Self-consistent hydrodynamic model of vortex plasma

Victor L. Mironov[1, a)]
Institute for physics of microstructures RAS, Nizhny Novgorod, 603950, Russia

(Dated: 6 January 2021)

We propose the system of self-consistent equations for vortex plasma in the framework of hydrodynamic two-fluid model. These equations describe both longitudinal flows and the rotation and twisting of vortex tubes taking into account internal electric and magnetic fields generated by fluctuations of plasma parameters. The main peculiarities of the proposed equations are illustrated with the analysis of electron and ion sound waves.

PACS numbers: 52.30.Cv, 52.35.Dm, 52.35.We
Keywords: Vortex plasma, hydrodynamic model, electron and ion sound waves

I. INTRODUCTION

The hydrodynamic description of plasma is based on two-fluid model, which includes Euler and continuity equations written separately for the electron and ion components, as well as Maxwell's equations for the electromagnetic field.[1,2] This model describes well the low-frequency properties of plasma, but it does not describe the vortex motion and associated disturbances of the electromagnetic field. In recent decades, much attention has been paid to the description of fluid dynamics by vector fields including vectors of fluid speed and vorticity, which satisfy symmetric Maxwell-type equations.[3–11] In particular, a similar approach is used to describe the plasma motion within the framework of a hydrodynamic two-fluid model.[12–15] However, in all mentioned works[3–15] an additional equation for the vortex motion is obtained by taking the "curl" operator from the Euler equation and therefore the resulting equation is not independent. Recently, we have developed an alternative approach based on droplet model of fluid introduced by Helmholtz,[16] and obtained a closed system of Maxwell-type equations for the vortex fluid taking into account the rotation and twisting of vortex tubes.[17] Here we apply these equations to develop the hydrodynamic description of vortex plasma.

II. HYDRODYNAMIC EQUATIONS FOR VORTEX FLOWS IN TWO-FLUID MODEL OF PLASMA

Recently we have shown[17] that the ideal fluid can be described by closed system of Maxwell-like equations for variables corresponding to the longitudinal motion and rotational vortex flows. In particular the free isentropic flow is described by the following symmetric equations:

$$\begin{aligned} &\frac{1}{s}\frac{d\mathbf{v}}{dt} + \nabla u + \nabla\times\mathbf{w} = 0, \\ &\frac{1}{s}\frac{du}{dt} + \nabla\cdot\mathbf{v} = 0, \\ &\frac{1}{s}\frac{d\mathbf{w}}{dt} + \nabla\xi - \nabla\times\mathbf{v} = 0, \\ &\frac{1}{s}\frac{d\xi}{dt} + \nabla\cdot\mathbf{w} = 0. \end{aligned} \tag{1}$$

Here s is the speed of sound, $\mathbf{v}$ is the velocity of the fluid, u is the enthalpy per unit mass, $\mathbf{w}$ is the variable characterizing the rotation of the vortex tubes and ξ is the variable characterizing the twisting of vortex tubes.[17] As usual, we assume the following expression for the material time derivative:

$$\frac{d}{dt} = \frac{\partial}{\partial t} + (\mathbf{v}\cdot\nabla)\,. \tag{2}$$

In the commonly used hydrodynamic approach, plasma is represented as a mixture of two fluids in which the particles have different masses and charges. We will consider only low-frequency excitations of neutral, fully ionized, nonradiative plasma, which propagate in the form of sound waves. In this approximation, the internal electric and magnetic fields are generated due to deviations of plasma parameters from equilibrium values. In fact these fields are quasi-static and also move together with the plasma. This representation is close to the concept of a frozen-in field applied in the description of Alfvén waves.[18–20] Here on the base of system (1) we construct self-consisted two-fluid model of vortex plasma. The hydrodynamic equations for electron and ion fluids with internal electromagnetic field can be obtained using the following substitutions[12] for variables in the system (1):

