Formulas of Feigenbaum Constants and Their Physical Meanings (viXra:2101.0187v2)

Gang Chen†, Tianman Chen, Tianyi Chen

Guangzhou Huifu Research Institute Co., Ltd., Guangzhou, P. R. China
7-20-4, Greenwich Village, Wangjianglu 1, Chengdu, P. R. China

†Correspondence to: gang137.chen@connect.polyu.hk

Dedicated to Prof. Albert Sun-Chi Chan on the occasion of his 70th birthday

Abstract

This paper supposes that Feigenbaum constants should be rational numbers in the world of nuclides, gives their formulas in fractional number format and exhibits the physical meanings of the factors in the formulas, especially their relationships with nuclides, the fine-structure constant and $2\pi$. This paper also supposes that there would be the third Feigenbaum constant and gives its two possible approximate values. Formulas of the fine-structure constant $\alpha_1$, Feigenbaum constant $\delta$ and $2\pi$ are also given, briefly to be $\alpha_1\delta^2(2\pi)\approx 1$, and their relationships with nuclides are illustrated.

Keywords: Feigenbaum constants; the fine-structure constant; $2\pi$; nuclides.

1. Introduction

Feigenbaum constants are characterizing constants in chaotic systems, it is assumed by scientific community that they would characterize chaos just as $2\pi$ stands for periodicity. The nuclei of nuclides except proton should be multiple-body systems and hence chaos should be the common state in the world of nuclei. So Feigenbaum constants should express some functions in nuclides.

There are two Feigenbaum constants, i. e., $\delta=4.6692...$ and $\alpha=2.5029...$, and mathematically they should be irrational numbers with infinite digits.

In our previous papers$^{1,2,3,4}$, we had already given formulas of the fine-structure constant and given two values of it, i. e., $\alpha_1=1/137.035999037435$ and $\alpha_2=1/137.035999111818$ which are rational numbers with 15 digits. The values and formulas of the fine-structure constant show strong relationships with nuclides, and hereby some relevant examples of these relationships are listed as follows. And it
shows that the values of the fine-structure constant express as integer numbers of 136, 137 and 138 in the world of nuclides.

In our previous paper \(^1\), we also exhibited the relationship of \(2\pi\) with nuclides, for example, \(2\pi = 6.28 = (4 \times 157)/100\) relates to nuclides as follows.

In this paper, we suppose Feigenbaum constants should also be rational numbers in the world of nuclides, give their formulas and exhibit their relationships with nuclides and hence with the fine-structure constant and \(2\pi\).

### 2. Formulas of Feigenbaum Constants in Fractional Numbers

It is supposed that Feigenbaum constants should be rational numbers with 15 digits and have the following fractional formulas and relationships with nuclides.

Note: \(136 = 8 \cdot 17\), \(138 = 6 \cdot 23\)

\[
\frac{1}{\delta} = \frac{1}{4.66920160910299} = 0.214169377062326
\]

\[
= \frac{1}{4} \cdot \frac{1}{27} + \frac{1}{4.9.23} - \frac{1}{2.3.7.23 \cdot (2.3 \cdot (4 \cdot 3 \cdot 11 - 1) + 1) + \frac{2.23}{3.19}}
\]

Note: ,

\[
\frac{1}{\alpha} = \frac{1}{2.50290787509589} = 0.399535280523135
\]

\[
= \frac{1}{2} - \frac{1}{9} + \frac{1}{31} - \frac{1}{23 \cdot (8 \cdot 3 \cdot 17 + 1)} + \frac{1}{17 \cdot 23 \cdot (8 \cdot 3 \cdot 11 + 1)} - \frac{1}{23 \cdot (8 \cdot 3 \cdot 11 + 1)}
\]
3. Formulas of Feigenbaum Constants in Continued Fractional Numbers

\[
\frac{1}{\delta} = \frac{1}{4.66920160910299} = 0.214169377062326 = \frac{3}{14 + \frac{1}{131 + \frac{1}{2 + \frac{1}{54 + \frac{1}{6 + \frac{1}{\frac{1}{5 + \frac{1}{18}}}}}}}}
\]

\[
\frac{1}{\delta} = \frac{1}{4.66920160910299} = 0.214169377062326 = \frac{3}{14 + \frac{1}{131 + \frac{1}{2 + \frac{1}{54 + \frac{1}{6 + \frac{1}{\frac{1}{5 + \frac{1}{18}}}}}}}}
\]

\[
\frac{1}{\delta} = \frac{1}{4.66920160910299} = 0.214169377062326 = \frac{3}{14 + \frac{1}{131 + \frac{1}{2 + \frac{1}{54 + \frac{1}{6 + \frac{1}{\frac{1}{5 + \frac{1}{18}}}}}}}}
\]

