Formulas of Feigenbaum Constants and Their Physical Meanings

Gang Chen†, Tianman Chen, Tianyi Chen

Guangzhou Huifu Research Institute Co., Ltd., Guangzhou, P. R. China
7-20-4, Greenwich Village, Wangjianglu 1, Chengdu, P. R. China

†Correspondence to: gang137.chen@connect.polyu.hk

Dedicated to Prof. Albert Sun-Chi Chan on the occasion of his 70th birthday

Abstract

This paper supposes that Feigenbaum constants should be rational numbers in the world of nuclides, gives their formulas in fractional number format and exhibits the physical meanings of the factors in the formulas, especially their relationships with nuclides, the fine-structure constant and 2π. This paper also supposes that there would be the third Feigenbaum constant and gives its two possible approximate values.

Keywords: Feigenbaum constants; the fine-structure constant; nuclides.

1. Introduction

Feigenbaum constants are characterizing constants in chaotic systems, it is assumed by scientific community that they would characterize chaos just as 2π stands for periodicity. The nuclei of nuclides except proton should be multiple-body systems and hence chaos should be the common state in the world of nuclei. So Feigenbaum constants should express some functions in nuclides.

There are two Feigenbaum constants, i. e., \( \delta = 4.6692\ldots \) and \( \alpha = 2.5029\ldots \), and mathematically they should be irrational numbers with infinite digits.

In our previous papers\(^{1,2,3}\), we had already given formulas of the fine-structure constant and given two values of it, i. e., \( \alpha_1 = 1/137.035999037435 \) and \( \alpha_2 = 1/137.035999111818 \) which are rational numbers with 15 digits. The values and formulas of the fine-structure constant show strong relationships with nuclides, and hereby some relevant examples of these relationships are listed as follows. And it
shows that the values of the fine-structure constant express as integer numbers of 136, 137 and 138 in the world of nuclides.

In our previous paper\textsuperscript{1,2,3}, we also exhibited the relationship of $2\pi$ with nuclides, for example, $2\pi \approx 6.28 = (4 \times 157)/100$ relates to nuclides as follows.

In this paper, we suppose Feigenbaum constants should also be rational numbers in the world of nuclides, give their formulas and exhibit their relationships with nuclides and hence with the fine-structure constant and $2\pi$.

2. Formulas of Feigenbaum Constants in Fractional Numbers

It is supposed that Feigenbaum constants should be rational numbers with 15 digits and have the following fractional formulas and relationships with nuclides.

Note: $136 = 8 \cdot 17$, $138 = 6 \cdot 23$

$$\frac{1}{\delta} = \frac{1}{4.66920160910299} = 0.214169377062326$$

$$\frac{1}{\alpha} = \frac{1}{2.50290787509589} = 0.399535280523135$$
3. Formulas of Feigenbaum Constants in Continued Fractional Numbers

\[ \frac{1}{\delta} = \frac{1}{4.66920160910299} = 0.214169377062326 = \frac{3}{14 + \frac{1}{131 + \frac{1}{2 + \frac{1}{54 + \frac{1}{6 + \frac{1}{18 + \frac{1}{131 + \frac{1}{2}}}}}}}} \]

\[ \frac{1}{\delta} = \frac{1}{4.66920160910299} = 0.214169377062326 = \frac{3}{14 + \frac{1}{131 + \frac{1}{2 + \frac{1}{54 + \frac{1}{6 + \frac{1}{18 + \frac{1}{131 + \frac{1}{2}}}}}}}} \]

\[ \frac{1}{\delta} = \frac{1}{4.66920160910299} = 0.214169377062326 = \frac{3}{14 + \frac{1}{131 + \frac{1}{2 + \frac{1}{54 + \frac{1}{6 + \frac{1}{18 + \frac{1}{131 + \frac{1}{2}}}}}}}} \]

\[ \frac{1}{\delta} = \frac{1}{4.66920160910299} = 0.214169377062326 = \frac{3}{14 + \frac{1}{131 + \frac{1}{2 + \frac{1}{54 + \frac{1}{6 + \frac{1}{18 + \frac{1}{131 + \frac{1}{2}}}}}}}} \]