$$\begin{aligned} &\mathbf{v} \Rightarrow \mathbf{v}_\alpha + a_\alpha\mathbf{A}_\alpha, \\ &u \Rightarrow u_\alpha + a_\alpha\varphi_\alpha, \\ &\mathbf{w} \Rightarrow \mathbf{w}_\alpha + a_\alpha\mathbf{M}_\alpha, \\ &\xi \Rightarrow \xi_\alpha + a_\alpha\phi_\alpha. \end{aligned} \tag{3}$$

a) Electronic mail: mironov@ipmras.ru

Here the index $\alpha \in \{e, i\}$, where e stands for electrons and i stands for ions; φ_α and $\mathbf{A}_\alpha$ are scalar and vector electric potentials; ϕ_α and $\mathbf{M}_\alpha$ are scalar and vector magnetic potentials respectively; the parameter a_α is

$$a_\alpha = \frac{q_\alpha}{m_\alpha s_\alpha}, \tag{4}$$

where q_α is particle charge, m_α is particle mass, s_α is corresponding speed of sound. Assuming the following definitions for internal field strengths

$$\begin{aligned} \mathbf{E}_\alpha &= -\frac{1}{s_\alpha}\left(\frac{\partial}{\partial t} + (\mathbf{v}_\alpha \cdot \nabla)\right)\mathbf{A}_\alpha - \nabla\varphi_\alpha - \nabla\times\mathbf{M}_\alpha, \\ \mathbf{B}_\alpha &= -\frac{1}{s_\alpha}\left(\frac{\partial}{\partial t} + (\mathbf{v}_\alpha \cdot \nabla)\right)\mathbf{M}_\alpha - \nabla\phi_\alpha + \nabla\times\mathbf{A}_\alpha, \end{aligned} \tag{5}$$

and taking the following gauge conditions:

$$\begin{aligned} &\frac{1}{s_\alpha}\left(\frac{\partial}{\partial t} + (\mathbf{v}_\alpha \cdot \nabla)\right)\varphi_\alpha + \nabla\cdot\mathbf{A}_\alpha = 0, \\ &\frac{1}{s_\alpha}\left(\frac{\partial}{\partial t} + (\mathbf{v}_\alpha \cdot \nabla)\right)\phi_\alpha + \nabla\cdot\mathbf{M}_\alpha = 0, \end{aligned} \tag{6}$$

from the system (1) we get

$$\begin{aligned} &\frac{1}{s_\alpha}\left(\frac{\partial}{\partial t} + (\mathbf{v}_\alpha \cdot \nabla)\right)\mathbf{v}_\alpha + \nabla u_\alpha + \nabla\times\mathbf{w}_\alpha = a_\alpha\mathbf{E}_\alpha, \\ &\frac{1}{s_\alpha}\left(\frac{\partial}{\partial t} + (\mathbf{v}_\alpha \cdot \nabla)\right)u_\alpha + \nabla\cdot\mathbf{v}_\alpha = 0, \\ &\frac{1}{s_\alpha}\left(\frac{\partial}{\partial t} + (\mathbf{v}_\alpha \cdot \nabla)\right)\mathbf{w}_\alpha + \nabla\xi_\alpha - \nabla\times\mathbf{v}_\alpha = a_\alpha\mathbf{B}_\alpha, \\ &\frac{1}{s_\alpha}\left(\frac{\partial}{\partial t} + (\mathbf{v}_\alpha \cdot \nabla)\right)\xi_\alpha + \nabla\cdot\mathbf{w}_\alpha = 0. \end{aligned} \tag{7}$$

Moreover, to close the system (7) we suppose that the internal fields satisfy the following equations:

$$\begin{aligned} &\nabla\cdot\mathbf{E}_\alpha = 4\pi e\left(n_i - n_e\right), \\ &\nabla\cdot\mathbf{B}_\alpha = 4\pi e\left(g_i - g_e\right), \\ &\left(\frac{\partial}{\partial t} + (\mathbf{v}_\alpha \cdot \nabla)\right)\mathbf{B}_\alpha + s_\alpha\nabla\times\mathbf{E}_\alpha = \\ &-4\pi e\left(n_i\mathbf{w}_i - n_e\mathbf{w}_e\right), \\ &\left(\frac{\partial}{\partial t} + (\mathbf{v}_\alpha \cdot \nabla)\right)\mathbf{E}_\alpha - s_\alpha\nabla\times\mathbf{B}_\alpha = \\ &-4\pi e\left(n_i\mathbf{v}_i - n_e\mathbf{v}_e\right), \end{aligned} \tag{8}$$

