

The Concept of Particle-Antiparticle and the Baryon Asymmetry of the Universe

Felix Lev

Email: felixlev314@gmail.com

Artwork Conversion Software Inc., 509 N. Sepulveda Blvd. Manhattan Beach, CA
90266, USA

Abstract

Following the results of our publications, in the first part of this letter, we explain that quantum theory based on finite mathematics (FQT) is more general (fundamental) than standard quantum theory based on Poincare invariance. Standard concept of particle-antiparticle is not universal because it arises as a result of symmetry breaking from FQT to standard quantum theory based on Poincare or standard anti-de Sitter symmetries. In FQT one irreducible representation of the symmetry algebra describes a particle and its antiparticle simultaneously, and there are no conservation laws of electric charge and baryon quantum number. Poincare and standard anti-de Sitter symmetry are good approximations at the present stage of the universe but in the early stages they cannot take place. Therefore, the statement that in such stages the numbers of baryons and antibaryons were the same, does not have a physical meaning, and the problem of baryon asymmetry of the universe does not arise. Analogously, the numbers of positive and negative electric charges at the present stage of the universe should not be the same, i.e., the total electric charge of the universe should not be zero.

PACS numbers: 02.20.Sv, 03.65.Ta, 11.30-j, 11.30.Cp, 11.30.Ly

Keywords: irreducible representations, particle-antiparticle, de Sitter symmetry, baryon asymmetry of the universe

1 Introduction

Modern fundamental particle theories (QED, QCD and electroweak theory) are based on Poincare symmetry, and in those theories the electric charge conservation and baryon number conservation are a must. In his famous paper "Missed Opportunities" [1] Dyson proves that de Sitter (dS) and anti-de Sitter (AdS) symmetries are more general (fundamental) than Poincare one, even from pure mathematical considerations, because dS and AdS groups are more symmetric than Poincare one. The transition from the former to the latter is described by a procedure called contraction when a parameter R (see below) goes to infinity. At the same time, since dS and AdS groups are semisimple, they have a maximum possible symmetry and cannot be obtained from more symmetric groups by contraction.

The paper [1] appeared in 1972 and, in view of Dyson's results, a question arises why the fundamental particle theories are still based on Poincare symmetry and not dS or AdS ones. The parameter R arises from particle theory but in the literature it is often interpreted as the radius of the universe. Probably, physicists believe that, since R is even much greater than sizes of stars, the dS and AdS symmetries can play an important role only in cosmology and there is no need to use them for describing elementary particles. We believe that this argument is not consistent because usually more general theories shed a new light on standard concepts, and the discussion in this paper is a good illustration of this point.

In Sec. 2 we describe the concept of symmetry on quantum level. In Sec. 3 we describe important properties of dS and AdS symmetries in standard quantum theory and in a quantum theory based on finite mathematics (FQT). Here we give a popular explanation why standard concepts of particle-antiparticle, electric charge and baryon number have only a limited meaning when the symmetry in FQT is broken to Poincare or standard anti-de Sitter symmetries. Following the results of our book [2], we also describe some applications of those results. The latter symmetries are good approximations at the present stage of the universe but in the early stages they cannot take place. Therefore, as explained in Sec. 4, the statement that in such stages the numbers of baryons and antibaryons were the same, does not have a physical meaning, and the problem of baryon asymmetry of the universe (BAU) does not arise. Finally, Sec. 5 is discussion.

In this paper we describe all physical quantities in units $c = \hbar = 1$.

2 Symmetry on quantum level

In relativistic quantum theory, the usual approach to symmetry on quantum level follows. Since the Poincare group is the group of motions of Minkowski space, quantum states should be described by representations of this group. This implies that the representation generators commute according to the commutation relations of the Poincare group Lie algebra:

$$[P^\mu, P^\nu] = 0, \quad [P^\mu, M^{\nu\rho}] = -i(\eta^{\mu\rho}P^\nu - \eta^{\mu\nu}P^\rho),$$

$$[M^{\mu\nu}, M^{\rho\sigma}] = -i(\eta^{\mu\rho}M^{\nu\sigma} + \eta^{\nu\sigma}M^{\mu\rho} - \eta^{\mu\sigma}M^{\nu\rho} - \eta^{\nu\rho}M^{\mu\sigma}) \quad (1)$$

where $\mu, \nu = 0, 1, 2, 3$, P^μ are the operators of the four-momentum and $M^{\mu\nu}$ are the operators of Lorentz angular momenta. This approach is in the spirit of Klein's Erlangen program in mathematics.

