A mass-without-mass model of protons and neutrons

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Abstract

This papers offers a zbw (mass-without-mass) model of neutrons and protons. The neutron model is based on the idea of the electromagnetic and nuclear force combining to keep two opposite charges apart and together at the same time. We develop the orbital energy equations. Finally, we offer an alternative particle classification based on form factors, and a few words of philosophy (ontology).

Contents

Introduction

'Mass without mass' models analyze elementary particles as harmonic oscillations whose total energy – at any moment (KE + PE) or over the *cycle* – is given by E = m·a^2·ω^2. One can calculate the radius or amplitude of the oscillation directly from the mass-energy equivalence and Planck-Einstein relations, as well as the tangential velocity formula—interpreting *c* as a tangential or orbital (escape) velocity.

$$\left.\begin{array}{r}\left.\begin{array}{r}\mathrm{E} = \mathrm{m}c^2\\ \mathrm{E} = \hbar\omega\end{array}\right\} \Rightarrow \mathrm{m}c^2 = \hbar\omega\\ c = a\omega \Leftrightarrow a = \dfrac{c}{\omega} \Leftrightarrow \omega = \dfrac{c}{a}\end{array}\right\} \Rightarrow \mathrm{m}a^2\omega^2 = \hbar\omega \Longrightarrow \mathrm{m}\frac{c^2}{\omega^2}\omega^2 = \hbar\frac{c}{a} \Leftrightarrow a = \frac{\hbar}{\mathrm{m}c}$$

Such models assume a centripetal force whose *nature*, in the absence of a charge at the center, can only be explained with a reference to the quantized energy levels we associate with atomic or molecular electron orbitals[1], and the *physical* dimension of the oscillation in space and time may effectively be understood as a *quantization* of spacetime.

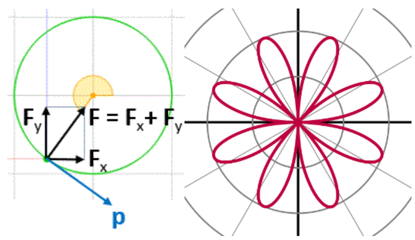

Figure 1: Circular and elliptical orbital motion[2]

The model is based on the assumption of a pointlike charge[3] with no other properties but its charge (zero rest mass). However, this zero-mass point charge acquires an *effective* mass which accounts for half of the energy of the elementary particle: the other half of the energy is in the (electromagnetic or nuclear) field which sustains the motion of the charge. As such, the pointlike *Zitterbewegung* (zbw) charge is photon-like but, unlike a photon, it carries (electric) charge.

The motion is not necessarily circular: one may imagine elliptical orbitals, such as depicted by the polar rose in the illustration.[4] The $r(\varphi) = a_0 \cdot \cos(k_0\varphi + \gamma_0)$ equation gives us the radial distance *r* as a function of

[1] See, for example, Feynman's analysis of quantized energy levels or his explanation of the size of an atom. As for the question why such elementary *currents* do not radiate their energy out, the answer is the same: persistent currents in a superconductor do not radiate their energy out either. The general idea is that of a *perpetuum mobile* (no external driving force or frictional/damping terms). For an easy mathematical introduction, see Feynman, Chapter 21 (the harmonic oscillator) and Chapter 23 (resonance).

[2] Illustrations made from Wikipedia templates. For the orbital equations, see the MIT OCW reference course on orbital motion.

[3] Pointlike but not of zero dimension. See our explanation of the anomaly in the magnetic moment of an electron in, for example, our paper on the basics.

[4] The Rutherford-Bohr model considers circular orbitals only. Schrödinger's wave equation adds non-circular (non-spherically symmetric) orbitals as solutions.

the phase $\varphi = \varphi(t) = \omega \cdot t$ of the oscillation. Thinking of r as a vector in 2D space (the plane of motion), we get a wavefunction:

$$\boldsymbol{r} = a_0 e^{i\cdot(\mathrm{k}_0\varphi+\gamma_0)} = a_0\cos(\mathrm{k}_0\varphi+\gamma_0) + i\cdot a_0\sin(\mathrm{k}_0\varphi+\gamma_0)$$

For an electron, we get the following energy-mass calculation:

$$\frac{\mathrm{E}}{\mathrm{m}} = c^2 = \frac{v^2}{2} - \frac{\mathrm{k_e q_e^2}}{\mathrm{m}r}$$

This yields the following equation:

$$c^2 = \frac{v^2}{2} - \frac{\mathrm{k_e q_e^2}}{\mathrm{m}r} \Leftrightarrow 1 = \frac{v^2}{2c^2} - \frac{\mathrm{k_e q_e^2}}{\mathrm{m}c^2 r} \Leftrightarrow \mathrm{E} = \mathrm{m}c^2 = \frac{1}{2}\mathrm{m}v^2 - \frac{\mathrm{k_e q_e^2}}{r}$$

This represents the E = KE + PE energy conservation equation. The velocity v is an orbital or *tangential* velocity[5] and the $\mathrm{m}v^2/2$ formula for the kinetic energy is, therefore, relativistically correct.

