A flat Earth model accounting for a pseudo-gravity effect

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1. Abstract

The misconception that the Earth is flat emerged in ancient times. In early Egyptian and Mesopotamian mythology, the world was portrayed as a disk floating in the ocean. Several pre-Socratic philosophers, including Thales, Leucippus and Democritus, believed the world was flat. Ironically, despite the scientific fact of Earth's sphericity, flat Earth conspiracy theories still persist nowadays and are spreading around the globe. Mike Huges, who was a well-known flat Earther, even attempted to prove the Earth is flat by launching his homemade rocket, but ended up dying tragically in a mission. In addition, many flat Earthers believe gravity is a hoax and does not exist, which disagrees with classical physics. In this article, I am going to demonstrate a flat Earth model that incorporates the effect of gravity well by replacing gravity with magnetic force, and subsequently discuss about the non-uniformity of magnetic field described by the model.

2. General set-up of model

As suggested by Dr. H [**1**], there could be magnets present at the bottom of Earth, which produce a magnetic field and attract objects on Earth, producing an effect similar to gravity. This brilliant idea enables me to replace gravity with magnetic force. The radius of Earth, R_e, is $6.371 \times 10^6 m$. The surface area of Earth is estimated as $4\pi {R_e}^2$. Thus, flat Earth can be a circular disk with radius $2R_e$ and thickness R, so that it has the same surface area as a spherical Earth. Tiny magnetic dipoles can be placed at the base of flat Earth, which is at a distance R below Earth surface. The flat Earth model should fulfill the following requirements:

(i) Density of magnetic dipole strength at the bottom of Earth should be fairly constant, otherwise forces on magnetic dipoles would be unbalanced, and these dipoles would redistribute themselves until forces on them are balanced.
(ii) Magnetic field at same distance z above Earth surface is uniform.
(iii) Magnetic force on objects obeys inverse square law.

According to my model, magnetic dipoles at the bottom of Earth are circular current loops with current I and differential area $rd\theta dr$. Without loss of generality, we assume current I flows anticlockwisely. Hence, each magnetic dipole, $\overrightarrow{m_e} = m_e\hat{k} = Irdrd\theta\hat{k}$. Moreover, all objects on Earth always carry a magnetic dipole $\overrightarrow{m_b} = m_b\hat{k}$. Interaction between magnetic field formed by magnetic dipoles beneath Earth and magnetic dipoles carried by objects on Earth leads to a pseudo-gravity effect.

3. Derivation of model

For convenience, I opt to use the cylindrical coordinate system in derivation below. The centre of Earth surface is located at origin, while the surface is coplanar with the xy-plane. We need to solve for R such that magnetic force acting on objects obeys inverse square law.

a) Magnetic field produced by a circular ring of magnetic dipoles with radius r and width dr in $\hat{k}$ direction

Firstly, we need to determine the magnetic field strength at $(w, 0, z)$ due to a magnetic dipole located at $(r, \theta, -R)$. We can then integrate the field from $\theta = 0$ to 2π to find the total magnetic field produced by the ring.

$\overrightarrow{r'} = (w^2 - 2wr\cos\theta + r^2)\hat{r} + \tan^{-1}\left(\frac{-r\sin\theta}{w - r\cos\theta}\right)\hat{\theta} + (z+R)\hat{k}$ is the position vector from $(r, \theta, -R)$ to $(w, 0, z)$. For simplicity, substitute $a = z + R$.

