On a Field Theory of Entropy

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Abstract
Starting from an argument on the impossibility of emergence of temperature within current physics, we propose to take temperature as a fundamental property of matter. As a consequence, this fundamental property of matter must produce a field of entropy according to the equation \( \Box S = -\frac{\kappa^2}{k_B^2} \theta \), where \( \kappa = \frac{8\pi G}{c^4} \) is Einstein’s constant, and \( \theta \) temperature density per unit volume.

Contents
1 Introduction 1
2 On the impossibility of emergence of temperature in current system of physics 2
   2.1 Rules of definitions in logic ...................... 2
   2.2 Definition of temperature ............................ 3
3 Incompatibility of Special Relativity and Fourier’s theory of Heat Conduction 4
4 Proposed Field Equation for Entropy 5
   4.1 Heuristic derivation of the proposed equation .......... 6
   4.2 Law of Conservation of Informedness ................. 7
5 Regarding Blackhole Thermodynamics 7

1 Introduction
Altogether there are two major theoretical problems with our overall understanding of heat and the related phenomena. The first is a logical problem lurking at the foundations of statistical physics which makes its goal of explanation of temperature as an emergent property of many particles impossible, and the second is the well-known Incompatibility of Special Relativity and Fourier’s theory of Heat Conduction. In this paper we shall propose an equation which solves both of these problems.

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2 On the impossibility of emergence of temperature in current system of physics

Since understanding the logical problem requires having a clear understanding of rules of logic to avoid being deceived by fallacies, we begin by a logical analysis of definitions.

Because the purpose of a definition is to construct a concept and to eliminate ambiguity while encompassing all the wanted examples of that concept that which we already know of, logic is what ought to be our guiding principle.

In logic, that which is being defined is called the *definiendum* and the words or images used to explain the meaning of the definiendum are called the *definiens*. For example,

A circle is a shape consisting of all points in a plane that are at a given distance from a given point.

In this example, circle is the concept that which we want to define, therefore the definiendum; And the rest of the sentence after ‘is’ are the definiens. Notice that this definition of circle does indeed encompass all the intended examples of ‘circle’ and no more unintended ones; This is among rules that we mention in the next section.

2.1 Rules of definitions in logic

In order to satisfy what definitions intend to do (i.e. clarification and conceptualization), we are bound by the rules of logic on how to define a term. Besides that a definition has to be precise and clear, a definition should not be negative where it can be affirmative and it must only state the essential attributes (not too broad not too narrow). There is a law that we are going to specially focus on, because it is most related to the subject matter of this paper: It is a logical necessity that a definition must not be circular. Synonyms and antonyms of the definiendum or any word, concept or image which in order to define or understand it, one has to refer back to the original definiendum must be absolutely avoided and not be re-stated in the definiens, otherwise, the definition has not fulfilled its intended purpose; it will not take any advance on conceptualization of the definiendum, and it will have no theoretical value and does not help us to understand the quiddity of the definiendum or get closer to understand it.

An infinite regress can also result from successive automatic substitutions of the definiens for the definiendum located in the definiens at each step of the regress[1]. For example,

A teacher is a person who teaches.

Which is equivalent to

A teacher is a person who does what a teacher does.

Which is equivalent to

A teacher is a person who does ‘what a person who does what a teacher does’, does *ad infinitum*.

This is a tautology formally known as the fallacy of circular definition.
2.2 Definition of temperature

According to statistical mechanics, the temperature of a system in equilibrium is defined by

\begin{equation}
\frac{1}{T} := \frac{\partial S}{\partial E}
\end{equation}

where \( S = k_B \log W(E) \). This established mathematical definition results in other interpretations of said definition, such as defining temperature by the average kinetic energy of particles

\( \frac{3}{2} N k_B T = \frac{1}{2} \langle m v^2 \rangle \)

But, there is a logical problem with this definition: Temperature is not ‘the average kinetic energy of a system of many particles’, it is the average kinetic energy of a system of many particles divided by the Boltzmann constant. The Boltzmann constant is not a dimensionless constant and is meaningless without its dimensions. Because the dimensions of the Boltzmann constant are an essential part of the definiens of it, and are necessary in the quiddity and what the Boltzmann constant even means, they shall not be omitted logically in statements of this constant; Here is where the logical fallacy becomes clear: In the current system of SI units, the units of the Boltzmann constant is Joules/Kelvin, but Kelvin is the base unit of measurement of temperature; this means, in order to define or to understand [temperature] (which we proved to be a necessary part of the definiens of ‘temperature’), one necessarily has to refer back to the original definiendum (temperature). Therefore, this circular-dependent relation in between the definiens and the definiendum, makes this definition circular.