We get the radius or amplitude of the oscillation from the $\mathrm{E} = \mathrm{m}\cdot a^2\cdot\omega^2$ equation:

$$a^2 = \frac{\mathrm{E}}{\mathrm{m}\omega^2} = \frac{\mathrm{E}}{\mathrm{m}(\frac{\mathrm{E}}{\hbar})^2} = \frac{\hbar^2}{\mathrm{m}^2c^2} \Leftrightarrow a = \frac{\hbar}{\mathrm{m}c}$$

We may interpret the positive and negative root of $\hbar^2/\mathrm{m}^2c^2$ as the two possibilities that correspond to the *direction* of angular momentum, which distinguishes an electron from a positron.[6]

This formula misses the 1/2 factor of the effective mass m_γ, which is *half* the total mass ($\mathrm{m} = \mathrm{E}/c^2$) of the elementary particle (m or E), and which explains why elementary (charged) particles are spin-1/2 particles, as shown by the calculation below:

$$L = I\cdot\omega = \mathrm{m}_\gamma\cdot a^2\cdot\omega = \frac{\mathrm{m_e}}{2}\cdot a^2\cdot\omega = \frac{\mathrm{m_e}}{2}\cdot\frac{\hbar^2}{\mathrm{m_e^2}\cdot c^2}\frac{\mathrm{E}}{\hbar} = \frac{\hbar}{2}$$

The momentum is, of course, orbital angular momentum only. As such, we are essentially modeling spin-zero (zero *spin* angular momentum) particles. We should, of course, note that a moving charge is a current, which explains the magnetic moment[7]:

$$\mu = \mathrm{I}\cdot\pi r_{\mathrm{C}}{}^2 = \frac{\mathrm{q_e}}{2\mathrm{m}}\hbar$$

[5] This velocity is sometimes referred to as the *escape* velocity, but the terms are not to be used interchangeably because they may refer to subtly different things (e.g. the velocity component with right angle to the semi-major/minor axis of the ellipse). Needless to say, m is the *relativistic* mass.

[6] The antimatter counterpart of an elementary particle has opposite angular momentum but shares the same form factor. This explains why a proton and an electron are *not* a matter-antimatter pair: their *form factors* are different. Positive and negative *charge* remain separate concepts, however: two electrons will, therefore, not annihilate each other but coexist as an electron *pair* through a *coupling* of their magnetic moments, thereby lowering total energy (which explains their stability as a pair). Formally, opposite angular momentum may also be modelled by inversing the time sign, but time goes in one direction only:

[7] The small anomaly in the magnetic moment may be explained by assuming the pointlike charge has a tiny (non-zero) dimension itself. See our [paper on the essentials](#).

The electron

The radius formula works perfectly well for the electron. It yields the electron's Compton radius $a = r_C$:

$$a = \frac{c}{\omega} = \frac{c \cdot \hbar}{\mathrm{m} \cdot c^2} = \frac{\hbar}{\mathrm{m} \cdot c} = \frac{\lambda_C}{2\pi} \approx 0.386 \times 10^{-12}\ \mathrm{m}$$

The idea here is that an orbital *cycle* of the pointlike charge in its *Zitterbewegung* does not only pack the electron's energy (E = m·c^2) but also Planck's quantum of action (S = *h*).[8] For an electron, we also get the following cycle time and *electric* current:

$$\mathrm{T} = \frac{h}{\mathrm{E}} \approx \frac{6.626 \times 10^{-34}\ \mathrm{J \cdot s}}{8.187 \times 10^{-14}\ \mathrm{J}} \approx 0.8 \times 10^{-20}\ \mathrm{s}$$

We can also calculate the electric current:

$$\mathrm{I} = \mathrm{q_e} f = \mathrm{q_e}\frac{\mathrm{E}}{h} \approx (1.6 \times 10^{-19}\ \mathrm{C})\frac{8.187 \times 10^{-14}\ \mathrm{J}}{6.626 \times 10^{-34}\ \mathrm{Js}} \approx 19.8\ \mathrm{A}$$

These values look rather phenomenal (we have a household-level current (almost 20 *ampere*) at the sub-atomic scale here), but they are what they are and far from the radius-energy values one gets for black holes (Schwarzschild radius).

