How is the Ehrenfest paradox resolved?

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Abstract

The Ehrenfest paradox relates to the relativistic description of the geometry of a rotating rigid disk. Ehrenfest has formulated this paradox in 1909 based on special relativity. I claim that the paradox can be resolved when taking into account both special and general relativity.

The Ehrenfest paradox relates to the relativistic description of the geometry of a rotating rigid disk. Ehrenfest has formulated this paradox in 1909 in the context of Einstein's special relativity theory (SRT) published in 1905. The paradox is that there is a discrepancy between the Euclidean circumference vs. the circumference calculated by SRT based on Lorentz concentration.

Since then, many (including Einstein) have tried to explain this paradox. However, there is no clear resolution to this paradox.

More details in Ehrenfest_paradox

I claim that this problem can be solved if one recognizes that it is an error to use only SRT formulas in the case of a rotating rigid disk. SRT has two postulates. The first postulate is:

The laws of physics take the same form in all inertial frames of reference.

(Note: The inertial frame of reference is a frame of reference that is not undergoing acceleration).

In the case of the rotating disk, there is a centripetal force exerted on every point of the disk. Or in other words, every point of the disk is undergoing acceleration. Therefore, for describing a rotating disk, in addition to SRT also General Relativity (GRT) effects must be included.

When considering SRT and GRT it is shown here that there is no length contraction of the rigid disk circumference That is, there is no paradox.

1. Length contraction according to SRT

$$L_{SR} = L_0 \cdot \sqrt{1-\frac{v^2}{c^2}}$$

Where:

L_{SR} - Contracted length

L_0 - Proper length of circumference

v- Tangential velocity at circumference

2. Time dialation according to GRT

A clock in a gravitational field runs more slowly according to gravitational time dialation:

$$T = \frac{T_0}{\sqrt{1-\frac{g \cdot h}{c^2}}}$$

Where:

g- Gravitational field

h- Distance source to measured body

In the disk there is an acceleration a rather than g. According to the equivalence principle g can be replaced by a.

Where: $a = \frac{v^2}{R}$; h=R

R – Radius of disk

Therefore:

$$T_{GR} = \frac{T_0}{\sqrt{1-\frac{g \cdot h}{c^2}}} = \frac{T_0}{\sqrt{1-\frac{a \cdot R}{c^2}}} = \frac{T_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

3. Length contraction according to GRT

$$L_{GR} = L_0 \cdot \sqrt{1-\frac{v^2}{c^2}}$$

Where:

L_{GR} - Contracted length

L_0 - Proper length of circumference

v- Tangential velocity at circumference

4. Conclusion: The total length contraction of the rigid disk is null:

$$\Delta L = L_{SR} - L_{GR} = L_0 \cdot \sqrt{1-\frac{v^2}{c^2}} - L_0 \cdot \sqrt{1-\frac{v^2}{c^2}} = 0$$