Norm of Four Acceleration in Flat Space Time

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Abstract

This article brings out a strange fact that the norm –square of the acceleration four vector in flat space time is not a real number in that it cannot be positive ,negative or zero.

Introduction

Considering four acceleration in the flat space time context we show that the norm-square of the stated four vector is neither positive ,negative or zero. This points to some inconsistency in theory.

Calculations

Four acceleration[1] in flat space time:

$$\left(c\frac{d^2t}{d\tau^2},\frac{d^2x}{d\tau^2},\frac{d^2y}{d\tau^2},\frac{d^2z}{d\tau^2}\right) \quad (1)$$

$\left[[ct]=[x]=[y]=[z]\right]$

Flat space time metric[2]

$$c^2d\tau^2=c^2dt^2-dx^2-dy^2-dz^2 \; (2.1)$$

$$c^2=c^2\left(\frac{dt}{d\tau}\right)^2-\left(\frac{dx}{d\tau}\right)^2-\left(\frac{dy}{d\tau}\right)^2-\left(\frac{dy}{d\tau}\right)^2 \quad (2.2)$$

Differentiating equation (2.1) with respect to proper time we have

$$c^2\frac{dt}{d\tau}\frac{d^2t}{d\tau^2}-\frac{dx}{d\tau}\frac{d^2x}{d\tau^2}-\frac{dy}{d\tau}\frac{d^2y}{d\tau^2}-\frac{dt}{d\tau}\frac{d^2y}{d\tau^2}=0 \; (3)$$

$$\Rightarrow v.a=0 \; (4)$$

For a particle spatially at rest :

$$c^2 \frac{dt}{d\tau}\frac{d^2t}{d\tau^2} = 0$$

Since $\frac{dt}{d\tau} = \gamma \neq 0$

$\frac{d^2t}{d\tau^2} = 0$ for a particle at rest[its energy does not change]

Now,

$$\frac{dx}{d\tau}\frac{d^2x}{d\tau^2} + \frac{dy}{d\tau}\frac{d^2y}{d\tau^2} + \frac{dt}{d\tau}\frac{d^2y}{d\tau^2} = c^2 \frac{dt}{d\tau}\frac{d^2t}{d\tau^2} \quad (5)$$

We define N such that

$$Nc^2 = c^2\left(\frac{d^2t}{d\tau^2}\right)^2 - \left(\frac{d^2x}{d\tau^2}\right)^2 - \left(\frac{d^2y}{d\tau^2}\right)^2 - \left(\frac{d^2z}{d\tau^2}\right)^2 \quad (6.1)$$

$$\left(\frac{d^2x}{d\tau^2}\right)^2 + \left(\frac{d^2y}{d\tau^2}\right)^2 + \left(\frac{d^2z}{d\tau^2}\right)^2 = c^2\left(\frac{d^2t}{d\tau^2}\right)^2 - Nc^2 \quad (6.2)$$

Applying the Cauchy Schwarz inequality[3] we have

$$\left(\left(\frac{d^2x}{d\tau^2}\right)^2 + \left(\frac{d^2y}{d\tau^2}\right)^2 + \left(\frac{d^2z}{d\tau^2}\right)^2\right)\left(\left(\frac{dx}{d\tau}\right)^2 + \left(\frac{dy}{d\tau}\right)^2 + \left(\frac{dy}{d\tau}\right)^2\right) \geq \left(\frac{dx}{d\tau}\frac{d^2x}{d\tau^2} + \frac{dy}{d\tau}\frac{d^2y}{d\tau^2} + \frac{dz}{d\tau}\frac{d^2z}{d\tau^2}\right)^2 \quad (7)$$

Applying (2.2)and (6.2) on equation (7) we obtain,

$$\left[c^2\left(\frac{d^2t}{d\tau^2}\right)^2 - Nc^2\right]\left[c^2\left(\frac{dt}{d\tau}\right)^2 - c^2\right] \geq c^4\left(\frac{d^2t}{d\tau^2}\right)^2\left(\frac{dt}{d\tau}\right)^2 \quad (8)$$

$$c^4\frac{d^2t}{d\tau^2}\left(\frac{dt}{d\tau}\right)^2 - c^4\left(\frac{d^2t}{d\tau^2}\right)^2 - Nc^4\left(\frac{dt}{d\tau}\right)^2 + Nc^4 \geq c^4\left(\frac{d^2t}{d\tau^2}\right)^2\left(\frac{dt}{d\tau}\right)^2$$

$$-c^4\left(\frac{d^2t}{d\tau^2}\right)^2 - Nc^4\left(\frac{dt}{d\tau}\right)^2 + Nc^4 \geq 0$$

$$\left(\frac{d^2t}{d\tau^2}\right)^2 + N\left(\frac{dt}{d\tau}\right)^2 \leq N \quad (9)$$

If $N < 0$, from (16.1)(6.1)

$$Nc^2 = c^2\left(\frac{d^2t}{d\tau^2}\right)^2 - \left(\frac{d^2x}{d\tau^2}\right)^2 - \left(\frac{d^2y}{d\tau^2}\right)^2 - \left(\frac{d^2z}{d\tau^2}\right)^2 < 0$$

For a particle at rest e have an impossible relation

$$\left(\frac{d^2t}{d\tau^2}\right)^2 < 0 \ (10)$$

If $N > 0$

$$N\left[1-\left(\frac{dt}{d\tau}\right)^2\right] > \left(\frac{d^2t}{d\tau^2}\right)^2 \ (11)$$

$$\Rightarrow N\left[1-\left(\frac{dt}{d\tau}\right)^2\right] > 0$$

$$\Rightarrow 1-\left(\frac{dt}{d\tau}\right)^2 > 0$$

$$\left(\frac{dt}{d\tau}\right)^2 < 1 \ (12)$$

But

$$\frac{dt}{d\tau} = \gamma \geq 1$$

$$\left(\frac{dt}{d\tau}\right)^2 \geq 1 \ (13)$$

Equations (20)and (21) run into a contradiction

if N=0 then from (9)

$$\left(\frac{d^2t}{d\tau^2}\right)^2 \leq 0 \ (14)$$

which is again impossible.

All three alternatives, N>0,N<0 and N=0 get ruled out. This points to inconsistencies in the theory.

Conclusions

We have derived that fact that the norm square of the acceleration four vector in fat space time[Cartesian coordinates] is neither positive, negative or zero.

References

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