$$
\begin{aligned}
dB &= \frac{\mu_0}{4\pi}\left[\frac{3\overrightarrow{r'}\cdot\left(\overrightarrow{m_e}\cdot\overrightarrow{r'}\right)}{\left|\overrightarrow{r'}\right|^5} - \frac{\overrightarrow{m_e}}{\left|\overrightarrow{r'}\right|^3}\right] \\
dB &= \frac{\mu_0}{4\pi}\left[\frac{3\left((w^2 - 2wr\cos\theta + r^2)\hat{r} + \tan^{-1}\left(\frac{-r\sin\theta}{w-r\cos\theta}\right)\hat{\theta} + a\hat{k}\right)(m_e a)}{\left|\overrightarrow{r'}\right|^5} - \frac{m_e\hat{k}}{\left|\overrightarrow{r'}\right|^3}\right] \\
dB_z &= \frac{\mu_0 m_e}{4\pi}\left[\frac{3a^2}{\left|\overrightarrow{r'}\right|^5} - \frac{1}{\left|\overrightarrow{r'}\right|^3}\right]\hat{k} \\
\left|\overrightarrow{B}_{z(ring)}\right| &= \int_0^{2\pi} |dB_z| \\
&= \frac{\mu_0 I r dr}{4\pi}\int_0^{2\pi}\left[\frac{3a^2}{(w^2 + r^2 + a^2 - 2wr\cos\theta)^{\frac{5}{2}}} - \frac{1}{(w^2 + r^2 + a^2 - 2wr\cos\theta)^{\frac{3}{2}}}\right]d\theta
\end{aligned}
$$

b) Total magnetic field at (w, 0, z) in $\hat{k}$ direction

In the previous session, we calculate the magnetic field produced by a circular ring of magnetic dipoles. After that, integrating $\left|\overrightarrow{B}_{z(ring)}\right|$ from $r = 0$ to $2R_e$ gives the total magnetic field.

Along cylindrical axis, $w = 0$.

$$
\begin{aligned}
\left|\overrightarrow{B}_{z(ring)}\right| &= \frac{\mu_0 I r dr}{4\pi}\int_0^{2\pi}\left[\frac{3a^2}{(r^2 + a^2)^{\frac{5}{2}}} - \frac{1}{(r^2 + a^2)^{\frac{3}{2}}}\right]d\theta \\
&= \frac{\mu_0 I r dr}{4\pi}\cdot\left[\frac{3a^2}{(r^2 + a^2)^{\frac{5}{2}}} - \frac{1}{(r^2 + a^2)^{\frac{3}{2}}}\right]\cdot 2\pi \\
&= \frac{\mu_0 I r}{2}\cdot\left[\frac{3a^2}{(r^2 + a^2)^{\frac{5}{2}}} - \frac{1}{(r^2 + a^2)^{\frac{3}{2}}}\right]\cdot dr \\
\left|\overrightarrow{B}_{z(total)}\right| &= \frac{\mu_0 I}{2}\int_0^{2R_e}\frac{3a^2 r}{(r^2 + a^2)^{\frac{5}{2}}} - \frac{r}{(r^2 + a^2)^{\frac{3}{2}}}\,dr \\
\left|\overrightarrow{B}_{z(total)}\right| &= \frac{\mu_0 I}{2}\left[\frac{-a^2}{(4{R_e}^2 + a^2)^{\frac{3}{2}}} + \frac{1}{(4{R_e}^2 + a^2)^{\frac{1}{2}}}\right] \quad ...(1)
\end{aligned}
$$

If $w \neq 0$, let $f(\theta) = \frac{3a^2}{(w^2+r^2+a^2-2wr\cos\theta)^{\frac{5}{2}}}$ and $g(\theta) = \frac{1}{(w^2+r^2+a^2-2wr\cos\theta)^{\frac{3}{2}}}$.

By symmetry, $\int_0^{2\pi} [f(\theta) - g(\theta)]\, d\theta = 2\int_0^{\pi} [f(\theta) - g(\theta)]\, d\theta$.

$$\begin{aligned}
\implies \left|\overrightarrow{B}_{z(ring)}\right| &= \frac{\mu_0 I r dr}{4\pi} \cdot 2\int_0^{\pi} [f(\theta) - g(\theta)]\, d\theta \\
&= \frac{\mu_0 I r dr}{2\pi} \int_0^{\pi} [f(\theta) - g(\theta)]\, d\theta \\
\left|\overrightarrow{B}_{z(total)}\right| &= \frac{\mu_0 I}{2\pi} \int_0^{2R_e} r \int_0^{\pi} [f(\theta) - g(\theta)]\, d\theta\, dr
\end{aligned}$$