where $e = |q_e|$ is the charge of electron. The plasma particle concentration n_α and parameter g_α are related to the plasma parameters u_α and ξ_α by the following relations:

$$\begin{aligned} n_\alpha &= \frac{n_{0\alpha}}{s_\alpha}u_\alpha, \\ g_\alpha &= \frac{n_{0\alpha}}{s_\alpha}\xi_\alpha. \end{aligned} \tag{9}$$

Thus, in this model, the electron and ion components of the plasma are characterized by the following set of scalar and vector parameters:

$$P_\alpha \in \left\{n_\alpha, g_\alpha, \mathbf{v}_\alpha, \mathbf{w}_\alpha, \mathbf{E}_\alpha, \mathbf{B}_\alpha\right\}. \tag{10}$$

III. LINEARIZED EQUATIONS FOR SOUND WAVES

Let us consider small fluctuations of plasma parameters near the equilibrium state

$$\begin{aligned} n_\alpha &= n_{0\alpha} + \tilde{n}_\alpha, \\ \mathbf{v}_\alpha &= \tilde{\mathbf{v}}_\alpha, \\ g_\alpha &= \tilde{g}_\alpha, \\ \mathbf{w}_\alpha &= \tilde{\mathbf{w}}_\alpha, \\ n_{0i} &= n_{0e} = n_0, \end{aligned} \tag{11}$$

which propagate as the different types of sound waves. Neglecting the convective derivative in the systems (7) and (8) we obtain the following linearaized equations for these sound waves:

$$\begin{aligned} &\frac{1}{s_\alpha}\frac{\partial\tilde{\mathbf{v}}_\alpha}{\partial t} + \frac{s_\alpha}{n_0}\nabla\tilde{n}_\alpha + \nabla\times\tilde{\mathbf{w}}_\alpha = a_\alpha\tilde{\mathbf{E}}_\alpha, \\ &\frac{1}{n_0}\frac{\partial\tilde{n}_\alpha}{\partial t} + \nabla\cdot\tilde{\mathbf{v}}_\alpha = 0, \\ &\frac{1}{s_\alpha}\frac{\partial\tilde{\mathbf{w}}_\alpha}{\partial t} + \frac{s_\alpha}{n_0}\nabla\tilde{g}_\alpha - \nabla\times\tilde{\mathbf{v}}_\alpha = a_\alpha\tilde{\mathbf{B}}_\alpha, \\ &\frac{1}{n_0}\frac{\partial\tilde{g}_\alpha}{\partial t} + \nabla\cdot\tilde{\mathbf{w}}_\alpha = 0, \end{aligned} \tag{12}$$

and

$$\begin{aligned} &\nabla\cdot\tilde{\mathbf{E}}_\alpha = 4\pi e\left(\tilde{n}_i - \tilde{n}_e\right), \\ &\nabla\cdot\tilde{\mathbf{B}}_\alpha = 4\pi e\left(\tilde{g}_i - \tilde{g}_e\right), \\ &\frac{\partial\tilde{\mathbf{B}}_\alpha}{\partial t} + s_\alpha\nabla\times\tilde{\mathbf{E}}_\alpha = -4\pi e n_0\left(\tilde{\mathbf{w}}_i - \tilde{\mathbf{w}}_e\right), \\ &\frac{\partial\tilde{\mathbf{E}}_\alpha}{\partial t} - s_\alpha\nabla\times\tilde{\mathbf{B}}_\alpha = -4\pi e n_0\left(\tilde{\mathbf{v}}_i - \tilde{\mathbf{v}}_e\right). \end{aligned} \tag{13}$$

