Norm of Four Acceleration in Flat Space Time

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Abstract

This article brings out a strange fact that the norm –square of the acceleration four vector in flat space time is not a real number in that it cannot be positive ,negative or zero.

Introduction

Considering four acceleration in the flat space time context we show that the norm-square of the stated four vector is neither positive ,negative or zero. This points to some inconsistency in theory.

Calculations

Four acceleration\(^{(1)}\) in flat space time:

\[
\begin{pmatrix}
\frac{c^2}{d\tau^2} & \frac{dx}{d\tau} & \frac{dy}{d\tau} & \frac{dz}{d\tau} \\
\end{pmatrix}
\]

\((1)\)

\[[c\tau] = [x] = [y] = [z]\]

Flat space time metric\(^{(2)}\)

\[c^2 dt^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2 \quad (2.1)\]

\[c^2 = c^2 \left(\frac{dt}{d\tau}\right)^2 - \left(\frac{dx}{d\tau}\right)^2 - \left(\frac{dy}{d\tau}\right)^2 - \left(\frac{dz}{d\tau}\right)^2 \quad (2.2)\]

Differentiating equation (2.1) with respect to proper time we have

\[c^2 \frac{dt}{d\tau} \frac{d^2 t}{d\tau^2} - \frac{dx}{d\tau} \frac{d^2 x}{d\tau^2} - \frac{dy}{d\tau} \frac{d^2 y}{d\tau^2} - \frac{dt}{d\tau} \frac{d^2 t}{d\tau^2} = 0 \quad (3)\]

\[\Rightarrow v \cdot a = 0 \quad (4)\]

For a particle spatially at rest:
\[ c^2 \frac{dt \, d^2 t}{d\tau \, d\tau^2} = 0 \]

Since \( \frac{dt}{d\tau} = \gamma \neq 0 \)

\( \frac{d^2t}{d\tau^2} = 0 \) for a particle at rest [its energy does not change]

Now,

\[ \frac{dx \, d^2x}{d\tau \, d\tau^2} + \frac{dy \, d^2y}{d\tau \, d\tau^2} + \frac{dt \, d^2t}{d\tau \, d\tau^2} = c^2 \frac{dt \, d^2 t}{d\tau \, d\tau^2} \]  \hspace{1cm} (5)

We define \( N \) such that

\[ Nc^2 = c^2 \left( \frac{d^2t}{d\tau^2} \right)^2 - \left( \frac{d^2x}{d\tau^2} \right)^2 - \left( \frac{d^2y}{d\tau^2} \right)^2 - \left( \frac{d^2z}{d\tau^2} \right)^2 \]  \hspace{1cm} (6.1)

\[ \left( \frac{d^2x}{d\tau^2} \right)^2 + \left( \frac{d^2y}{d\tau^2} \right)^2 + \left( \frac{d^2z}{d\tau^2} \right)^2 = c^2 \left( \frac{d^2t}{d\tau^2} \right)^2 - Nc^2 \]  \hspace{1cm} (6.2)

Applying the Cauchy Schwarz inequality \(^{[3]}\) we have

\[ \left( \frac{d^2x}{d\tau^2} \right)^2 + \left( \frac{d^2y}{d\tau^2} \right)^2 + \left( \frac{d^2z}{d\tau^2} \right)^2 \geq \left( \frac{dx}{d\tau} \right)^2 + \left( \frac{dy}{d\tau} \right)^2 + \left( \frac{dz}{d\tau} \right)^2 \]  \hspace{1cm} (7)

Applying (2.2) and (6.2) on equation (7) we obtain,

\[ \left[ c^2 \left( \frac{d^2t}{d\tau^2} \right)^2 - Nc^2 \right] \left[ c^2 \left( \frac{dt}{d\tau} \right)^2 - c^2 \right] \geq c^4 \left( \frac{d^2t}{d\tau^2} \right)^2 \left( \frac{dt}{d\tau} \right)^2 \]  \hspace{1cm} (8)

\[ c^4 \frac{d^2t}{d\tau^2} \left( \frac{dt}{d\tau} \right)^2 - c^4 \left( \frac{d^2t}{d\tau^2} \right)^2 - Nc^4 \left( \frac{dt}{d\tau} \right)^2 + Nc^4 \geq c^4 \left( \frac{d^2t}{d\tau^2} \right)^2 \left( \frac{dt}{d\tau} \right)^2 \]

\[ -c^4 \left( \frac{d^2t}{d\tau^2} \right)^2 - Nc^4 \left( \frac{dt}{d\tau} \right)^2 + Nc^4 \geq 0 \]

\[ \left( \frac{d^2t}{d\tau^2} \right)^2 + N \left( \frac{dt}{d\tau} \right)^2 \leq N \]  \hspace{1cm} (9)

If \( N < 0 \), from (16.1)(6.1)

\[ Nc^2 = c^2 \left( \frac{d^2t}{d\tau^2} \right)^2 - \left( \frac{d^2x}{d\tau^2} \right)^2 - \left( \frac{d^2y}{d\tau^2} \right)^2 - \left( \frac{d^2z}{d\tau^2} \right)^2 < 0 \]
For a particle at rest we have an impossible relation

\[
\left( \frac{d^2 t}{d\tau^2} \right)^2 < 0 \quad (10)
\]

If \( N > 0 \)

\[
N \left[ 1 - \left( \frac{dt}{d\tau} \right)^2 \right] > \left( \frac{d^2 t}{d\tau^2} \right)^2 \quad (11)
\]

\[\Rightarrow N \left[ 1 - \left( \frac{dt}{d\tau} \right)^2 \right] > 0\]

\[\Rightarrow 1 - \left( \frac{dt}{d\tau} \right)^2 > 0\]

\[\left( \frac{dt}{d\tau} \right)^2 < 1 \quad (12)\]

But

\[\frac{dt}{d\tau} = \gamma \geq 1\]

\[\left( \frac{dt}{d\tau} \right)^2 \geq 1 \quad (13)\]

Equations (20) and (21) run into a contradiction

if \( N=0 \) then from (9)

\[\left( \frac{d^2 t}{d\tau^2} \right)^2 \leq 0 \quad (14)\]

which is again impossible.

All three alternatives, \( N>0, N<0 \) and \( N=0 \) get ruled out. This points to inconsistencies in the theory.

**Conclusions**

We have derived that fact that the norm square of the acceleration four vector in flat space-time [Cartesian coordinates] is neither positive, negative or zero.

**References**

2. Griffiths D J, Introduction to Electrodynamics., Pearson Education, India, Eighth Impression, p505 The Invariant Interval

3. Cauchy Schwarz Inequality
   https://en.wikipedia.org/wiki/Cauchy%E2%80%93Schwarz_inequality