Cosmological General Theory of Relativity

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ABSTRACT

In expanded universe, we found gravity field equation and solution. We found Schwarzschild solution, Kerr-Newman solution in expanded universe. Hence, We found new general relativity theory-Cosmological General Theory of Relativity(CGTR).

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1. Introduction

Our article's aim is that we make Cosmological General theory of Relativity (CGRT).

At first, Robertson-Walker metric is

$$d\tau^2 = dt^2 - \frac{1}{c^2}\Omega^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right] \tag{1}$$

According to ΛCDM model, our universe's k is zero. In this time, if t_0 is cosmological time,[2]

$$k = 0, t = t_0 >> \Delta t\ ,\ \ \Delta t \text{ is period of matter's motion} \tag{2}$$

Hence, the proper time is in cosmological time,

$$\begin{aligned} d\tau^2 &= dt^2 - \frac{1}{c^2}\Omega^2(t_0)[dr^2 + r^2 d\Omega^2] \\ &= dt^2 - \frac{1}{c^2}\Omega^2(t_0)[dx^2 + dy^2 + dz^2] \\ &= dt^2(1 - \frac{1}{c^2}\Omega^2(t_0)V^2), \quad V^2 = \frac{dx^2 + dy^2 + dz^2}{dt^2} \end{aligned} \tag{3}$$

In this time,

$$d\bar{t} = dt, d\bar{x} = \Omega(t_0)dx, d\bar{y} = \Omega(t_0)dy, d\bar{z} = \Omega(t_0)dz \tag{4}$$

In Cosmological General theory of Relativity(CGTR)'s differential operators are

$$\frac{1}{c}\frac{\partial}{\partial \bar{t}} = \frac{1}{c}\frac{\partial}{\partial t}, \frac{\partial}{\partial \bar{x}} = \frac{\partial}{\partial x}\frac{1}{\Omega(t_0)}, \frac{\partial}{\partial \bar{y}} = \frac{\partial}{\partial y}\frac{1}{\Omega(t_0)}, \frac{\partial}{\partial \bar{z}} = \frac{\partial}{\partial z}\frac{1}{\Omega(t_0)} \tag{5}$$

Hence,

$$\frac{1}{c^2}\frac{\partial^2}{\partial \bar{t}^2} - \bar{\nabla}^2 = \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \frac{1}{\Omega^2(t_0)}\{(\frac{\partial}{\partial x})^2 + (\frac{\partial}{\partial y})^2 + (\frac{\partial}{\partial z})^2\} \tag{6}$$

2. Newtonian Gravity in Expanded Universe

Newton's Gravity is built in static universe. Hence, for making our cosmological theory, we modified Newtonian Gravity in expanded universe.

At first, Newton's gravity acceleration is

$$\bar{\vec{a}} = \vec{a}\Omega(t_0) = -\bar{\vec{\nabla}}\bar{\phi} = -\frac{1}{\Omega^2(t_0)}\vec{\nabla}\phi \tag{7}$$

$$\bar{\phi} = \phi\frac{1}{\Omega(t_0)} = -\frac{GM}{r\Omega(t_0)} \tag{8}$$

or

$$\bar{\phi} = \phi\frac{1}{\Omega(t_0)} = \frac{1}{2}\frac{GM}{R^3\Omega^3(t_0)}r^2\Omega^2(t_0) = \frac{1}{2}\frac{GM}{R^3}r^2\frac{1}{\Omega(t_0)} \tag{9}$$

In this time, if Newton's gravity potential is

$$\bar{\phi} = \phi\frac{1}{\Omega(t_0)} = -\frac{GM}{r\Omega(t_0)} \tag{10}$$

Eq(7) is

$$\bar{\vec{a}} = \vec{a}\Omega(t_0) = -\frac{1}{\Omega^2(t_0)}\vec{\nabla}\phi = -\frac{GM}{r^3}\vec{r}\frac{1}{\Omega^2(t_0)} \tag{11}$$

If Newton's gravity potential is

$$\bar{\phi} = \phi\frac{1}{\Omega(t_0)} = \frac{1}{2}\frac{GM}{R^3}r^2\frac{1}{\Omega(t_0)} \tag{12}$$

Poisson equation is in expanded universe,

$$\bar{\nabla}^2\bar{\phi} = \frac{1}{\Omega^3(t_0)}\nabla^2\phi = 4\pi G\bar{\rho}\,,\ \ \bar{\rho} = \frac{\rho}{\Omega^3(t_0)} \tag{13}$$

Newton force is in expanded universe,

$$\bar{\vec{F}} = m_0\bar{\vec{a}} = m_0\vec{a}\Omega(t_0) = \vec{F}\Omega(t_0) \tag{14}$$

3. Cosmological General Theory of Relativity

Einstein's geodesic equation is in expanded universe,

$$\frac{d^2\bar{x}^\mu}{d\tau^2} + \bar{\Gamma}^\mu_{\alpha\beta}\frac{d\bar{x}^\alpha}{d\tau}\frac{d\bar{x}^\beta}{d\tau} = 0 \tag{15}$$