\[ \frac{1}{\delta} = \frac{1}{4.66920160910299} = 0.214169377062326 = \frac{3}{14 + \frac{1}{131 + \frac{1}{2 + \frac{1}{54 + \frac{1}{6 + \frac{1}{18 + \frac{1}{131 + \frac{1}{2}}}}}}}} \]

\[ \frac{1}{\delta} = \frac{1}{4.66920160910299} = 0.214169377062326 = \frac{3}{14 + \frac{1}{131 + \frac{1}{2 + \frac{1}{54 + \frac{1}{6 + \frac{1}{18 + \frac{1}{131 + \frac{1}{2}}}}}}}} \]

\[ \frac{1}{\delta} = \frac{1}{4.66920160910299} = 0.214169377062326 = \frac{3}{14 + \frac{1}{131 + \frac{1}{2 + \frac{1}{54 + \frac{1}{6 + \frac{1}{18 + \frac{1}{131 + \frac{1}{2}}}}}}}} \]
\[
\frac{1}{\alpha} = \frac{1}{2.50290787509589} = 0.399535280523135
\]

\[
= \frac{2}{5 + \frac{1}{171 + \frac{1}{1 + \frac{1}{17 + \frac{1}{1 + \frac{1}{4 + \frac{1}{1 + \frac{1}{7 + \frac{1}{1 + \frac{1}{16}}}}}}}}}}
\]

\[
= \frac{2}{5 + \frac{1}{171 + \frac{1}{1 + \frac{1}{17 + \frac{1}{1 + \frac{1}{4 + \frac{1}{1 + \frac{1}{7 + \frac{1}{1 + \frac{1}{16}}}}}}}}}}
\]
\[
\frac{1}{\alpha} = \frac{1}{2.50290787509589} = 0.399535280523135
\]

\[
= \frac{1}{5 + \frac{1}{171 + \frac{1}{1 + \frac{1}{17 + \frac{11 \cdot 37}{4 + 13 \cdot 59}}}}}
\]

\[
\frac{1}{\alpha} = \frac{1}{2.50290787509589} = 0.399535280523135
\]

\[
= \frac{1}{5 + \frac{1}{171 + \frac{1}{1 + \frac{1}{25 \cdot (4 \cdot 5 \cdot 7 - 1)}}}}
\]

\[
\frac{1}{\alpha} = \frac{1}{2.50290787509589} = 0.399535280523135
\]

\[
= \frac{2}{5 + \frac{1}{171 + \frac{25 \cdot (4 \cdot 5 \cdot 7 - 1)}{2 \cdot 3 \cdot 7 \cdot 101}}}
\]

\[
\frac{1}{\alpha} = \frac{1}{2.50290787509589} = 0.399535280523135
\]

\[
= \frac{2}{5 + \frac{97 \cdot (6 \cdot 137 + 1)}{2 \cdot 5 \cdot (2 \cdot 113 + 1)}}
\]

\[
\frac{1}{\alpha} = \frac{1}{2.50290787509589} = 0.399535280523135
\]

\[
= \frac{2}{5 + \frac{97 \cdot (6 \cdot 137 + 1)}{2 \cdot 5 \cdot (2 \cdot 113 + 1)}}
\]

\[
\frac{1}{\alpha} = \frac{1}{2.50290787509589} = 0.399535280523135
\]

\[
= \frac{2}{5 + \frac{97 \cdot (6 \cdot 137 + 1)}{2 \cdot 5 \cdot (2 \cdot 113 + 1)}}
\]

\[
\frac{1}{\alpha} = \frac{1}{2.50290787509589} = 0.399535280523135
\]
4. The Third Feigenbaum Constant

There would be the third Feigenbaum constant $\gamma$, and we could only give some guesses at present stage as follows.