From the systems (12) and (13) we have the following wave equations for the electron and ion concentrations

$$\begin{aligned} \left(\frac{\partial^2}{\partial t^2} - s_i^2\Delta + \omega_{ip}^2\right)\tilde{n}_i &= \omega_{ip}^2\tilde{n}_e, \\ \left(\frac{\partial^2}{\partial t^2} - s_e^2\Delta + \omega_{ep}^2\right)\tilde{n}_e &= \omega_{ep}^2\tilde{n}_i. \end{aligned} \tag{14}$$

Here Δ is Laplace operator, ω_{ip} is the ion plasma frequency and ω_{ep} is the electron plasma frequency:

$$\omega_{ip}^2 = \frac{4\pi n_0 e^2}{m_i}, \tag{15}$$

$$\omega_{ep}^2 = \frac{4\pi n_0 e^2}{m_e}. \tag{16}$$

Let us define the following second-order differential operators

$$\Box_i = \frac{\partial^2}{\partial t^2} - s_i^2\Delta, \tag{17}$$

$$\Box_e = \frac{\partial^2}{\partial t^2} - s_e^2 \Delta. \quad (18)$$

Then the equations (14) can be separated as:

$$\left\{ \left(\Box_e + \omega_{ep}^2\right) \left(\Box_i + \omega_{ip}^2\right) - \omega_{ip}^2 \omega_{ep}^2 \right\} \tilde{n}_i = 0, \quad (19)$$

$$\left\{ \left(\Box_e + \omega_{ep}^2\right) \left(\Box_i + \omega_{ip}^2\right) - \omega_{ip}^2 \omega_{ep}^2 \right\} \tilde{n}_e = 0. \quad (20)$$

The same type of equations we obtain for the remaining variables. If we denote the generalized plasma parameter as

$$\tilde{P}_\alpha \in \left\{ \tilde{n}_\alpha, \tilde{g}_\alpha, \tilde{\mathbf{v}}_\alpha, \tilde{\mathbf{w}}_\alpha, \tilde{\mathbf{E}}_\alpha, \tilde{\mathbf{B}}_\alpha \right\}, \quad (21)$$

then the generalized sound wave equation can be written as

$$\left\{ \left(\Box_e + \omega_{ep}^2\right) \left(\Box_i + \omega_{ip}^2\right) - \omega_{ip}^2 \omega_{ep}^2 \right\} \tilde{P}_\alpha = 0. \quad (22)$$

These waves have the following dispersion relation

$$\left(\omega^2 - s_e^2 k^2 - \omega_{ep}^2\right) \left(\omega^2 - s_i^2 k^2 - \omega_{ip}^2\right) - \omega_{ip}^2 \omega_{ep}^2 = 0, \quad (23)$$

where ω is the frequency and $\mathbf{k}$ is the wave vector ($k = |\mathbf{k}|$). The schematic plots illustrating this dispersion relation are represented in FIG. 1. If $k = 0$, then we have two roots of equation (23)

$$\begin{aligned} &\omega = 0, \\ &\omega = \omega_* = \sqrt{\omega_{ep}^2 + \omega_{ip}^2}. \end{aligned} \quad (24)$$

If $k \to \infty$ we have two asymptotes

$$\omega = s_e k, \quad (25)$$

and

$$\omega = s_i k. \quad (26)$$

The upper curve in FIG. 1 corresponds to the electron sound, while the lower curve corresponds to the ion sound. The group velocity of ion sound in the long wave limit ($k \to 0$) is

$$v_{ig} = \frac{d\omega}{dk} = \sqrt{\frac{s_e^2 \omega_{ip}^2 + s_i^2 \omega_{ep}^2}{\omega_{ep}^2 + \omega_{ip}^2}}. \quad (27)$$

IV. ELECTRON SOUND WAVES

Let us suppose that ion fluid is motionless

$$\begin{aligned} &n_i = n_{0i} = n_{0e} = n_0, \\ &\mathbf{v}_i = 0, \\ &g_i = 0, \\ &\mathbf{w}_i = 0, \end{aligned} \quad (28)$$

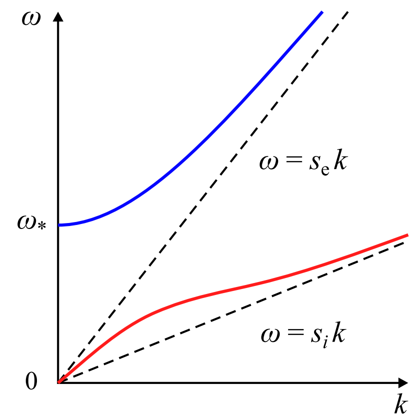

FIG. 1. The schematic plots of dispersion curves for sound waves.