We get the classical electron radius from the formula above:

$$\mathrm{U} = \frac{1}{2}\frac{\mathrm{e}^2}{r_\mathrm{e}} = \frac{1}{2}\frac{\mathrm{q_e^2}}{4\pi\varepsilon_0}\frac{1}{r_\mathrm{e}} \Leftrightarrow r_\mathrm{e} = \frac{1}{2}\frac{\mathrm{q_e^2}}{4\pi\varepsilon_0 \mathrm{U}} = \alpha\frac{\hbar c}{2\mathrm{m}_\gamma c^2} = \alpha\frac{\hbar}{\mathrm{m_e} c} = \alpha r_\mathrm{C} \approx 2.82 \times 10^{-15}\ \mathrm{m}$$

This illustrates the interpretation of the fine-structure constant as a scaling parameter: $r_e = \alpha r_C$.[9] The harmonic oscillator model can be used to show that the elasticity or stiffness parameter k or, expressed per unit mass, k/m in the **F** = –k***x*** formula is equal to:

$$\left.\begin{array}{l} \mathrm{E} = \mathrm{m}a^2\omega^2 \Leftrightarrow \omega = \sqrt{\dfrac{\mathrm{E}}{\mathrm{m}a^2}} = \sqrt{\dfrac{\mathrm{m}c^2}{\mathrm{m}a^2}} \\ \omega = \sqrt{\dfrac{\mathrm{k}}{\mathrm{m}}} \end{array}\right\} \Rightarrow \omega^2 = \frac{\mathrm{k}}{\mathrm{m}} = \frac{c^2}{a^2} = \frac{a^2\omega^2}{a^2} = \omega^2$$

This equation shows various oscillatory *modes* are possible: these modes are characterized by the frequency (or its square[10]) which, in turn, depends on the speed of light (*c*) and the radius or range parameter (*a*).

[8] The idea of an oscillation packing some amount of physical action may not be very familiar. In the context of our model we think of physical action as the product of (*i*) the force that keeps the *zbw* charge in its orbit, (*ii*) the distance along the loop, and (*iii*) the orbital cycle time.

[9] The 2019 revision of the system of SI units incorporates these new physics, which amount to what we refer to as a *realist* interpretation of quantum physics. Needless to say, the fine-structure constant has other interpretations as well. See our paper on the meaning of the fine-structure constant.

[10] The energy in an oscillation is proportional to the *square* of the frequency (and the *square* of its amplitude too).

We can also easily calculate the magnitude of the centripetal force $|\mathbf{F}|$ = F from Newton's force law[11]:

$$\mathrm{F} = \mathrm{m}_\gamma a_\mathrm{c} = \frac{\mathrm{m_e}}{2} a\omega^2 = \frac{\mathrm{m_e}}{2} a \frac{\mathrm{E}^2}{\hbar^2} = \frac{\mathrm{m_e^3}c^4}{2\hbar^2}\frac{\hbar}{\mathrm{m_e}c} = \frac{\mathrm{m_e^2}c^3}{2\hbar} \approx 0.106\ \mathrm{N}$$

This is a *huge* force at the sub-atomic scale: it is equivalent to a force that gives a mass of about 106 *gram* (1 g = 10^{-3} kg) an acceleration of 1 m/s per second! It is, however, an electromagnetic force only.

The muon-electron

The same formulas apply to the muon-electron as well but suggest the centripetal force is of an entirely different nature. The muon carries 105.66 MeV (about 207 times the electron energy) and has a (mean) lifetime which is *much* shorter than that of a free neutron[12] but longer than that of other unstable particles: about 2.2 *micro*seconds (10^{-6} s).[13] This may explain why we get a sensible result when using the Planck-Einstein relation to calculate its frequency and/or radius.[14]

$$a = c/\omega = c\frac{\hbar}{\mathrm{E}} = \frac{\hbar c}{\mathrm{m}c^2} = \frac{\hbar}{\mathrm{m}c} \approx 1.87\ \mathrm{fm}$$

The *ratio* of the centripetal forces which keep the charge in its orbital for the electron and muon respectively is equal to:

$$\frac{\mathrm{F_\mu}}{\mathrm{F_e}} = \frac{\frac{\mathrm{m_\mu^2}c^3}{2\hbar}}{\frac{\mathrm{m_e^2}c^3}{2\hbar}} = \frac{\mathrm{m_\mu^2}}{\mathrm{m_e^2}} \approx 42{,}753$$

If a force of 0.106 N is rather humongous, then a force that is about 42,753 times as strong, may *surely* be referred to as a strong force, right? Is it the nuclear force? It underscores the point about the *modes* of the elementary oscillation depending on the radius or amplitude $a = \hbar/\mathrm{m}c$ of the oscillation. We may, indeed, rewrite the force ratio as[15]:

[11] See p. 14-15 of our paper on the electron model for the formula for the centripetal acceleration ($a_c = a\cdot\omega^2 = v_t^2/a$). In the same paper, we also comment on the rather particular behavior of the momentum function p = $\mathrm{m}_\gamma v$, which resembles the mathematical particularities of the Dirac function.

