$S=\frac{4}{13}\sum_{n=0}^{\infty}\frac{c_n}{2n+1}\left(\frac{1}{52}\right)^n \quad (10)$$

where

$$c_{n+2}=-8c_{n+1}-208c_n \quad ,c_0=1,c_1=-8 \quad (11)$$

$$S=\frac{i}{\sqrt{3}}\sum_{n=0}^{\infty}\frac{(-1)^n}{2n+1}\left(\frac{1}{\left(1+2i\sqrt{3}\right)^{n+1}}-\frac{1}{\left(1-2i\sqrt{3}\right)^{n+1}}\right) \quad (12)$$

Remarks: $F(a,b,c,x)$ is the hypergeometric function , $i=\sqrt{-1}$, $\frac{\pi}{4}=\sum_{n=0}^{\infty}\frac{(-1)^n}{2n+1}$.

References

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