Schwarzschild solution (vacuum solution) is in expanded universe,

$$ds^2 = -c^2\left(1-\frac{2GM}{\bar{r}c^2}\right)dt^2 + \frac{d\bar{r}^2}{1-\frac{2GM}{\bar{r}c^2}} + \bar{r}^2d\theta^2 + \bar{r}^2\sin^2\theta d\varphi^2$$

$$= -c^2\left(1-\frac{2GM}{r\Omega(t_0)c^2}\right)dt^2 + \Omega^2(t_0)\left[\frac{dr^2}{1-\frac{2GM}{r\Omega(t_0)c^2}} + r^2(d\theta^2 + \sin^2\theta d\varphi^2)\right] \tag{16}$$

Hence, Newtonian approximation is by Eq(11)

$$\bar{a}_r = \frac{d^2r}{d\tau^2}\Omega(t_0) \approx -\bar{\Gamma}^1_{00}c^2\left(\frac{dt}{d\tau}\right)^2 \approx \frac{1}{2}c^2\frac{\partial\bar{g}_{00}}{\partial\bar{r}} = \frac{1}{2}\frac{c^2}{\Omega(t_0)}\frac{\partial g_{00}}{\partial r} = -\frac{GM}{r^2}\frac{1}{\Omega^2(t_0)} \tag{17}$$

Hence, the gravity field equation of Einstein in expanded universe,

$$\bar{R}_{\mu\upsilon} - \frac{1}{2}\bar{g}_{\mu\upsilon}\bar{R} = \frac{8\pi G}{c^4}\bar{T} \tag{18}$$

In this time,

$$\bar{T}_{00} = \bar{\rho} c^2 = \frac{\rho}{\Omega^3(t_0)} c^2 = \frac{T_{00}}{\Omega^3(t_0)} \qquad (19)$$

Einstein’s general solution- Kerr-Newman solution is in expanded universe,[1]

$$ds^2 = \bar{g}_{\mu\upsilon} d\bar{x}^{\mu} d\bar{x}^{\upsilon}$$

$$= -c^2 \left(1 - \frac{2c^2 GM\bar{r} - kGQ^2}{c^4 \bar{\Sigma}}\right) dt^2 - 2(2c^2 MG\bar{r} - kGQ^2) \frac{\bar{a} \sin^2\theta}{c^4 \bar{\Sigma}} c\,dt\,d\varphi$$

$$+ \frac{c^4 \bar{\Sigma}}{\bar{r}^2 - c^2 2GM\bar{r} + \bar{a}^2 + kGQ^2} d\bar{r}^2 + \bar{\Sigma} d\theta^2$$

$$+ \sin^2\theta [\bar{r}^2 + \bar{a}^2 + (2c^2 GM\bar{r} - kGQ^2) \frac{\bar{a}^2 \sin^2\theta}{c^4 \bar{\Sigma}}] d\varphi^2$$

$$\bar{\Sigma} = \bar{r}^2 + \bar{a}^2 \cos^2\theta = (r^2 + a^2 \Omega^2(t_0) cos^2\theta) \Omega^2(t_0)$$

$$= \Sigma' \Omega^2(t_0) \ , \quad \Sigma' = r^2 + a^2 \Omega^2(t_0) cos^2\theta \qquad (20)$$

Hence, Kerr-Newman solution is expanded universe,

$$ds^2 = \bar{g}_{\mu\upsilon} d\bar{x}^{\mu} d\bar{x}^{\upsilon}$$

$$= -c^2 \left(1 - \frac{2c^2 GMr\Omega(t_0) - kGQ^2}{c^4 \Sigma' \Omega^2(t_0)}\right) dt^2$$

$$-2(2c^2 MGr\Omega(t_0) - kGQ^2) \frac{a\Omega^2(t_0) \sin^2\theta}{c^4 \Sigma' \Omega^2(t_0)} c\,dt\,d\varphi$$

$$+\Omega^2(t_0) \{ \frac{c^4 \Sigma' \Omega^2(t_0)}{r^2 \Omega^2(t_0) - c^2 2GMr\Omega(t_0) + a^2 \Omega^4(t_0) + kGQ^2} dr^2 + \Sigma' d\theta^2$$

$$+ \sin^2\theta [r^2 + a^2 \Omega^2(t_0) + (2c^2 GMr\Omega(t_0) - kGQ^2) \frac{a^2 \Omega^4(t_0) \sin^2\theta}{\Omega^4(t_0) c^4 \Sigma'}] d\varphi^2 \}$$

$$\bar{\Sigma} = \bar{r}^2 + \bar{a}^2 \cos^2\theta = (r^2 + a^2 \Omega^2(t_0) cos^2\theta) \Omega^2(t_0)$$

$$= \Sigma' \Omega^2(t_0) \ , \quad \Sigma' = r^2 + a^2 \Omega^2(t_0) cos^2\theta \qquad (21)$$

Robertson-Walker solution is Minkowski space-time by Einstein gravity field equation in CGTR-Eq(18) in expanded universe.

$$ds^2 = -c^2dt^2 + [d\bar{r}^2 + \bar{r}^2 d\Omega^2] = -c^2dt^2 + \Omega^2(t_0)[dr^2 + r^2 d\Omega^2] \qquad (22)$$

Eq(22) is equal to Eq(3). Eq(1) is derived by normal Einstein gravity field equation.