[12] The mean lifetime of a neutron in the open (outside of the nucleus) is almost 15 minutes!

[13] The tau-electron is just a *resonance* (as opposed to a *transient* particle) with an extremely short lifetime (2.9×10^{-13} s only). Hence, the Planck-Einstein relation does not apply: it is *not* an equilibrium state. We think the conceptualization of both the muon- as well as the tau-electron in terms of particle *generations* is unproductive. Likewise, charged pions ($\pi^\pm$) has no resemblance whatsoever with the neutral pion (see footnote 14).

[14] The longevity of the muon-electron should not be exaggerated, however: the mean lifetime of charged pions, for example, is about 26 *nano*seconds (10^{-9} s), so that is only 85 times less. As for the 1.87 fm value, this is a radius and, hence, should be multiplied by 2π to get the CODATA value (more or less) for the Compton *wavelength* of the muon ($1.173444110\times10^{-14}$ m $\pm\,0.000000026\times10^{-14}$ m).

[15] We did not find any easy interpretation of this ratio in terms of the fine-structure constant, however. Hence, the mass or energy of the elementary particles may be considered to be fundamental constants of Nature themselves.

$$\frac{F_\mu}{F_e} = \frac{m_\mu^2}{m_e^2} = \frac{\frac{\hbar^2}{a_\mu^2 c^2}}{\frac{\hbar^2}{a_e^2 c^2}} = \frac{a_e^2}{a_\mu^2} = \left(\frac{a_e}{a_\mu}\right)^2 \approx (206.332256\ldots)^2$$

The proton

The proton mass is about 8.88 times that of the muon, and it is about 2.22 times smaller: we have a rather mysterious 1/4 factor here, which needs explaining. Indeed, when applying the $a = \hbar/mc$ radius formula to a proton, we get a value which is 1/4 of the *measured* proton radius: about 0.21 fm, as opposed to the 0.83-0.84 fm charge radius which was established by Professors Pohl, Gasparan and others over the past decade.[16]

$$a_p = c/\omega = c\frac{\hbar}{E} = \frac{\hbar c}{m_p c^2} = \frac{\hbar}{m_p c} \approx 0.21 \text{ fm}$$

The 1/4 factor is puzzling, and there may be no obvious way to explain it. However, geometry offers a way out. We have a 1/4 factor between the volume of a sphere ($V = 4\pi r^2$) and the surface area of a circle ($A = \pi r^2$) and, hence, one might intuitively think of an oscillation in three rather than just two dimensions only. In other words, the oscillator would be driven by two (perpendicular) forces rather than just one. We can model this by thinking of *two* oscillators which, according to the equipartition theorem, should each pack *half* of the total energy of the proton. This spherical view of a proton – as opposed to the planar picture of an electron – fits nicely with packing models for nucleons.

The frequency of *each* of the oscillators would be equal to $\omega = E/2\hbar = mc^2/2\hbar$: each of the two perpendicular oscillations would, therefore, pack one *half*-unit of only.[17] This, then, gives us the experimentally established value for the proton radius:

$$\frac{E}{m_p} = c^2 = a^2\omega^2 = a^2\left(\frac{m_p c^2}{2\hbar}\right)^2 \Leftrightarrow a^2 = c^2\frac{4\hbar^2}{m_p^2 c^4} \Leftrightarrow a = \frac{4\hbar}{m_p c}$$

The force along one of the two axes or planes of oscillation inside of a proton is equal to:

$$F_p = \frac{1}{2}m_p a\omega^2 = \frac{1}{2}m_p a\frac{c^2}{a^2} = \frac{1}{2}\frac{m_p c^2}{a} = \frac{1}{2}\frac{{m_p}^2 c^3}{4\hbar} = \frac{1}{8}\frac{{m_p}^2 c^3}{\hbar} \approx 89{,}349 \text{ N}$$

Hence, we get a force of 4,532 N inside of a muon and a force of 89,349 N inside of a proton. Compensating for the 1/4 factor (which we loosely refer to as the different *form factor* of the proton oscillation), we find the force inside of a proton is almost 5 times stronger than the force inside of a muon. Hence, we may conclude that the force inside of a muon-electron and a proton (and neutron, which we think of as a proton-electron combination[18]) are of the same *nature*. However, a muon-electron is, clearly, not the

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[16] For the exact references and contextual information on the (now solved) 'proton radius puzzle', see our paper on it: https://vixra.org/abs/2002.0160, in which we also make some remarks on the (anomalous) magnetic moment of the proton.

