# Mass spectrum of the charmonium tetraquarks 

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This paper reveals an easy and accurate way to calculate the 3d masses of tetraquarks, which involves taking into consideration their higher dimensional structure. A mass spectrum correlating 18 experimental tetraquark masses with their predicted theoretical masses is displayed, along with an explanation of how to calculate the factoring unit employed. The underlying theory that the factoring is based on is also discussed.

## Mass spectrum of the charmonium tetraquarks specified by (S5) ${ }^{4} \mathrm{~h} / 49000$ Factoring

|  | Factoring |  | Thr Mass | Exp Mass | +/- | Meson | Source |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 54.5 | (S5) 4h/49000 | $=3536.14742$ |  |  |  |  |
|  | 55 | (S5) 4h/49000 | $=3568.58914$ |  |  |  |  |
|  | 55.5 | (S5) 4h/49000 | $=3601.03086$ |  |  |  |  |
|  | 56 | (S5) 4h/49000 | $=3633.47258$ | 3633.6 | $1.7 / 0.6$ | $\mathrm{nc}(2 \mathrm{~S})$ | [1] |
|  | 56.5 | (S5) 4h/49000 | $=3665.91430$ | 3666 | 10 |  | [2] |
|  | 57 | (S5) 4h/49000 | $=3698.35602$ |  |  |  |  |
|  | 57.5 | (S5) 4h/49000 | $=3730.79774$ |  |  |  |  |
| 4 | 58 | (S5) 4h/49000 | $=3763.23946$ |  |  |  |  |
|  | 58.25 | (S5) 4h/49000 | $=3779.46031$ | 3780 | 7 |  | [2] |
|  | 58.5 | (S5) 4h/49000 | $=3795.68118$ |  |  |  |  |
|  | 59 | (S5) 4h/49000 | $=3828.12290$ | 3831 | 10/16 |  | [2] |
|  | 59.5 | (S5) 4h/49000 | $=3860.56462$ |  |  |  |  |
|  | 60 | (S5) 4h/49000 | $=3893.00634$ | 3893.0 | 2.3/19.9 | Zc (3900) o | [ 4 ] |
|  | 60.5 | (S5) 4h/49000 | $=3925.44805$ |  |  |  |  |
|  | 61 | (S5) 4h/49000 | $=3957.88977$ |  |  |  |  |
|  | 61.5 | (S5) 4h/49000 | $=3990.33149$ |  |  |  |  |
| 4 | 62 | (S5) 4h/49000 | $=4022.77321$ | 4022.9 | $0.8 / 2.7$ | X (4020) | [1] |
|  | 62.5 | (S5) 4h/49000 | $=4055.21493$ |  |  |  |  |
|  | 63 | (S5) 4h/49000 | $=4087.65665$ |  |  |  |  |
|  | 63.125 | (S5) 4h/49000 | $=4095.76708$ | 4096 | 20 | X (4100) | [1] |
|  | 63.5 | (S5) 4h/49000 | $=4120.09837$ |  |  |  |  |
|  | 64 | (S5) 4h/49000 | $=4152.54009$ | 4152.5 | $1.7 / 6.2$ | Xc1 (4140) | [1] |
|  | 64.5 | (S5) 4h/49000 | $=4184.98181$ |  |  |  |  |
|  | 64.625 | (S5) 4h/49000 | $=4193.09224$ | 4193 | 7 | Y(4160) | [1] |
|  | 64.875 | (S5) 4h/49000 | $=4209.31310$ | 4209.5 | 7.4/1.4 | Y(4230) | [1] |
|  | 65 | (S5) 4h/49000 | $=4217.42353$ | 4218 | $5.5 / 4.5$ | Y (4230) | [1] |
|  | 65.5 | (S5) 4h/49000 | $=4249.86525$ |  |  |  |  |
| 4 | 65.875 | (S5) 4h/49000 | $=4274.19653$ | 4274.4 | $8.4 / 6.7$ | Xc1 (4274) | [1] |
|  | 66 | (S5) 4h/49000 | $=4282.30697$ |  |  |  |  |
|  | 66.5 | (S5) 4h/49000 | $=4314.74869$ |  |  |  |  |
|  | 67 | (S5) 4h/49000 | $=4347.19041$ | 4347 | $6 / 3$ | Y(4360) | [1] |
|  | 67.5 | (S5) 4h/49000 | $=4379.63213$ | 4380 | 8/29 | Pc(4380) | [5] |
|  | 68 | (S5) 4h/49000 | $=4412.07385$ | 4412 | 15 | Y(4415) | [1] |
|  | 68.5 | (S5) 4h/49000 | $=4444.51557$ |  |  |  |  |
|  | 69 | (S5) 4h/49000 | $=4476.95729$ | 4478 | 15/18 | Zc (4430) | [1] |
|  | 69.125 | (S5) 4h/49000 | $=4485.06771$ | 4485 | 2 |  | [3] |
|  | 69.5 | (S5) 4h/49000 | $=4509.39900$ |  |  |  |  |
| 4 | 70 | (S5) 4h/49000 | $=4541.84072$ |  |  |  |  |
|  | 70.5 | (S5) 4h/49000 | $=4574.28244$ |  |  |  |  |
|  | 71 | (S5) 4h/49000 | $=4606.72416$ |  |  |  |  |
|  | 71.5 | (S5) 4h/49000 | $=4639.16588$ |  |  |  |  |
|  | 72 | (S5) 4h/49000 | $=4671.60760$ |  |  |  |  |
|  | 72.5 | (S5) 4h/49000 | $=4704.04932$ | 4704 | 10/14 | Xc0 (4700) | [1] |
|  | 73 | (S5) 4h/49000 | $=4736.49104$ |  |  |  |  |
|  | 73.5 | (S5) 4h/49000 | $=4768.93276$ |  |  |  |  |