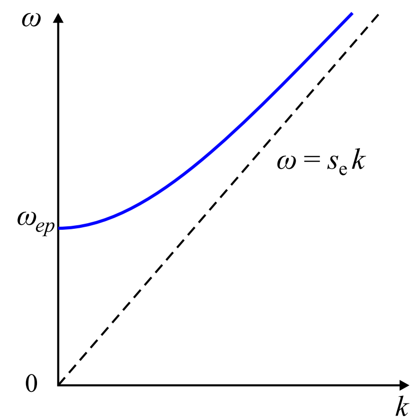

FIG. 2. The schematic plot of dispersion curve for electron sound waves.

while electron fluid makes small oscillations near the equilibrium state

$$\begin{aligned} &n_e = n_0 + \tilde{n}_e, \\ &\mathbf{v}_e = \tilde{\mathbf{v}}_e, \\ &g_e = \tilde{g}_e, \\ &\mathbf{w}_e = \tilde{\mathbf{w}}_e. \end{aligned} \quad (29)$$

Then we have the following linearized equations

$$\begin{aligned} &\frac{\partial \tilde{\mathbf{v}}_e}{\partial t} + \frac{s_e^2}{n_0} \nabla \tilde{n}_e + s_e \nabla \times \tilde{\mathbf{w}}_e = -\frac{e}{m_e} \tilde{\mathbf{E}}_e, \\ &\frac{1}{n_0} \frac{\partial \tilde{n}_e}{\partial t} + \nabla \cdot \tilde{\mathbf{v}}_e = 0, \\ &\frac{1}{n_0} \frac{\partial \tilde{g}_e}{\partial t} + \nabla \cdot \tilde{\mathbf{w}}_e = 0, \\ &\frac{\partial \tilde{\mathbf{w}}_e}{\partial t} + \frac{s_e^2}{n_0} \nabla \tilde{g}_e - s_e \nabla \times \tilde{\mathbf{v}}_e = -\frac{e}{m_e} \tilde{\mathbf{B}}_e, \end{aligned} \quad (30)$$

and

$$
\begin{aligned}
&\nabla\cdot\tilde{\mathbf{E}}_e = -4\pi e\tilde{n}_e,\\
&\nabla\cdot\tilde{\mathbf{B}}_e = -4\pi e\tilde{g}_e,\\
&\frac{\partial\tilde{\mathbf{B}}_e}{\partial t} + s_e\nabla\times\tilde{\mathbf{E}}_e = 4\pi e n_0\tilde{\mathbf{w}}_e,\\
&\frac{\partial\tilde{\mathbf{E}}_e}{\partial t} - s_e\nabla\times\tilde{\mathbf{B}}_e = 4\pi e n_0\tilde{\mathbf{v}}_e.
\end{aligned}
\tag{31}
$$

This simple model corresponds, for example, to the case of electron fluid in ideal metal.

From the system (30) taking into account (31) we obtain the following generalized equations for sound waves in electron fluid:

$$\left(\Box_e + \omega_{ep}^2\right)\tilde{P}_e = 0. \tag{32}$$

The corresponding dispersion relation is

$$\omega^2 - s_e^2 k^2 - \omega_{ep}^2 = 0. \tag{33}$$

The schematic plot of dispersion relation (33) is represented in FIG. 2.