Note: $136=8 \cdot 17$, $138=6 \cdot 23$

\[
\frac{1}{\alpha} = \frac{1}{2.5029078787509589} = 0.399535280523135 = f(2,3,17,23,11,\ldots)
\]
\[
= \frac{1}{2} - \frac{1}{9} + \frac{1}{3 \cdot 31} - \frac{1}{23 \cdot (8 \cdot 3 \cdot 17 + 1)} + \frac{1}{17 \cdot 23 \cdot (8 \cdot 3 \cdot 11^4 - 1)}
\]
\[
= \frac{2}{5 + \frac{1}{171 + \frac{1}{97 \cdot (6 \cdot 137 + 1)}}}
\]
\[
\frac{1}{\delta} = \frac{1}{4.66920160910299} = 0.214169377062326 = f(2,3,23,11,\ldots)
\]
\[
= \frac{1}{4} - \frac{1}{27} + \frac{1}{4 \cdot 9 \cdot 23} - \frac{1}{2 \cdot 3 \cdot 7 \cdot 23 \cdot (2 \cdot 3 \cdot (4 \cdot 3 \cdot 11 - 1) + 1)} + \frac{2 \cdot 23}{3 \cdot 19}
\]
\[
= \frac{3}{14 + \frac{1}{131 + \frac{1}{9 \cdot 83 \cdot 109}}}
\]
\[
\frac{1}{\gamma} = f(2,3,17,\ldots) \approx 1 - \frac{1}{3} + \frac{1}{8 \cdot 17} = \frac{25 \cdot 11}{8 \cdot 3 \cdot 17} \approx \frac{2}{3} - \frac{1}{31 + \frac{1}{2}} = \frac{2 \cdot 9 \cdot 7}{11 \cdot 17} \approx \frac{1}{1.484}
\]

or
\[
\frac{1}{\gamma} = f(2,3,17,\ldots) \approx 1 - \frac{1}{8} + \frac{1}{8 \cdot 3 \cdot 17} = \frac{317}{2 \cdot 81 \cdot 17} \approx \frac{3}{26 + \frac{1}{15}} = \frac{9 \cdot 5 \cdot 7}{17 \cdot 23} \approx \frac{1}{8.689}
\]

2021/1/27 – 28

\[
\frac{1}{\gamma} = f(2,3,17,\ldots) \approx 1 - \frac{1}{8} + \frac{3 \cdot 31}{8 \cdot 101} = \frac{3}{26 + \frac{1}{151}} = \frac{151}{7 \cdot 11 \cdot 17} \approx \frac{1}{8.6689}
\]

\[
\frac{1}{\gamma} = f(2,3,17,\ldots) \approx 1 - \frac{1}{8} + \frac{5 \cdot 19}{8 \cdot 103} \approx \frac{3}{26 + \frac{1}{157}} = \frac{157}{6 \cdot 227 - 1} \approx \frac{1}{8.6688}
\]

2021/1/30 – 31
Formula of \( \alpha \) in fractional number format contains the factors of 2, 3, 17, 23 and 11. Formula of \( \delta \) in fractional number format contains the factors of 2, 3, 23 and 11, so it is supposed that there would be the third Feigenbaum constant \( \gamma \) which should relate to factors of 2, 3, 17 and 11.

5. Bifurcation Diagram and Feigenbaum Constants

There are three features in a typical bifurcation diagram, which are “point”, “tine” and “gap”. These three features should correspond to three Feigenbaum constants \( \delta, \alpha \) and \( \gamma \) (Fig. 1), so there should be three Feigenbaum constants.
References:

Acknowledgements
Yichang Huifu Silicon Material Co., Ltd., Guangzhou Huifu Research Institute Co., Ltd. and Yichang Huifu Nanometer Material Co., Ltd. have been giving Dr. Gang Chen a part-time employment since Dec. 2018. Thank these companies for their financial support. Specially thank Dr. Yuelin Wang and other colleagues of these companies for their appreciation, support and help.

Thank Prof. Wenhao Hu, the dean of School of Pharmaceutical Sciences, Sun Yet-Sen University, for providing us an apartment in Shanghai since January of 2021 and hence facilitating the process of writing this paper.

Appendix I: Research History

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
<th>Date</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>2</td>
<td>2021/1/23-24</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>3-5</td>
<td>2021/1/20-22.28</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>6-7</td>
<td>2021/1/27-28,30-31</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>7</td>
<td>2021/1/28-30</td>
<td></td>
</tr>
<tr>
<td>Preparing this paper</td>
<td>1-8</td>
<td>2021/1/20-31</td>
<td></td>
</tr>
</tbody>
</table>