A LOWER LIMIT $\Delta H^Z_{vap}$ FOR THE LATENT HEAT OF VAPORIZATION $\Delta H_{vap}$ WITH RESPECT TO THE PRESSURE AND THE VOLUME CHANGE OF THE PHASE TRANSITION.

ALEXIS ZAGANIDIS

Abstract. We derive a lower limit $\Delta H^Z_{vap}$ for the latent heat of vaporization $\Delta H_{vap}$ with respect to the pressure and the volume change of the phase transition from the study of a heat engine using water as working fluid with an infinitesimal variation of the temperature $\delta T$ and an infinitesimal variation of the pressure $\delta P$ and in the vanishing limit of the massive flow rate $Q_m$. We calculate the latent heat index $h^Z = \Delta H^Z_{vap}/\Delta H_{vap}$ for few gas at $P = 100 \text{ kPa}$.

We derive a lower limit expression for the latent heat of vaporization $\Delta H_{vap}$ from the study of a heat engine using water as working fluid with an infinitesimal variation of the temperature $\delta T$ and an infinitesimal variation of the pressure $\delta P$ and in the vanishing limit of the massive flow rate $Q_m$:

Figure 1. Heat engine with $\delta T < \delta T$ and $\delta P < \delta P$.

Case: 2: $\delta P = 0$

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The efficiency of that heat engine is:

\[ \eta = 1 - \frac{T}{T+\delta T} = \frac{\delta T}{T} + \mathcal{O}(\delta^2 T) \]

Since the irreversible processes are negligible with infinitesimal variations for the temperature and the volume and .

The input power of that heat engine is: \( P_{in} = Q_m \Delta H_{vap} + \mathcal{O}(\delta P) \)

The output power of that heat engine is: \( P_{out} = \frac{Q_m}{\rho_g} \delta P \)

Therefore, we derive the efficiency of that heat engine in a second way:

\[ \eta = \frac{P_{out}}{P_{in}} = \frac{\delta P}{\rho_g \Delta H_{vap}} + \mathcal{O}(\delta^2 P) \]

Finally the case 1 is impossible since it breaks the well known Clausius–Clapeyron relation:

\[ \frac{\delta P}{\delta T} = \frac{\Delta H_{vap}}{T(\nu_g-\nu_l)} \]

Case: 1: \( \delta T = 0 \)

We start from \( \bar{\delta}P < \delta P \):

\[ 1 > \frac{\bar{\delta}P}{\delta P} = \frac{\delta T}{\delta P} \frac{\bar{\delta}P}{\delta T} = \frac{T(\nu_g-\nu_l)}{\Delta H_{vap}} \frac{\delta P}{\delta T} \]

From there, we express \( \bar{\delta}P \) with respect to \( \frac{\delta V}{\delta P} \) with the help of the following equation of state:

\[ \delta V = \left( \frac{\partial V}{\partial T} \right)_P \delta T + \left( \frac{\partial V}{\partial P} \right)_T \bar{\delta}P = V \left( \alpha \delta T - \beta_T \bar{\delta}P \right) \]

\[ \frac{\delta P}{\delta V} = \frac{P_0}{\frac{\partial P}{\partial T} + \frac{\partial P}{\partial V}} \]

\[ \frac{\delta P}{\delta V} = \frac{P}{T} \frac{\gamma_a T \alpha}{\gamma_a P \beta_T - 1} \]
\[ \frac{V}{\Delta P} = \frac{\gamma_a T}{\gamma_a P - 1} P (\nu_g - \nu_l) < \Delta H_{\text{vap}} \]

Finally, we derive a lower limit expression for the latent heat of vaporization \( \Delta H_{\text{vap}} \):

\[ \Delta H^Z_{\text{vap}} = \frac{\gamma_a T}{\gamma_a P^T - 1} P (\nu_g - \nu_l) < \Delta H_{\text{vap}} \]

We can define some latent heat index \( h^Z \) ranging between 0 and 1:

\[ h^Z = \frac{\Delta H^Z_{\text{vap}}}{\Delta H_{\text{vap}}} = \frac{\gamma_a T}{\gamma_a P^T - 1} \frac{P (\nu_g - \nu_l)}{\Delta H_{\text{vap}}} \]

From the above equation and the definition of an adiabatic expansion:

\[ \frac{dV}{dT} = \alpha dT - \beta T dP \]
\[ -P \frac{dV}{P} = (c_P - P \alpha) dT + (P \beta_T - T \alpha) dP \]

we derive an expression for the adiabatic index \( \gamma_a \) with respect to the heat capacity ratio \( \gamma \):

\[ \gamma_a = -\frac{P}{V} \frac{dV}{P} = \frac{1}{P \beta_T - \frac{1}{T} T \alpha} = \frac{\gamma}{P \beta_T} \]

To conclude the theoretical part, we develop \( \gamma_a \) with respect to the heat capacity ratio \( \gamma \) inside the previous results:

\[ \Delta H^Z_{\text{vap}} = \frac{\gamma_a T}{\gamma_a P^T - 1} P (\nu_g - \nu_l) < \Delta H_{\text{vap}} \]

\[ h^Z = \frac{\Delta H^Z_{\text{vap}}}{\Delta H_{\text{vap}}} = \frac{\gamma_a T}{\gamma_a P^T - 1} \frac{P (\nu_g - \nu_l)}{\Delta H_{\text{vap}}} \]

To conclude this paper, we calculate the latent heat index \( h^Z \) for the following gas:

\[ h^Z_{\text{H}_2O} = 1/2256540 \times 1.69402 \times 100000 \times 1.324/0.324 = 0.306774 \]
\[ h^Z_{\text{CO}_2} = 1/574000/1.977 \times 100000 \times 1.310/0.310 = 0.372384 \]
\[ h_{N_2O_4} = 1/415000/2.853 \times 100000 \times 1.262/0.262 = 0.4068262 \]

\[ h_{C_2H_6O} = 1/855000/1.627 \times 100000 \times 1.135/0.135 = 0.604378 \]

\[ h_{He} = 1/20500/16.9 \times 100000 \times 1.66/0.66 = 0.725978 \]

, at \( P = 100 \) kPa with the approximations: \( v_g - v_l \cong v_g \) and \( T_\alpha \cong P \beta_T \cong 1 \).

References