[17] This explanation is similar to our explanation of one-photon Mach-Zehnder interference, in which we assume a photon is the superposition of two orthogonal *linearly polarized* oscillations (see p. 32 of our paper on basic quantum physics, which summarizes an earlier paper on the same topic).

[18] See our paper on the nuclear force and the neutron hypothesis.

antimatter counterpart of a proton !

The orbital energy equation for the nuclear field is given by[19]:

$$\frac{\mathrm{E}}{\mathrm{m}} = c^2 = \frac{v^2}{2} + \frac{a\mathrm{k_e q_e^2}}{\mathrm{m}r^2}$$

Can we calculate *a* out of this? Maybe. How? Perhaps by evaluating potential and kinetic energy at the *periapsis*, where the distance between the charge and the center of the radial field is closest. However, the limit values $v_\pi = c$ (for $r_\pi \to 0$) and $r_\pi = 0$ (for $v_\pi \to c$) are never reached and should, therefore, not be used: neither the kinetic nor the potential energy seems to reach the zero value and we can, therefore, probably not simplify any further.[20] We, therefore, prefer the simpler *zbw* approach as outlined above:

$$\frac{\mathrm{E}}{\mathrm{m}} = c^2 = a^2\omega^2 = a^2\left(\frac{\mathrm{m}c^2}{2\hbar}\right)^2 \Leftrightarrow a = \frac{4\hbar}{\mathrm{m}c} \approx 0.84 \text{ fm}$$

We may now substitute this value for *a* in the orbital energy equation[21]:

$$\frac{\mathrm{E}}{\mathrm{m}} = c^2 = \frac{v^2}{2} + \frac{4\hbar\mathrm{k_e q_e^2}}{\mathrm{m}^2 c r^2} = \frac{v^2}{2} + \frac{4\alpha\hbar^2}{\mathrm{m}^2 r^2}$$

Re-arranging yields:

$$c^2 = \frac{v^2}{2} + \frac{4\alpha\hbar^2}{\mathrm{m}^2 r^2} \Leftrightarrow 1 - \frac{\beta^2}{2} = \frac{4\alpha\hbar^2}{\mathrm{m}^2 c^2 r^2}$$

It is a nice formula. In the next section, we will find a similar formula for the neutron.

The neutron

We think of the (free) neutron as a composite (non-stable) particle consisting of a 'proton' and an 'electron'. However, we will soon qualify this statement: the reader should effectively think in terms of *pointlike charges* here—rather than in terms of a massive proton and a much less massive electron! Both the 'proton' and the 'electron' carry the elementary (electric) charge but we think *both are bound in a nuclear as well as in an electromagnetic oscillation*. In order to interpret *v* as an orbital or tangential velocity, we must, of course, choose a reference frame. Let us first jot down the orbital energy equation for the nuclear field, however[22]:

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[19] The reader can/should check the physical dimensions:

$$\left[\frac{v^2}{2} - \frac{a\mathrm{k_e q_e^2}}{\mathrm{m}r^2}\right] = \frac{\mathrm{m}^2}{\mathrm{s}^2} - \frac{\frac{\mathrm{Nm}^3}{\mathrm{C}^2}\mathrm{C}^2}{\mathrm{kg}\cdot\mathrm{m}^2} = \frac{\mathrm{m}^2}{\mathrm{s}^2} - \frac{\frac{\mathrm{Nm}^2}{\mathrm{C}^2}\mathrm{C}^2}{\mathrm{N}\frac{\mathrm{s}^2}{\mathrm{m}}\mathrm{m}} = \frac{\mathrm{m}^2}{\mathrm{s}^2}$$

Note that we have a *plus* (+) sign in the equation because the potential energy in the orbital energy equation is zero at the center and, therefore, always positive. For more details, see our paper on the nuclear force.

[20] One can, however, calculate other interesting properties of the orbitals, such as the eccentricity (see the above-mentioned MIT OCW reference course on orbital motion).