The same is the issue with the equation (1).

From the perspective of dimensional analysis, the equation (1) of the definiens reads: [temperature] and our definiendum reads [temperature] It is true that this is the condition of dimensional analysis: the dimensions of both sides of the equation have to be the same; but there is more to this, for this oughts to be a definition of temperature; there is not a problem at all with the dimensions being equal per se, the problem occurs when one claims this equation as a definition for temperature, because it overlooks the logical necessity of avoidance of circular logic.

This is, however, not the case with other definitions in physics. For example, Newton’s second law of motion and the definition of ‘force’

\[ \mathbf{F} := m \mathbf{a} \]

Here, the dimensional analysis reads

\[ \text{Newton} = kg m/s^2; \]

In which a ‘Newton’ is defined by \( kgm/s^2 \). This equation sets meaning on what the dimension of newton means, and no already-synonymous concept of the definiendum (‘Newton’) is in the definiens.
As the unit of temperature (kelvin) is one of the basic units of measurement in physics, this observation suggests that within a physics constructed upon a given set of basic units, none of the basic units can be explained to be emergent.

The logical conclusion of the above discussion is that

In the current system of physics, we propose to take temperature as a fundamental property of matter. Therefore, just like the case for mass or electric charge, a single particle can possess temperature; on the precise amount of this temperature for individual elementary particles we shall remain conservative. But from a combination of the theorem of equipartition of energy and special relativity we can say

<table>
<thead>
<tr>
<th>Particle Type</th>
<th>Equipartition Energy</th>
<th>Associated temperature</th>
</tr>
</thead>
<tbody>
<tr>
<td>Massive</td>
<td>$E = (D/2)Nk_BT$</td>
<td>$mc^2 = 2k_BT$</td>
</tr>
<tr>
<td>Ultra-relativistic</td>
<td>$E = DNk_BT$</td>
<td>$pc = 4k_BT$</td>
</tr>
</tbody>
</table>

shall give an estimate for the rest temperature $T_0$ of an uncharged particle. As we are in the realm of relativity, all dimensions must be treated equally and according to special relativity, a particle in fact moves in four dimensions, therefore in the theorem of equipartition of energy\(^1\) we have let $D = 4$ and $N = 1$.

An interesting instance is the rest temperature of proton,

$$T_{0,p} = \frac{m_0pc^2}{2k_B} \approx 5.444 \times 10^{12} \text{ Kelvin}$$

which is statistically the same temperature achieved at ALICE\([5]\).

3 Incompatibility of Special Relativity and Fourier’s theory of Heat Conduction

It is well known that the Fourier equation is incompatible with the theory of relativity\([6]\) for at least one reason: it admits infinite speed of propagation of heat signals within the continuum field. For example, consider a pulse of heat at the origin; then according to Fourier equation, it is felt (i.e. temperature changes) at any distant point, instantaneously. The speed of information propagation is faster than the speed of light in vacuum, which is inadmissible within the framework of relativity.

To proceed with overcoming this incompatibility, we must answer a question:

\(^1\)In a second reading of this paper there is a possible objection: The theorem of equipartition of energy is usually derived from the partition function, but our proposal denies any ontological basis for the statistical treatment of temperature. Our response is that the choice of the factor remains arbitrary unless a direct experimental test of the theorem of equipartition can select the proper factor; something which is extremely difficult, if not epistemologically impossible. On the other hand, it may be the case that this theorem turn out to be a principle, not derivable from prior assumptions, so we better not risk contradicting a possible principle of nature. In any case, our approach is the most consistent and safest approach that one can pursue to maintain coherency of old and new theories.
• What are these heat signals? In other words, what is the field of heat?