V. ELECTRON SOUND WAVES IN AN EXTERNAL MAGNETIC FIELD

Let us consider electron fluid in a homogeneous external magnetic field applied in Z direction

$$\mathbf{B}_0 = B_0\mathbf{e}_z, \tag{34}$$

where $\mathbf{e}_z$ is the unit vector in Z direction. The system of hydrodynamic equations (30) taking into account Lorentz force is written as

$$
\begin{aligned}
&\frac{\partial\tilde{\mathbf{v}}_e}{\partial t} + \frac{s_e^2}{n_0}\nabla\tilde{n}_e + s_e\nabla\times\tilde{\mathbf{w}}_e = -\frac{e}{m_e}(\tilde{\mathbf{E}}_e + \frac{1}{c}\tilde{\mathbf{v}}_e\times\mathbf{B}_0),\\
&\frac{1}{n_0}\frac{\partial\tilde{n}_e}{\partial t} + \nabla\cdot\tilde{\mathbf{v}}_e = 0,\\
&\frac{1}{n_0}\frac{\partial\tilde{g}_e}{\partial t} + \nabla\cdot\tilde{\mathbf{w}}_e = 0,\\
&\frac{\partial\tilde{\mathbf{w}}_e}{\partial t} + \frac{s_e^2}{n_0}\nabla\tilde{g}_e - s_e\nabla\times\tilde{\mathbf{v}}_e = -\frac{e}{m_e}\tilde{\mathbf{B}}_e,
\end{aligned}
\tag{35}
$$

and

$$
\begin{aligned}
&\nabla\cdot\tilde{\mathbf{E}}_e = -4\pi e\tilde{n}_e,\\
&\nabla\cdot\tilde{\mathbf{B}}_e = -4\pi e\tilde{g}_e,\\
&\frac{\partial\tilde{\mathbf{B}}_e}{\partial t} + s_e\nabla\times\tilde{\mathbf{E}}_e = 4\pi e n_0\tilde{\mathbf{w}}_e,\\
&\frac{\partial\tilde{\mathbf{E}}_e}{\partial t} - s_e\nabla\times\tilde{\mathbf{B}}_e = 4\pi e n_0\tilde{\mathbf{v}}_e.
\end{aligned}
\tag{36}
$$

Directly from the equations (35) and (36) we obtain

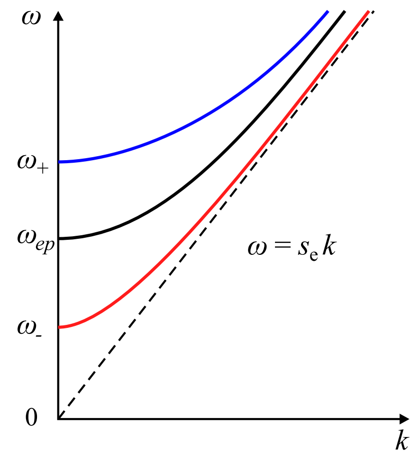

FIG. 3. The schematic plots of dispersion curves for electron sound waves in an external magnetic field.

$$\left(\Box_e + \omega_{ep}^2\right)\tilde{g}_e = 0, \tag{37}$$

and

$$\left(\Box_e + \omega_{ep}^2\right)\tilde{\mathbf{B}}_e = 0. \tag{38}$$

For the speed components we have

$$
\begin{aligned}
&\left(\Box_e + \omega_{ep}^2\right)\tilde{v}_{ez} = 0,\\
&\left(\left(\Box_e + \omega_{ep}^2\right)^2 + \omega_{ec}^2\frac{\partial^2}{\partial t^2}\right)\tilde{v}_{ex} = 0,\\
&\left(\left(\Box_e + \omega_{ep}^2\right)^2 + \omega_{ec}^2\frac{\partial^2}{\partial t^2}\right)\tilde{v}_{ey} = 0.
\end{aligned}
\tag{39}
$$

Here ω_{ec} is electron cyclotron frequency

$$\omega_{ec} = \frac{e}{cm_e}B_0, \tag{40}$$

where c is speed of light in a vacuum. For electron concentration we obtain

$$\left(\Box_e + \omega_{ep}^2\right)\left(\left(\Box_e + \omega_{ep}^2\right)^2 + \omega_{ec}^2\frac{\partial^2}{\partial t^2}\right)\tilde{n}_e = 0. \tag{41}$$