[21] We use the *definition* (cf. the 2019 revision of SI units) of the fine-structure constant: $\alpha = \frac{\mathrm{k_e q_e^2}}{\hbar c}$ formula.

[22] A dimensional check of the equation yields:

$$\frac{\mathrm{E}_N}{\mathrm{m}_N} = \frac{v^2}{2} + \frac{a\mathrm{k_e q_e^2}}{\mathrm{m}_N r^2}$$

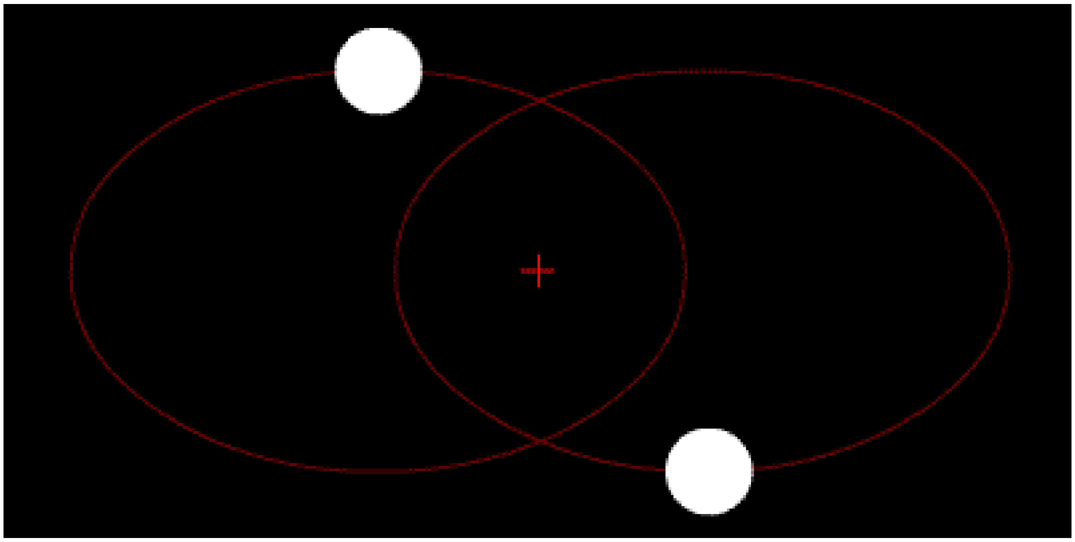

Figure 2: Two opposite charges in elliptical orbitals around the center of mass[23]

The mass factor m_N is the equivalent mass of the energy in the oscillation[24], which is the sum of the kinetic energy and the potential energy between the two charges. The velocity *v* is the velocity of the two charges ($\mathrm{q_e}^+$ and $\mathrm{q_e}^-$) as measured in the center-of-mass (*barycenter*) reference frame[25] and may be written as a vector $\boldsymbol{v} = \boldsymbol{v}(\boldsymbol{r}) = \boldsymbol{v}(x, y, z) = \boldsymbol{v}(r, \theta, \varphi)$, using either Cartesian or spherical coordinates.

We have a plus sign for the potential energy term ($\mathrm{PE} = a\mathrm{k_e q_e}^2/\mathrm{m}r^2$) because we assume the two charges are being kept *separate* by the nuclear force.[26] The electromagnetic force which keeps them *together* is the Coulomb force:

$$\frac{\mathrm{E}_C}{\mathrm{m}_C} = \frac{v^2}{2} + \frac{\mathrm{k_e q_e^2}}{\mathrm{m}_C r}$$

The total energy in the oscillation is given by the sum of nuclear and Coulomb energies and we may, therefore, write:

$$\left[\frac{v^2}{2} + \frac{a\mathrm{k_e q_e^2}}{\mathrm{m}r^2}\right] = \frac{\mathrm{m^2}}{\mathrm{s^2}} + \frac{\frac{\mathrm{Nm^3}}{\mathrm{C^2}}\mathrm{C^2}}{\mathrm{kg\cdot m^2}} = \frac{\mathrm{m^2}}{\mathrm{s^2}} + \frac{\frac{\mathrm{Nm^2}}{\mathrm{C^2}}\mathrm{C^2}}{\mathrm{N}\frac{\mathrm{s^2}}{\mathrm{m}}\mathrm{m}} = \frac{\mathrm{m^2}}{\mathrm{s^2}}$$

We recommend the reader to regularly check our formulas: we do make mistakes sometimes!

[23] Illustration taken from Wikipedia. For the orbital equations, see the MIT OCW reference course on orbital motion.

[24] We will use the subscripts x_N and x_C to distinguish nuclear from electromagnetic mass/energy.