It has been long thought that the temperature scalar field \( T(x, t) \) is the carrier of heat signals; in which case the heat equation needs to be modified, but all such attempts of Hyperbolic heat conduction\(^7\)\(^8\)\(^9\)

\[
\frac{1}{C^2} \frac{\partial^2 T}{\partial t^2} + \frac{1}{\alpha} \frac{\partial T}{\partial t} = \nabla^2 T
\]

are still unsatisfactory in that they contain both first and second derivatives, thus still not expressible in a special relativistic-invariant form.

### 4 Proposed Field Equation for Entropy

Our fundamental proposition\(^2\) which solves both of the aforementioned problems at once, is that \( S \), having dimension of entropy\(^3\), is the potential function of the heat signals and these signals propagate at the speed of light\(^4\) which is governed by the following equation

\[
\left( \frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \nabla^2 \right) S = \square S = -\frac{4\pi G k_B^2}{c^4} \theta
\]

where \( \theta \) is temperature density (per unit volume), \( G \) the universal Gravitational constant, \( k_B \) Boltzmann’s constant and \( c \) speed of light in vacuum.

Taken most seriously, this equation represents a new fundamental force, the third inverse-square law of physics after that of Newton and Coulomb, and the first to combine constants from the previous two. As \( \Delta S = -\Delta I \) where \( I \) is information\(^11\), we absorb the minus in \( S \) and take

\[
\square I = \frac{4\pi G k_B^2}{c^4} \theta
\]

to be the law of information propagation in space-time. Accordingly, temperature of a particle maybe renamed as the informedness, \( \iota \) of the particle.\(^5\)

In the next subsection we shall present an inductive arguments of the proposed equation (2) based on the fundamental proposal that a single particle does possess temperature.

\(^2\)The whole discussion of this paper is restricted to vacuum. We shall not deal with the analogue of ‘electromagnetism in matter’.\(^3\)

It need not, however, to be defined as \( S = k_B \log W \), as with the case for Bekenstein-Hawking entropy,

\[ S_{BH} = \frac{k_B A}{4l_P^2}. \]

We shall say more on this in a forthcoming paper.\(^4\)

There is not much choice about the speed of propagation when one considers a fundamental field; special relativity brute-forces all such fields to propagate at \( c \), including the field of gravity (Newtonian gravitational potential).\(^5\)

It might be useful to distinguish \( \iota \) and \( \theta \) by taking \( \iota \) as \(-\theta\), but as \( \theta \) is always positive in the absolute scale, that would mean an ever-negative \( \iota \), which lacks formalistic harmony; we do not, however, deny that possibility that informedness like electric charge may be both positive and negative and it must remain open to empirical judgement.
4.1 Heuristic derivation of the proposed equation

If we posit that a particle possesses a fundamental property, it is an inductive conclusion that the property must be the source of a new interaction; for example we know that mass is a fundamental property and it is the source for gravitational force, or electric charge is a fundamental property and it is the source for electric force. Therefore according to our fundamental proposal that temperature is a fundamental property of (elementary) single particles, it must as well be the source for a new force which we now proceed to find. Let us begin by the observation that if we add mass to the source term (current density) of electromagnetism, we can construct the gravitational potential and from the field equations we are led to the Poisson equation for gravity. To see this, we first construct the five-current

\[ J^\mu := (c\sigma_0\mu, c\rho, J), \]

where \( \mu \) is mass density, \( \rho \) electric charge density and \( J \) electric current density and \( \sigma_0 := \sqrt{4\pi\epsilon_0 G} \).

To construct the five-potential, we first note how the four-potential

\[ A^\mu := (\varphi/c, A), \]

is constructed from the four-current: divide the component of potential by the coefficient of the corresponding component of current, i.e. divide the electric potential by \( c \), therefore the candidate for the gravitational component of current density would be \( \varphi/(c\sigma_0) \), where \( \varphi \) is the (Newtonian) gravitational potential. To validate this candidate it only remains to verify that \( \varphi/(c\sigma_0) \) has the units of Volts/c, which indeed it has. It only remains to observe that the field equation

\[ \Box A^\mu = -\mu_0 J^\mu \]

yields

\[ \left( \frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \nabla^2 \right) \phi = -4\pi G \mu \]

assuming the field is time-independent, we are led to the Poisson equation for gravity

\[ \nabla^2 \phi = 4\pi G \mu. \]

