Quantization of Gravitational Wave by Klein-Gordon Equation

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ABSTRACT

In the general relativity theory, we find gravitational matter wave by Klein-Gordon wave equation. Specially, this article is that Quantization of gravitational wave is made by Klein-Gordon wave equation. We assume this matter wave as Dark Matter.

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1. Introduction

In the general relativity theory, our article's aim is that we find the quantization of gravitational wave by Klein-Gordon wave equation.

At first, gravitational wave equation is

$$(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2})h_{\mu\upsilon} = 0 \quad (1)$$

The solution, gravitational wave function $h_{\mu\upsilon}(x)$ is

$$h_{\mu\upsilon}(x) = a_{\mu\upsilon}\exp(ik_\lambda x^\lambda) + a^*_{\mu\upsilon}\exp(-ik_\lambda x^\lambda) \quad (2)$$

In this time,

$$k_\lambda k^\lambda = -\frac{\omega_0{}^2}{c^2} + k_0{}^2 = 0, \ k_\lambda = (\frac{\omega_0}{c}, \vec{k}_0), k^\lambda = (-\frac{\omega_0}{c}, \vec{k}_0) \quad (3)$$

The constant tensor $a_{\mu\upsilon}$ is the polarization tensor.

$$a_{\mu\upsilon} = a_{\upsilon\mu} \quad (4)$$

Harmonic coordinate condition is

$$(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2})\varepsilon_\upsilon = \frac{\partial h^\mu{}_\upsilon}{\partial x^\mu} - \frac{1}{2}\frac{\partial h^\mu{}_\mu}{\partial x^\upsilon} \quad (5)$$

$$k_\mu a^\mu{}_\upsilon = \frac{1}{2}k_\upsilon a^\mu{}_\mu \quad (6)$$

The coordinate transformation is

$$x'^\mu = x^\mu + i\varepsilon^\mu\exp(ik_\lambda x^\lambda) - i\varepsilon^{*\mu}\exp(-ik_\lambda x^\lambda) \quad (7)$$

According to Eq(7), the transformation of the polarization tensor is

$$a'_{\mu\upsilon} = a_{\mu\upsilon} + k_\mu\varepsilon_\upsilon + k_\upsilon\varepsilon_\mu \quad (8)$$

2. Quantization of Gravitational Wave by Klein-Gordon Equation

The speed of Gravitational wave is light speed. If we make matter by Gravitational space-time, this matter moves as the usual matter. We consider the matter interacting only gravity. Hence, we assume this matter as Dark Matter.

At first, gravitational matter wave equation is

$$(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2})h_{\mu\upsilon} = \frac{m_0{}^2 c^2}{\hbar^2}h_{\mu\upsilon} \quad (9)$$

The solution, gravitational matter wave function $h_{\mu\upsilon}(x)$ is

$$h_{\mu\upsilon}(x) = a_{\mu\upsilon} \exp(ik_\lambda x^\lambda) + a^*{}_{\mu\upsilon} \exp(-ik_\lambda x^\lambda) \tag{10}$$

In this time,

$$-k_\lambda k^\lambda = \frac{{\omega_0}^2}{c^2} - {k_0}^2 = \frac{{m_0}^2 c^2}{\hbar^2}, E = \hbar\omega_0, \vec{p} = \hbar\vec{k}_0,$$

$$k_\lambda = (\frac{\omega_0}{c}, \vec{k}_0), k^\lambda = (-\frac{\omega_0}{c}, \vec{k}_0) \tag{11}$$

The constant tensor $a_{\mu\upsilon}$ is the polarization tensor.

$$a_{\mu\upsilon} = a_{\upsilon\mu} \tag{12}$$

Harmonic coordinate condition is in gravitational matter wave by Eq(5),Eq(6)

$$k_\mu a^\mu{}_\upsilon - \frac{1}{2} k_\upsilon a^\mu{}_\mu = \frac{{m_0}^2 c^2}{\hbar^2} \varepsilon_\upsilon = (\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2})\varepsilon_\upsilon \tag{13}$$

3. Conclusion

We find the gravitational matter wave by Klein-Gordon wave equation. We find the quantization of gravitational wave by Klein-Gordon wave equation.

References

[1]S.Yi, “Electromagnetic Wave Functions of CMB and Schwarzschild Space-Time”, International Journal of Advanced Research in Physical Science,**6**,3,(2019)

[2]S.Yi, “Klein-Gordon Equation and Wave Function for Free Particle in Rindler Space-Time”, International Journal of Advanced Research in Physical Science,**7**,9,(2020)

[3]S.Weinberg,Gravitation and Cosmology(John wiley & Sons,Inc,1972)

[4]W.Rindler, Am.J.Phys.**34**.1174(1966)

[5]P.Bergman,Introduction to the Theory of Relativity(Dover Pub. Co.,Inc., New York,1976),Chapter V

[6]C.Misner, K,Thorne and J. Wheeler, Gravitation(W.H.Freedman & Co.,1973)

[7]S.Hawking and G. Ellis,The Large Scale Structure of Space-Time(Cam-bridge University Press,1973)

[8]R.Adler,M.Bazin and M.Schiffer,Introduction to General Relativity(McGraw-Hill,Inc.,1965)

[9]A. Einstein, “ Zur Elektrodynamik bewegter K¨orper”, Annalen der Physik. 17:891(1905)

[10]J.D. Bjorken & S. D. Drell, Relativistic Quantum Field(McGraw- Hill Co., 1965)

[11]P.Bergman,Introduction to the Theory of Relativity(Dover Pub. Co.,Inc., New York,1976),Chapter V

[12]R.L.Liboff, Quantum Mechanics(Addison-Wesley Publishing Co., Inc.,1990)