\[
\frac{1}{\delta} = \frac{1}{4.66920160910299} = 0.214169377062326 = \frac{3}{14 + \frac{1}{131 + \frac{1}{2 + \frac{1}{54 + \frac{1}{6 + \frac{1}{\frac{1}{5 + \frac{1}{18}}}}}}}}
\]
\[
\frac{1}{\alpha} = \frac{1}{2.50290787509589} = 0.399535280523135 \\
= \frac{2}{5 + \frac{1}{1 + \frac{1}{17 + \frac{1}{1 + \frac{1}{4 + \frac{1}{1 + \frac{1}{7 + \frac{1}{1 + \frac{1}{15}}}}}}}}}
\]
\[
\frac{1}{\alpha} = \frac{1}{2.50290778509589} = 0.399535280523135 \\
= \frac{2}{5 + \frac{1}{171 + \frac{1}{1 + \frac{1}{17 + \frac{1}{1 + \frac{11\cdot37}{13\cdot59}}}}}}
\]

\[
\frac{1}{\alpha} = \frac{1}{2.5029077509589} = 0.399535280523135 \\
= \frac{2}{5 + \frac{1}{171 + \frac{1}{1 + \frac{1}{17 + \frac{25\cdot(4\cdot5\cdot7-1)}{2\cdot3\cdot7\cdot101}}}}}
\]

\[
1 = \frac{1}{\alpha} = \frac{1}{2.50290778509589} = 0.399535280523135
\]

\[
= \frac{2}{5 + \frac{1}{171 + \frac{1}{1 + \frac{1}{17 + \frac{97\cdot(6\cdot137+1)}{2\cdot5\cdot(2\cdot113+1)\cdot(32\cdot27\cdot7-1)}}}}}
\]

\[
\frac{1}{\alpha} = \frac{1}{2.5029077509589} = 0.399535280523135 \\
= \frac{2}{5 + \frac{1}{171 + \frac{1}{1 + \frac{1}{17 + \frac{97\cdot(6\cdot137+1)}{2\cdot5\cdot(2\cdot113+1)\cdot(32\cdot27\cdot7-1)}}}}}
\]

\[
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\]

\[
\frac{1}{\alpha} = \frac{1}{9.7\cdot13\cdot5\cdot3\cdot(2\cdot7\cdot113+1)} = 68713281
\]

\[
1 \text{ to } 22
\]

\[
\frac{1}{\alpha} = \frac{4\cdot5\cdot(2\cdot2\cdot19-1+1)\cdot(32\cdot27\cdot7-1)}{68713281} = 27453380
\]
4. **The Third Feigenbaum Constant**

There would be the third Feigenbaum constant $\gamma$, and we could only give some guesses at present stage as follows.

Note: $136=8\cdot17$, $138=6\cdot23$

\[
\frac{1}{\alpha} = \frac{1}{2.50290787509589} = 0.399535280523135 = f(2,3,17,23,11,\cdots)
\]

\[
= \frac{1}{2} - \frac{1}{9} + \frac{1}{3\cdot31} - \frac{1}{23\cdot(8\cdot3\cdot17+1)} + \frac{1}{17\cdot23\cdot(8\cdot3\cdot11^4-1)}
\]

\[
= \frac{2}{5} + \frac{1}{171 + \frac{269\cdot281}{97\cdot(6\cdot137+1)}}
\]

\[
\frac{1}{\delta} = \frac{1}{4.66920160910299} = 0.214169377062326 = f(2,3,23,11,\cdots)
\]

\[
= \frac{1}{4} - \frac{1}{27} + \frac{1}{4\cdot9\cdot23} - \frac{1}{2\cdot3\cdot7\cdot23\cdot(2\cdot3\cdot(4\cdot3\cdot11-1)+1)+\frac{2\cdot23}{3\cdot19}}
\]

\[
= \frac{3}{14} + \frac{1}{131 + \frac{13\cdot29\cdot107}{9\cdot83\cdot109}}
\]

\[
\frac{1}{\gamma} = f(2,3,17,\cdots) \approx 1 + \frac{1}{3\cdot8\cdot17} = \frac{25\cdot11}{8\cdot3\cdot17} \approx 2\cdot\frac{1}{3} - \frac{1}{31 + \frac{1}{2}} = \frac{9\cdot7}{17\cdot23} \approx \frac{1}{8.6689}
\]

or

\[
\frac{1}{\gamma} = f(2,3,17,\cdots) \approx \frac{1}{8} - \frac{1}{81} + \frac{1}{8\cdot3\cdot17} = \frac{317}{2\cdot81\cdot17} \approx \frac{3}{26 + \frac{1}{15}} = \frac{1}{8.6689}
\]

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\[
\frac{1}{\gamma} = f(2,3,17,\cdots) \approx \frac{1}{8} - \frac{1}{101} = \frac{3\cdot31}{8\cdot101} \approx \frac{3}{26 + \frac{1}{151}} = \frac{1}{8.6689}
\]

\[
\frac{1}{\gamma} = f(2,3,17,\cdots) \approx \frac{1}{8} - \frac{1}{103} = \frac{5\cdot19}{8\cdot103} \approx \frac{3}{26 + \frac{1}{157}} = \frac{1}{8.6688}
\]

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Formula of $\alpha$ in fractional number format contains the factors of 2, 3, 17, 23 and 11, Formula of $\delta$ in fractional number format contains the factors of 2, 3, 23 and 11, so it is supposed that there would be the third Feigenbaum constant $\gamma$ which should relate to factors of 2, 3, 17 and 11.