[25] This relates to the point we made in regard to the nature of the 'proton' in the neutron: it is *not* like the massive proton at the center of the hydrogen atom, with the electron orbiting around it.

[26] We have a minus sign in the same formula in our paper on the nuclear force because the context considered two *like* charges (e.g. two protons). As for the plus (+) sign for the potential energy in the electromagnetic orbital energy, we take the reference point for zero potential energy to be the center-of-mass and we, therefore, have positive potential energy here as well.

$$\frac{\mathrm{E}_C}{\mathrm{m}_C} = c^2 = \frac{\mathrm{E}_C}{\mathrm{m}_C} + \frac{\mathrm{E}_N}{\mathrm{m}_N} = \frac{v^2}{2} + \frac{\mathrm{k_e q_e^2}}{\mathrm{m}_C r} + \frac{v^2}{2} + \frac{a\mathrm{k_e q_e^2}}{\mathrm{m}_N r^2} \Leftrightarrow$$

$$c^2 - v^2 = \frac{\mathrm{k_e q_e^2}}{\mathrm{m}_C r} + \frac{a\mathrm{k_e q_e^2}}{\mathrm{m}_N r^2} = \mathrm{k_e q_e^2}\frac{\mathrm{m}_N r + \mathrm{m}_C a}{\mathrm{m}_N \mathrm{m}_N r^2} \Leftrightarrow$$

$$c^2 = v^2 + \mathrm{k_e q_e^2}\frac{\mathrm{m}_N r + \mathrm{m}_C a}{\mathrm{m}_N \mathrm{m}_N r^2} = v^2 + \alpha\hbar c\frac{\mathrm{m}_N r + \mathrm{m}_C a}{\mathrm{m}_N \mathrm{m}_N r^2}$$

The latter substitution uses the definition of the fine-structure constant once more.[27] Dividing both sides of the equation by c^2, and substituting m_N and m_C for m/2 using the energy equipartition theorem, yields:

$$1 - \beta^2 = \frac{\alpha\hbar(r + a)}{\mathrm{m}cr^2} = \frac{\alpha\hbar}{\mathrm{m}c}\frac{r + a}{r^2}$$

It is a beautiful formula, and we could/should probably play with it some more by, for example, evaluating potential and kinetic energy at the *periapsis*, where the distance between the charge and the center of the radial field is closest. However, the limit values $v_\pi = c$ (for $r_\pi \to 0$) and $r_\pi = 0$ (for $v_\pi \to c$) are never reached and should, therefore, not be used. We are sure one of our readers will find ways to get a *specific* value for the radius *a*, which should be, *hopefully*, very near to 0.84 fm (the proton/neutron diameter. It should, in fact, be slightly larger because of the energy difference between a proton and a neutron, which is of the order of about 1.3 MeV, which is about 2.5 times the energy of a free electron.[28]

Till then, we must assume we may apply the mass-without-mass formula for the proton radius to the neutron too:

$$\frac{\mathrm{E}}{\mathrm{m}} = c^2 = a^2\omega^2 = a^2\left(\frac{\mathrm{m}c^2}{2\hbar}\right)^2 \Leftrightarrow a = \frac{4\hbar}{\mathrm{m}c} \approx 0.84 \text{ fm}$$

Conclusions

Mass-without-mass models of elementary particles model the oscillation of a pointlike charge. Potentials and forces depend on and/or act on a charge: the elementary charge $e^\pm$ (e, e_μ, e_p) or its antimatter counterpart. We think there are only two forces/potentials: electromagnetic and nuclear—or some combination thereof.[29]

We also think forces/potentials/particles have a field- or light-particle counterpart: the photon or the neutrino (as applicable to proton/neutron *Verwandlung* reactions).

Of course, we are very much aware that we offer a non-conventional analysis here which breaks away from common ideas on several critical points. Most importantly, perhaps, we think leptons do partake in

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[27] One easily obtains the $\mathrm{k_e q_e}^2 = \alpha\hbar c$ identity from the $\alpha = \frac{\mathrm{k_e q_e^2}}{\hbar c}$ formula.

[28] We note there is no CODATA value for the neutron radius. This may or may not be related to the difficulty of measuring the radius of a decaying neutral particle. As for the instability of the free neutron, its lifetime is *very* long as compared to the muon-electron, so we may effectively assume that the oscillation must *very nearly* pack one unit of Planck's quantum of action.

[29] We develop a model for the deuteron nucleus in the above-mentioned paper too!

nuclear interactions, which explains deep electron orbitals[30], our model of the muon as a (potentially) *pure* nuclear oscillation (neutrinos carry the (excess) energy of a muon decay reaction) and, perhaps, why low-energy nuclear reactions (transmutation of nucleons by laser irradiation) can possibly take place.