Using small angle approximation, $\cos\theta \approx 1 - \frac{\theta^2}{2}$. Thus,

$$f(\theta) \approx \frac{3a^2}{((w-r)^2 + a^2)^{\frac{5}{2}}} \cdot \frac{1}{\left(1 + \frac{wr}{(w-r)^2+a^2}\theta^2\right)^{\frac{5}{2}}},\ g(\theta) \approx \frac{1}{((w-r)^2 + a^2)^{\frac{3}{2}}} \cdot \frac{1}{\left(1 + \frac{wr}{(w-r)^2+a^2}\theta^2\right)^{\frac{3}{2}}}$$

Let $K = \frac{wr}{(w-r)^2+a^2}$.

$$\begin{aligned}
\int_0^{\pi} [f(\theta) - g(\theta)]\, d\theta &\approx \frac{3a^2}{((w-r)^2 + a^2)^{\frac{5}{2}}} \int_0^{\pi} \frac{1}{(1+K\theta^2)^{\frac{5}{2}}}\, d\theta - \frac{\pi}{((w-r)^2 + a^2)^{\frac{3}{2}}} \int_0^{\pi} \frac{1}{(1+K\theta^2)^{\frac{3}{2}}}\, d\theta \\
&= \frac{3a^2}{((w-r)^2 + a^2)^{\frac{5}{2}}} \cdot \frac{\pi(2\pi^2 K + 3)}{3(\pi^2 K + 1)^{\frac{3}{2}}} - \frac{\pi}{((w-r)^2 + a^2)^{\frac{3}{2}}} \cdot \frac{1}{(\pi^2 K + 1)^{\frac{1}{2}}}
\end{aligned}$$

Therefore,

$$\left|\overrightarrow{B}_{z(total)}\right| \approx \frac{\mu_0 I}{2\pi} \int_0^{2R_e} \frac{3a^2 r}{((w-r)^2 + a^2)^{\frac{5}{2}}} \cdot \frac{\pi(2\pi^2 K + 3)}{3(\pi^2 K + 1)^{\frac{3}{2}}} - \frac{\pi r}{((w-r)^2 + a^2)^{\frac{3}{2}}} \cdot \frac{1}{(\pi^2 K + 1)^{\frac{1}{2}}}\, dr \ ...\ (2)$$

c) First order approximation of gravity

According to Newton's law of universal gravitation, gravitational force acting on a mass m_g at a distance z above Earth surface is:

$$\overrightarrow{F_g} = -\frac{GM_e m_g}{(R_e + z)^2}\hat{k}, \text{ where } M_e \text{ is Earth's mass}$$

If $z \ll R$, by Taylor expansion,

$$\overrightarrow{F_g} = -m_g g\left(1 - \frac{2z}{R_e}\right)\hat{k}$$

d) Comparing magnetic force with the first order approximation of gravity

Magnetic force acting on an object above Earth surface at $(0, \theta, z)$ is:

$$\begin{aligned}
\overrightarrow{F_b} &= \overrightarrow{\nabla} \cdot (\overrightarrow{m_b} \cdot \overrightarrow{B}_{z(total)}) \\
&= m_b \left(\frac{\partial}{\partial z} \left| \overrightarrow{B}_{z(total)} \right| \right) \hat{k} \\
&= \frac{\mu_0 I m_b}{2} \frac{\partial}{\partial z} \left[\frac{-(z+R)^2}{(4{R_e}^2 + (z+R)^2)^{\frac{3}{2}}} + \frac{1}{(4{R_e}^2 + (z+R)^2)^{\frac{1}{2}}} \right] \hat{k} \quad \text{(by (1))} \\
&= -\frac{\mu_0 I m_b}{2} \left[\frac{-(z+R)^3}{(4{R_e}^2 + (z+R)^2)^{\frac{5}{2}}} + \frac{8{R_e}^2(z+R)}{(4{R_e}^2 + (z+R)^2)^{\frac{5}{2}}} - \frac{z+R}{(4{R_e}^2 + (z+R)^2)^{\frac{3}{2}}} \right] \hat{k}
\end{aligned}$$