5. Bifurcation Diagram and Feigenbaum Constants

There are three features in a typical bifurcation diagram, which are “point”, “tine” and “gap”. These three features should correspond to three Feigenbaum constants $\delta$, $\alpha$ and $\gamma$ (Fig. 1), so there should be three Feigenbaum constants.

6. Integrated Formulas of $\alpha_1$, $\delta$ and $2\pi$

$$
(2\pi)_{\text{Chen}} = e^2 \left( \frac{2}{1} \right)^3 \left( \frac{3}{2} \right)^5 \left( \frac{4}{3} \right)^7 \cdots \left( \frac{k+1}{k} \right)^{2k+1}
$$

$$
(2\pi)_{\text{Wallis}} = 4 \cdot \left( \frac{2}{3} \cdot \frac{4}{5} \cdot \frac{6}{5} \cdot \cdots \frac{2k}{2k+1} \right)
$$

$$
(2\pi)_{\text{GL}} = 8 \cdot (1 + \frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \cdots + \frac{1}{2k+1})
$$

(GL means Gregory-Leibniz)
\[(2\pi)_{NC}^{-4} = 6 + \sum_{n=1}^{k} \frac{(-1)^{n+1}}{n(n+1/2)(n+1)} \quad \text{(NC means Nilakantha-Chen)}\]

\[
\alpha_1 = \frac{1}{137.035990373455} = \frac{1}{4.66920160910299^2 \cdot e^2 \cdot e^2 \cdot e^2 \cdot e^2 \cdot e^2 \cdot e^2 \cdot e^2 \cdot e^2 \cdot e^2} + \frac{1}{128 \cdot 9 \cdot (2 \cdot 3 \cdot 173 + 1) + \frac{7}{2 \cdot 5}}
\]

\[
(\alpha_1)_{NC} = \frac{1}{163 \cdot (6 \cdot 11^2 \cdot 97 + 1)} = \frac{1}{4.66920160910299^3 \cdot 8 \cdot (1 + \frac{1}{3} - \frac{1}{5} + \frac{1}{7} - \cdots - \frac{1}{2 \cdot 2 \cdot 11 \cdot 17 + 1})}
\]

\[
\alpha_1 = \frac{1}{137.035990373455} = \frac{1}{4.66920160910299^2 \cdot (6 + \sum_{n=1}^{3} \frac{(-1)^{n+1}}{n(n+1/2)(n+1)})}
\]

\[
(\alpha_1)_{NC} = \frac{1}{169 \cdot 2 \cdot 3 \cdot 5 \cdot 7 \cdot 11 + 1} = \frac{1}{169 \cdot 2 \cdot 3 \cdot 5 \cdot 7 \cdot 11 + 1}
\]
The Fine-structure Constant: \( \alpha_1 = 1/137.035999037435 \)
\( \alpha_2 = 1/137.035999111818 \)

The Speed of Light in Vacuum in Atomic Units:
\[ c_{au} = \frac{1}{\alpha_c} = \frac{1}{\sqrt{\alpha_1 \alpha_2}} = 137.035999074626 \]

Feigenbaum Constants:
\( \delta = 4.66920160910299 \)
\( \alpha = 2.50290787509589 \)

\[ \alpha_1 \delta^2 (2\pi)_{Chen-25,17} = 1 + \frac{1}{128 \cdot 9 \cdot (2 \cdot 3 \cdot 173 + 1) + \frac{7}{2 \cdot 5}} \approx 1 \]

\[ \alpha_1 \delta^2 (2\pi)_{Wallis-9,71} = 1 + \frac{1}{2 \cdot 9 \cdot (8 \cdot 3 \cdot 25 \cdot 7 \cdot 59 - 1) - \frac{2}{5}} \approx 1 \]

\[ \alpha_1 \delta^2 (2\pi)_{GL-22,37} = 1 + \frac{1}{163 \cdot (6 \cdot 11^2 \cdot 97 + 1)} \approx 1 \]

\[ \alpha_1 \delta^2 (2\pi)_{NC-3} = 1 + \frac{1}{3 \cdot (2 \cdot 7 \cdot 31 + 1)} - \frac{1}{13 \cdot 89 \cdot (2 \cdot 37 \cdot 53 + 1) - \frac{5}{8}} \approx 1 \]

\( \alpha_1 \delta^2 (2\pi) \approx 1 \)
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References:


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Appendix I: Research History

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