For the components of electric field we have

$$
\begin{aligned}
&\left(\Box_e + \omega_{ep}^2\right)\tilde{E}_{ez} = 0,\\
&\left(\Box_e + \omega_{ep}^2\right)\left(\left(\Box_e + \omega_{ep}^2\right)^2 + \omega_{ec}^2\frac{\partial^2}{\partial t^2}\right)\tilde{E}_{ex} = 0,\\
&\left(\Box_e + \omega_{ep}^2\right)\left(\left(\Box_e + \omega_{ep}^2\right)^2 + \omega_{ec}^2\frac{\partial^2}{\partial t^2}\right)\tilde{E}_{ey} = 0,
\end{aligned}
\tag{42}
$$

and for rotation variable we get

$$
\begin{aligned}
&\left(\Box_e + \omega_{ep}^2\right)^2\left(\left(\Box_e + \omega_{ep}^2\right)^2 + \omega_{ec}^2\frac{\partial^2}{\partial t^2}\right)\tilde{w}_{ez} = 0,\\
&\left(\Box_e + \omega_{ep}^2\right)\left(\left(\Box_e + \omega_{ep}^2\right)^2 + \omega_{ec}^2\frac{\partial^2}{\partial t^2}\right)\tilde{w}_{ex} = 0,\\
&\left(\Box_e + \omega_{ep}^2\right)\left(\left(\Box_e + \omega_{ep}^2\right)^2 + \omega_{ec}^2\frac{\partial^2}{\partial t^2}\right)\tilde{w}_{ey} = 0.
\end{aligned}
\tag{43}
$$

Thus for different variables of the electron sound waves propagating in an external homogeneous magnetic field we obtain the following set of the dispersion relations:

$$\left(\omega^2 - s_e^2 k^2 - \omega_{ep}^2\right) = 0, \tag{44}$$

$$\left(\omega^2 - s_e^2 k^2 - \omega_{ep}^2\right)^2 - \omega^2 \omega_{ec}^2 = 0. \tag{45}$$

If $k = 0$, then we get three roots of the equations (44) and (45)

$$\omega = \omega_{ep}, \tag{46}$$

$$\omega = \omega_+ = \frac{\sqrt{\omega_{ec}^2 + 4\omega_{ep}^2}}{2} + \frac{\omega_{ec}}{2}, \tag{47}$$

$$\omega = \omega_- = \frac{\sqrt{\omega_{ec}^2 + 4\omega_{ep}^2}}{2} - \frac{\omega_{ec}}{2}. \tag{48}$$

If $k \to \infty$ we have the following asymptote

$$\omega = s_e k. \tag{49}$$

The schematic plots illustrating the dispersion relations (44) and (45) are represented in FIG. 3.

VI. CONCLUSION

Thus, we propose the system of self-consistent equations (7) and (8), which describes the vortex plasma within the framework of two-fluid hydrodynamic model. It is shown that the internal electric and magnetic fields generated by fluctuations of the mechanical parameters of the plasma can be taken into account separately for the electronic and ionic components. These fields satisfy modified Maxwell-like equations (8), which show that the fields are incorporated in plasma and propagate at the speed of sound.

Linearized equations (12) and (13) form a closed system that describes sound waves, in which variables n_α and $\mathbf{v}_\alpha$ describe longitudinal expansion-compression waves, and variables g_α and $\mathbf{w}_\alpha$ describe vortex twisting waves.[17] System (12)-(13) is reduced to fourth-order wave equations (22), in which the spectrum of eigenwaves has two branches corresponding to the hybridization of electron and ion sound waves. Note that the dynamics of $\mathbf{E}_\alpha$ and $\mathbf{B}_\alpha$ fields is described by the same wave equations.

In the case of the model of stationary ions, which corresponds to the model of electron fluid in ideal metal, we have purely electron sound waves for all plasma parameters with the known dispersion law (33). In addition, it was shown that in an external magnetic field the spectrum of eigenwaves splits and except the usual electronic sound waves for the variables g_e and $\mathbf{B}_e$, we have a superposition of electron sound waves and electron-cyclotron sound waves for the rest parameters of plasma.

The proposed equations can be potentially applied to describe turbulent plasma motion[4] in external electromagnetic field.