Now let us find the component of five-potential whose source is temperature. In this case the current five-vector would be

\[ J^\mu := (\theta k_B \sigma_0 / c, c\rho, J), \]

where \( \theta \) is temperature density per unit volume and \( k_B \) Boltzmann constant. To find the corresponding component of five-potential we must follow the above-mentioned recipe, except that now we do not know the corresponding potential for temperature and it is in fact our goal to find it. Let us therefore suppose that the corresponding potential is denoted by \( \xi \). Applying our recipe and dimensional analysis simultaneously, we must have

\[ \frac{\xi c}{k_B \sigma_0} = \frac{\text{volt}}{[c]} \]
immediately it would follow that

\[ [\xi] = \text{Joules/Kelvin} \]

which is the same dimensions as that of entropy, therefore the corresponding potential for temperature is entropy. The field equation

\[ \Box A^\mu = -\mu_0 J^\mu \]

would now yield the proposed equation (2).

### 4.2 Law of Conservation of Informedness

\[ \frac{\partial \iota(x, t)}{\partial t} = -\nabla \cdot (\iota \mathbf{v}) \quad (3) \]

Although similar in name, this conservation is a classical law and has nothing to do with the famous statement from quantum mechanics that the quantum-mechanical wave function is the carrier of information, in other words, complete information about a system is encoded in its wave function up to when the wave function collapses. That quantum mechanical statement is theoretically vague for it is based on the informational interpretation of quantum mechanics, so it is dependent upon interpretation in a theory whose advocates abhor interpretation in favour of — although fictitious— mathematical transparency and well-definedness, so the dogma of conventional quantum mechanics calls for some equation, but is unable to provide any such equation and the statement by their standards remains in the level of an statement, not a principle or law.

### 5 Regarding Blackhole Thermodynamics

Our proposed fundamental law (2) suggests that temperature (or informedness) is not a statistical quantity but a fundamental property of elementary particles, very much like mass or electric charge. If (2) is a true law of nature, then entropy is re-defined without any reference to microstates; the only example of such encounter we already know in theoretical physics is the Bekenstein-Hawking entropy of a Schwarzschild blackhole[10],

\[ S_{BH} = \frac{k_B A}{4 l_P^2} \quad (4) \]

where \( A \) is the area of blackhole’s event-horizon and \( l_P = \sqrt{\hbar G/c^3} \) is the Planck length.

With (2) in mind in one hand, and a statistical definition of entropy \( S = k_B \log W \) on the other hand, we are faced with a dichotomy:

\textbf{A})  According to the statistical definition of entropy, the entropy of a Schwarzschild blackhole (4) must come from an ensemble of microstates, whereas

\textbf{B})  Since a Schwarzschild blackhole is formally no different from an uncharged spin-0 elementary particle\(^6\), according to (2) a (Schwarzschild)
blackhole does fundamentally possess temperature, rendering any talk of microstates redundant, if not meaningless.

The only simple and rather-satisfactory treatment along A is the work of Rovelli[12] with the assumption of the quantisation of area, ‘[...] based on the idea that the entropy of the hole originates from the microstates of the horizon that correspond to a given macroscopic configuration’, Rovelli took it that ‘in a thermal context, the Schwarzschild metric represents the coarse-grained description of a microscopically fluctuating geometry.’

There is a serious logical objection to the work of Rovelli, which finally leads to an infinite regress or a contradiction:

Since a Schwarzschild blackhole and an uncharged spin-0 elementary particle are formally the same, The mere acceptance of this statement is a contradiction with the statistical definition of temperature: it means that one has accepted that a single particle possesses temperature! To see how this leads to an infinite regress let us review what has happened to this position in the argument: One came defending the position that temperature is a property of many particles, then admitted it can be attributed to a single particle only to defer its cause to ‘microstates of geometry’; but logically continuing, those microstates themselves must possess temperature coming from microstates of geometry, of geometry and so on (infinite regress), unless there is a lower limit to the geometry of microstates of that elementary particle, for instance Higgs boson, which has to be the Schwarzschild radius of the particle

\[ r_{s,H^0} = \frac{2GM_{H^0}}{c^2} \approx 3.2 \times 10^{-52} \approx 8.3 \times 10^{-23} l_P \]

which is astronomically smaller than the Planck length and no conventional theory of quantum gravity can ever hope to probe that scale, given that theories such as the loop quantum gravity are promised to work in orders of \( l_P \), therefore this alternative has led to a contradiction too.

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References