If $z \ll R$, we can neglect $O(z^2)$ terms. By Taylor expansion,

$$\overrightarrow{F_b} \approx -\frac{\mu_0 I m_b}{2} \left[\frac{8R{R_e}^2 - R^3}{(4{R_e}^2 + R^2)^{\frac{5}{2}}} - \frac{R}{(4{R_e}^2 + R^2)^{\frac{3}{2}}} + \frac{(32{R_e}^4 - 44R^2{R_e}^2 + 2R^4)z}{(4{R_e}^2 + R^2)^{\frac{7}{2}}} + \frac{(2R^2 - 4{R_e}^2)z}{(4{R_e}^2 + R^2)^{\frac{5}{2}}} \right] \hat{k}$$

Let $A = \frac{32{R_e}^4 - 44R^2{R_e}^2 + 2R^4}{(4{R_e}^2+R^2)^{\frac{7}{2}}} + \frac{2R^2 - 4{R_e}^2}{(4{R_e}^2+R^2)^{\frac{5}{2}}}$ and $B = \frac{R}{(4{R_e}^2+R^2)^{\frac{3}{2}}} - \frac{8R{R_e}^2 - R^3}{(4{R_e}^2+R^2)^{\frac{5}{2}}}$.
Then, $\overrightarrow{F_b} \approx -\frac{\mu_0 I m_b}{2}(-B + Az)\hat{k}$.

In order for $\overrightarrow{F_b}$ to match with the first order approximation of gravity, we must have $\frac{A}{B} = \frac{2}{R_e}$.
Let $R = xR_e$, after solving for x we can complete the flat Earth model.

$$\begin{aligned}
\frac{A}{B} &= \frac{2}{R_e} \\
\implies \left(\frac{32 - 44x^2 + 2x^4}{(x^2+4)^{\frac{7}{2}}} + \frac{2x^2 - 4}{(x^2+4)^{\frac{5}{2}}} \right) \left(\frac{x}{(x^2+4)^{\frac{3}{2}}} - \frac{8x - x^3}{(x^2+4)^{\frac{5}{2}}} \right)^{-1} &= 2
\end{aligned}$$

Let $p(x) = \left(\frac{32-44x^2+2x^4}{(x^2+4)^{\frac{7}{2}}} + \frac{2x^2-4}{(x^2+4)^{\frac{5}{2}}} \right) \left(\frac{x}{(x^2+4)^{\frac{3}{2}}} - \frac{8x-x^3}{(x^2+4)^{\frac{5}{2}}} \right)^{-1}$. Obviously, $p(x)$ is continuous on $(0, \sqrt{2})$. Moreover, $\lim_{x \to 0^+} p(x) = -\infty$ and $\lim_{x \to \sqrt{2}^-} p(x) = +\infty$. By Intermediate value theorem, $\exists x \in (0, \sqrt{2})$ such that $p(x) = 2$. After some algebraic mainpulation, we get $x = 1$.

We can further prove $x = 1$ is the unique root of $p(x) = 2$ when x is positive.
(i) $f'(x) > 0 \ \forall x \in (0, \sqrt{2})$
(ii) $f(x) < 2 \ \forall x \in (\sqrt{2}, 5.594]$
(iii) $f'(x) \leq 0 \ \forall x \in [5.594, +\infty)$
By (i), (ii) and (iii), $x = 1$ is the unique root of $p(x) = 2$ for positive x. Hence, $R = R_e$.