## Introduction

Theorists, using wave equations, have been able to estimate the 3d masses of some of the charmonium tetraquarks to within one or two MeV . [3] This paper reveals a much easier and more accurate way to calculate the 3d masses of tetraquarks, which involves taking into consideration their higher dimensional structure. Tetraquarks may be constructed of four intersecting hypersphere surfaces of energy. That's what the good agreement between experimental and theoretical mass values shown in the mass spectrum above seems to affirm, because the unit of factorization used to generate the mass spectrum above involves the surface volume of a 5 -sphere raised to the fourth power. Can there be other interpretations of these correlations? Perhaps, but none more logical. (Most other mesons, said to consist of two 'quarks', are constructed of two intersecting hypersphere surfaces of energy that range in dimension from 3 to 9 .) How to calculate the factoring unit used to construct the mass spectrum shown above, and the theory behind it, will be explained in this paper.

## How to Calculate the Unit of Factorization for Tetraquarks

To calculate the factorization unit for tetraquarks, multiply the equation for the surface volume of a unit radius 5 -sphere together four times, then multiply that by the coefficient of Planck's constant ( $\mathrm{h}=6.62607015$ ), and, finally divide by 49000 or 343000 . The factor of 1000 in the divisor is used to reduce the factoring unit to an optimal size. About half of the known tetraquarks will factor with the 49000 divisor and the rest will factor with the 343000 divisor. The 49000 divisor was used for the mass spectrum shown here for brevity. Also for the sake of brevity, the resolution of the displayed mass spectrum is .500 of the factoring unit, which equals 32.44171946 MeV . As can be seen, a few tetraquarks that factor to multiples of .125 of the factoring unit have been inserted. Increasing the resolution of the displayed mass spectrum to .125 of the factoring unit, which equals 8.110429864 MeV , would result in many more matches with experimental tetraquark masses, but would make it four times longer. Why multiply by the coefficient of Planck's constant? It seems to operate as a conversion factor that, in this case, converts hypersphere surface volume energy (hypermass) to 3 d mass in units of MeV . In the derivation that follows, S 5 stands for the surface volume of a 5 -sphere.

$$
\begin{aligned}
\mathbf{S 5} & =8 / 3 \pi^{2} \mathrm{r}^{4} \\
(\mathrm{~S} 5)^{4} & =4096 / 81 \pi^{8} \mathrm{r}^{16}
\end{aligned}
$$

We are using unit radius hyperspheres, so must set $r=1$ :

$$
(\mathbf{S} 5)^{4}=4096 / 81 \boldsymbol{\pi}^{\mathbf{8}}
$$

Now multiplying by ' $\mathrm{h}=6.62607015$ ' converts hypermass to the 3 d mass we observe.

$$
\begin{aligned}
& (\mathbf{S 5})^{\mathbf{4}} \mathrm{h}=4096 / 81 \boldsymbol{\pi}^{\mathbf{8}} \mathrm{h} \\
& (\mathbf{S 5})^{4} \mathrm{~h}=3179288.507 \mathrm{MeV}
\end{aligned}
$$

This is a large number compared to tetraquark masses. It's about 750 times larger than the average charmoium tetraquark's mass expressed in units of MeV , so divide by 1000 to get small integer results from factoring. But this is still not the best divisor, because its use will result in powers of seven fractions. To avoid that and to get powers of two fractions in the decimal expansions of factoring results divide also by 49. So 49000 is a good divisor. ( 343000 is an even better divisor, because its use will factor more tetraquarks to small integers and powers of two fractions, but results in a longer mass spectrum.)

$$
(\mathrm{S} 5)^{4} \mathrm{~h} / 49000=64.88343892 \mathrm{MeV}
$$

This is the factoring unit used in the mass spectrum above.

## The Theory Behind the Factoring

It appears, from the results of hypersphere surface volume factoring that has been carried out on the experimental masses of numerous tetraquarks and other mesons, that mesons are constructed of matter in higher dimensional states. At least, that hypothesis is consistent with the factoring evidence. But how can a meson, which is constructed of two intersecting 5 -sphere surfaces - as is thought to be the case with cc mesons - exist seemingly totally immersed in our 3d space? You can't put a higher dimensional object in a lower dimensional space. It's because they are not totally immersed in our 3d space. Some of their structure protrudes out into nspace.

How is that possible? Because our 3d space has zero thickness in the fourth and higher dimensional directions. Just as 2-space (a plane) has zero thickness in the third dimensional direction, 3 -space (our space) has zero thickness in the fourth dimensional direction (and higher dimensional directions). So, higher dimensional objects can appear to exist totally immersed in our 3d space, because the part of the object we observe 'totally immersed' is not the whole object. What we observe of it is just its intersection with our 3d space. Most of a meson's structure resides in the n -space immediately adjacent to our 3 -space.

## Conclusion

If the correct interpretation of successful hypersphere surface volume factoring of meson masses is that mesons have higher dimensional parts, then mesons, and most likely, all the other subatomic particles, are much more complicated than previously thought. And since, also from factoring results, it is seen that mesons are constructed of matter of many different dimensional states, it can be surmised that matter has the ability, under certain extreme conditions, to move from one dimensional state to another. In other words, matter exists in more than one dimensional state, and can change from lower to higher dimensional states and vice versa..

## References

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