[1] F. E. Chen *"Introduction to Plasma and Controlled Fusion"*, vol. 1: Plasma Physics, Plenum Press, New York, 1984.
[2] V. Krishan, *"Two-Fluid Description of Plasmas"*. In: Astrophysical Plasmas and Fluids. Astrophysics and Space Science Library, vol 235. Springer, Dordrecht, 1999.
[3] J. G. Logan, "Hydrodynamic analog of the classical field equations", Physics of Fluids, **5**, 868-869 (1962).
[4] H. Marmanis, "Analogy between the Navier-Stokes equations and Maxwells equations: Application to turbulence, Physics of Fluids", **10**, 1428-1437 (1998).
[5] T. Kambe, "A new formulation of equation of compressible fluids by analogy with Maxwell's equations", Fluid Dynamics Research, **42**, 055502 (2010).
[6] T. Kambe, "On fluid Maxwell equations", In Springer Proceedings in Physics, vol. 145, "Frontiers of fundamental physics and physics education research", Eds. B.G. Sidharth, M. Michelini, L. Santi. p. 287-296. Springer, Heidelberg, 2014.
[7] S. Demir, M. Tanışlı, "Spacetime algebra for the reformulation of fluid field equations", International Journal of Geometric Methods in Modern Physics, **14**, 1750075 (2017).
[8] R. J. Thompson, T. M. Moeller, "Numerical and closed-form solutions for the Maxwell equations of incompressible flow", Physics of Fluids, **30**, 083606 (2018).
[9] A. C. R. Mendes, F. I. Takakura, E. M. C. Abreu, J. A. Neto, "Compressible fluids with Maxwell-type equations, the minimal coupling with electromagnetic field and the Stefan-Boltzmann law", Annals of Physics, **380**, 12-22 (2017).
[10] A. C. R. Mendes, F. I. Takakura, E. M. C. Abreu, J. A. Neto, P. P. Silva, J. V. Frossad, "Helicity and vortex generation", Annals of Physics, **398**, 146-158 (2018).
[11] M. Tanışlı, S. Demir, N. Şahin, "Octonic formulations of Maxwell type fluid equations", Journal of Mathematical Physics, **56**, 091701 (2015).
[12] R. J. Thompson, T. M. Moeller, "A Maxwell formulation for the equations of a plasma", Physics of Plasmas, **19**, 010702 (2012).
[13] R. J. Thompson, T. M. Moeller, "Classical field isomorphisms in two-fluid plasmas", Physics of Plasmas, **19**, 082116 (2012).
[14] B. C. Chanyal, M. Pathak, "Quaternionic approach to dual Magneto-hydrodynamics of dyonic cold plasma", Advances in High Energy Physics, **13**, 7843730 (2018).
[15] S. Demir, M. Tanışlı, N. Şahin, M.E. Kansu, "Biquaternionic reformulation of multifluid plasma equations", Chinese journal of physics, **55**(4), 1329 (2017).
[16] H. Helmholtz, "Über Integrale der hydrodynamischen Gleichungen, welche den Wirbelbewegungen entsprechen", Journal für die Reine und Angewandte Mathematik, **55**, 25—55 (1858). Translation. P. G. Tait, "On integrals of the hydrodynamical equations, which express vortex-motion", Philosophical Magazine, **33**(4), 485-512 (1867).
[17] V. L. Mironov, S. V. Mironov, "Generalized sedeonic equations of hydrodynamics", European Physical Journal Plus, **135**(9), 708 (2020).
[18] H. Alfvén, "Existence of electromagnetic-hydrodynamic waves", Nature, **150**, 405 (1942).
[19] K. I. Ilin, Y. L. Trakhinin, V. A. Vladimirov, "The stability of steady magnetohydrodynamic flows with current-vortex sheets", Physics of Plasmas, **10**, 2649 (2003).
[20] G. L. Eyink, H. Aluie, "The breakdown of Alfvén's theorem in ideal plasma flows: Necessary conditions and physical conjectures", Physica D: Nonlinear Phenomena, **223**(1), 82 (2006).