We effectively think the classification of particles into generations or into baryons and leptons are too general to be useful: we just have two forces/potentials, and combinations thereof. Incidentally, we also think the quark hypothesis might not be very useful: at best, they are temporary non-equilibrium states and, as such, mathematical abstractions. We get the following table of elementary matter- and light-particles[31]:

Table 1: Elementary particle classification according to form factors

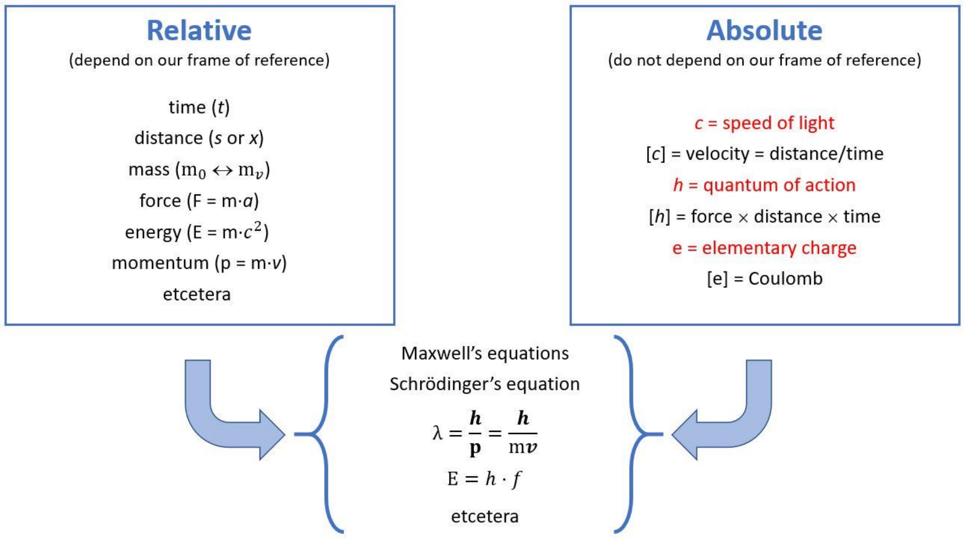
	2D oscillation	**3D oscillation**
electromagnetic force	$e^{\pm}$ (electron/positron)	orbital electron (e.g: ^1H)
nuclear force	$\mu^{\pm}$ (muon-electron/antimuon)	$p^{\pm}$ (proton/antiproton)
composite	pions ($\pi^{\pm}$/ π^0)?	e.g: n (neutron), D^+ (deuteron)
corresponding field particle	γ (photon)	ν (neutrino)

This is nice and complete: each theoretical/mathematical/logical possibility corresponds to a physical reality, with spin distinguishing matter from antimatter for particles with the same *form factor*.

So what is our theory of reality, then? We think physic reality and our logical representation of it blends as part of the sensemaking process. Wittgenstein was wrong on language, but his intuition was quite correct: *Die Welt ist die Gesamtheit der Tatsachen, nicht der Dinge.* (Wittgenstein, TLP, 1.1) We, therefore, stick to classical quantum physics or, as we refer to it, to a *realist* interpretation of it. Indeed, we think the criticism of H.A. Lorentz of the new theories, before he left the scene, was quite apt:

> "I would like to draw your attention to the difficulties in these theories. We are trying to represent phenomena. We try to form an image of them in our mind. Till now, we always tried to do using the ordinary notions of space and time. These notions may be innate; they result, in any case, from our personal experience, from our daily observations. To me, these notions are clear, and I admit I am not able to have any idea about physics without those notions. The image I want to have when thinking physical phenomena has to be clear and well defined, and it seems to me that cannot be done without these notions of a system defined in space and in time."[32]

[30] See, for example, Andrew Meulenberg and Jean-Luc Paillet, *Deep Electron Orbitals and the Dirac Equation*, January 2020.
[31] We think of the tau-lepton as a resonance or a *very* short-lived transient. It is, therefore, not an elementary particle in our view but only an intermediary reaction product.
[32] See our *brief history of quantum-mechanical ideas*.



Die Welt ist die Gesamtheit der Tatsachen, nicht der Dinge.
(Wittgenstein, TLP, 1.1)

We conclude with Wittgenstein's last and final proposition in his *Tractatus Logico-Philosophicus* (TLP):

"*Wovon man nicht sprechen kann, darüber muss man schweigen*."

Brussels, 8 February 2021