4. Conclusion

We find Cosmological General theory of Relativity. We obtain solution of Einstein gravity field equation in expanded universe..

Reference

[1]S.Yi, ”PMBH Theory of Representation of Gravity Field Equation and Solution, Hawking Radiation in Data General Relativity Theory”, International Journal of Advanced Research in Physical Science,**5**,9,(2018),pp36-45

[2]S.Yi, ”Cosmological Special Theory of Relativity”, International Journal of Advanced Research in Physical Science,**7**,11,(2020),pp4-9

[3]S.Yi, ”Spherical Solution of Classical Quantum Gravity”, International Journal of Advanced Research in Physical Science,**6**,8,(2019),pp3-6

[4]A. Einstein, “ Zur Elektrodynamik bewegter K¨orper”, Annalen der Physik. 17:891(1905)

[5]Friedman-Lemaitre-Robertson-Walker metric-Wikipedia

[6]A.Miller, Albert Einstein’s Special Theory of Relativity(Addison-Wesley Publishing Co., Inc., 1981)

[7]Lambda-CDM model -Wikipedia

[8]S. Weinberg, Gravitation and Cosmology(John wiley & Sons,Inc,1972)

[9]C. Misner, K, Thorne and J. Wheeler, Gravitation (W.H.Freedman & Co.,1973)

[10]S. Hawking and G. Ellis,The Large Scale Structure of Space-Time(Cam-bridge University Press,1973)

[11]R. Adler,M.Bazin and M.Schiffer,Introduction to General Relativity(McGraw-Hill,Inc.,1965)

[12]P. Roll, R. Krotkov and R. Dicke, Ann. Phys.(U.S.A) **26**,442(1964)

[13]V. Braginsky and V. Panov, Zh. Eksp. & Teor. Fiz.**61**, 873(1971)(English translation, Sov. Phys.-JETP **34**,464(1971)

[14]L.Einsenhart, Riemannian Geometry(Princeton University Press, 1926)

[15]J. Schouten, Ricci-Calculus(Springer-Verlag, Berlin, 1954)

[16]A. Einstein,”The Foundation of General Relativity”, Ann. Phys. (Germany)**49**(1916)

[17]G.Birkoff, Relativity and Modern Physics (Harvard University Press,1923),p253

[18]A.Raychaudhuri, Theorical Cosmology(Oxpord University Press,1979)

[19]E. Kasner, Am. J. Math. **43**, 217(1921)

[20]D.N.Page. ”Hawking Radiation and Black hole Thermodynamics.”.arXiv:hep-th/040924

[21]M.Rabinowitz.”Gravitaational Tunneling Radiation”. Physics Essays. **12**(2):346-357.arxiv:astro-ph/0212249(2000)

[22]S,Giddings and S.Thomas ”High energy colliders as black hole factories: The end of short distant physics”. Physical Review D. **65**(5)(2002)

[23]S.Dimoploulus and G.Landsberg “Black holes at the Large Hardron Collider”. Physical Review Letter**.** **87**(16):161602.arxiv:hep-th/0106295(2001)

[24]F.Belgiorno ,S.Cacciatori, M.Clerici, V.Gorini, G. Ortenzi, L.Rizzi ,E.Rubbino, V.Sala D. Faccio.”Hawking Radiation from ultrashort laser pulse filaments”.Phys. Rev. Lett. **105**(20):203901(2010):arxiv:1009.4634

[25]K. Kumar,B. Kiranagi,C.Begewadi,”Hawking Radiation- An Augmentation Attrition Model”.Adv.Nat.Sci**.5**.(2):14-33(2012)

[26]A.Helfer”Do black-holes radiate?”.Reports on Progress in Physics. **66**(6):943-1008(2003):arxiv:gr-qc/0304042.(2003)

[27]R.Brout,S.Massar,R.Parentani,P.Spindel”Hawking radiation without tras-Plankian frequencies”.Phiscal Review D.**52**(8):4559-4568(1995):arxiv:hep-th/9506121

[28]D.Page,”Particle emission rates from a black-hole:Massless particles from an unchrge,nonrotating hole”.Physical Review D.**13**(2):198-206(1976)

[29]T.Jacobson,”Black hole-evaporation and ultrashorts distances” Physical Review D**.44**(6):1731-1739(1991)

[30]J.Kapusta,”The Last Eight Minutes of a Primordial Black-hole”(1999):Arxiv:astro-ph/9911309

[31]A.Ashtekar,J.Baez,A.Corichi,K.Krasnov,”Quantum Geometry and Black Hole Entropy”Phys.Rev.Lett.**80**(5):904-907(1998):arxiv:gr-qc/9710007