$$\begin{aligned}
\overrightarrow{F_b} &\approx -\frac{\mu_0 I m_b}{2} \left[\frac{7{R_e}^3}{(5{R_e}^2)^{\frac{5}{2}}} - \frac{R_e}{(5{R_e}^2)^{\frac{3}{2}}} - \frac{10{R_e}^4 z}{(5{R_e}^2)^{\frac{7}{2}}} - \frac{2{R_e}^2 z}{(5{R_e}^2)^{\frac{5}{2}}} \right] \hat{k} \\
&= \frac{-\sqrt{5}\mu_0 I m_b {R_e}^2}{125} \left(1 - \frac{2z}{R_e} \right) \hat{k}
\end{aligned}$$

Finally, we need to determine current I. By $\overrightarrow{F_b} = \overrightarrow{F_g}$, we obtain $I = \frac{25\sqrt{5} m_g g}{\mu_0 m_b {R_e}^2}$.

4. Non-uniformity of magnetic field

We have just derived a flat Earth model that obeys inverse square law when $z \ll R$. We should also examine whether the magnetic field of the model is uniform at same distance z above Earth surface. Due to cylindrical symmetry, magnetic field is independent of θ, we only need to check if magnetic field varies with w, ie. the radial distance of an object located at $(w, 0, z)$.

At $(0, \theta, 0)$, by (1),

$$\begin{aligned}\left|\overrightarrow{B}_{z(total)}\right| &= \frac{\mu_0 I}{2}\left[\frac{-{R_e}^2}{(4{R_e}^2+{R_e}^2)^{\frac{3}{2}}}+\frac{1}{(4{R_e}^2+{R_e}^2)^{\frac{1}{2}}}\right] \\ &\approx \mu_0 I(2.81\times 10^{-8})\end{aligned}$$

At $(2R_e, \theta, 0)$, by (2),

$$\left|\overrightarrow{B}_{z(total)}\right| \approx \frac{\mu_0 I}{2\pi}\int_0^{2R_e} \frac{3{R_e}^2 r}{((2R_e-r)^2+{R_e}^2)^{\frac{5}{2}}}\cdot\frac{\pi(2\pi^2K+3)}{3(\pi^2K+1)^{\frac{3}{2}}}-\frac{\pi r}{((2R_e-r)^2+{R_e}^2)^{\frac{3}{2}}}\cdot\frac{1}{(\pi^2K+1)^{\frac{1}{2}}}\,dr$$

Using numerical integration, we get:

$$\begin{aligned}\left|\overrightarrow{B}_{z(total)}\text{ at }(2R_e,\theta,0)\right| &\approx \mu_0 I(1.12\times 10^{-8}) \\ &\approx 0.4\left|\overrightarrow{B}_{z(total)}\text{ at }(0,\theta,0)\right|\end{aligned}$$

This counterexample shows magnetic field at same distance z above Earth surface is non-uniform.

5. Conclusion

It is theoretically possible to build a flat Earth that replaces gravity with magnetic force, such that the magentic force acting on objects still obeys inverse square law when $z \ll R$. However, the magnetic field is only uniform at central region of Earth near $w = 0$, and at $w \neq 0$ there is a net radial field. Two methods can be applied to eliminate the non-uniformity of magnetic field as well as the net radial field at $w \neq 0$:

(i) Changing the density of magnetic dipole strength at the bottom of Earth, but this contradicts with the assumption that the density should remain fairly constant, and we need another mechanism to explain why magnetic dipoles do not redistribute themselves.
(ii) Extending the base of Earth, but the flat Earth would become like a frustum instead of a circular disk.

Both methods (i) and (ii) inevitably increase complexity of the flat Earth model. Rather than developing an overcomplicated flat Earth model, it is much easier for flat Earthers to embrace Earth's sphericity and accept Newton's law of universal gravitation:

$$F = \frac{Gm_1m_2}{R^2}$$

Reference

[**1**]Dr. H is an expert in some electronics and modern physics. Since he is unwilling to disclose his identity, I replace his name with 'Dr. H'.