However, as discussed in detail in Refs. [2, 3, 4], in quantum theory the concept of space-time background does not have a physical meaning. Although for constructing modern theories of elementary particles local Lagrangians are used, the goal of the theory is to construct the S-matrix in momentum space, and, when this construction has been accomplished, one can forget about space-time background. This is in the spirit of the Heisenberg S-matrix program according to which in quantum theory one can describe only transitions of states from the infinite past when $t \rightarrow -\infty$ to the distant future when $t \rightarrow +\infty$.

As argued in Refs. [2, 5], the approach should be the opposite. Each system is described by a set of linearly independent operators. By definition, the rules how they commute with each other define the symmetry algebra. In particular, *by definition*, Poincare symmetry on quantum level means that the operators commute according to Eq. (1). This definition does not involve Minkowski space at all. I am very grateful to Leonid Avksent'evich Kondratyuk for explaining me this definition during our collaboration.

For illustration, let us consider the concept of particle-antiparticle. Historically, this concept arose because the Dirac equations have solutions with both positive and negative energies. It is a great success of the Dirac equations that in the approximation $(v/c)^2$ they reproduce the fine structure of the hydrogen atom with a very high accuracy. However, in higher order approximations, non-quantized Dirac spinors do not have a physical meaning for several reasons. First, as noted below, in systems consisting of particles and antiparticles the energy sign of all of them should be the same. Also, in higher order approximations in v/c , the probabilistic interpretation of non-quantized Dirac spinors is lost because the coordinate description implies that they are described by representations induced from non-unitary representations of the Lorentz algebra. On the other hand, in Poincare invariant particle theories, the concept of particle-antiparticle is a consequence of the fact that elementary particles are described by irreducible representations (IRs) of the algebra (1), such that in each IR energies can be either strictly positive or strictly negative but there are no IRs where energies have different signs. Then objects described by positive-energy IRs are called particles, and objects described by negative-energy IRs are called antiparticles. As explained in detail in the next section, energies of both particles and antiparticles become positive after second quantization.

By analogy with the definition of Poincare symmetry on quantum level, the definition of dS symmetry on quantum level should not involve the fact that the dS group is the group of motions of dS space. Instead, *the definition* is that the operators M^{ab} ($a, b = 0, 1, 2, 3, 4$, $M^{ab} = -M^{ba}$) describing the system under consideration satisfy the commutation relations of the dS Lie algebra, i.e.,

$$[M^{ab}, M^{cd}] = -i(\eta^{ac}M^{bd} + \eta^{bd}M^{ac} - \eta^{ad}M^{bc} - \eta^{bc}M^{ad}) \quad (2)$$

where η^{ab} is the diagonal metric tensor such that $\eta^{00} = -\eta^{11} = -\eta^{22} = -\eta^{33} = -\eta^{44} = 1$. The *definition* of AdS symmetry on quantum level is given by the same equations but $\eta^{44} = 1$.

The procedure of contraction from dS and AdS symmetries to Poincare one is defined as follows. If we *define* the operators P^ν as $P^\nu = M^{\nu 4}/R$ where R is a parameter with the dimension *length* then in the formal limit when $R \rightarrow \infty$, $M^{\nu 4} \rightarrow \infty$ but the quantities P^ν are finite, Eqs. (2) become Eqs. (1). This procedure is the same for the dS and AdS symmetries.

The above contraction is analogous to the contraction from Poincare symmetry to Galilei one, where the parameter of contraction is c . On quantum level, R and c are only the parameters describing the relations between Lie algebras of higher and lower symmetries. On classical level, the physical meaning of c is well-known, while R is the radius of the dS or AdS space. A detailed discussion of the both contractions is described in a vast literature, in particular, in Refs. [2, 3, 4] where it has been proposed the following

Definition: *Let theory A contain a finite nonzero parameter and theory B be obtained from theory A in the formal limit when the parameter goes to zero or infinity. Suppose that, with any desired accuracy, theory A can reproduce any result of theory B by choosing a value of the parameter. On the contrary, when the limit is already taken, one cannot return back to theory A, and theory B cannot reproduce all results of theory A. Then theory A is more general (fundamental) than theory B and theory B is a special degenerate case of theory A.*

As proved in Refs. [2, 3, 4], dS and AdS symmetries are more general (fundamental) than Poincare symmetry. The latter is a special degenerate case of the former in the formal limit $R \rightarrow \infty$. As noted above, in contrast to Dyson's approach based on Lie groups, our approach is based on Lie algebras. Then, as proved in Refs. [2, 3, 4], classical theory is a special degenerate case of quantum one in the formal limit $\hbar \rightarrow 0$, and nonrelativistic theory (NT) is a special degenerate case of relativistic one (RT) in the formal limit $c \rightarrow \infty$. In the literature the above facts are explained from physical considerations but, as shown in Refs. [2, 3, 4] they can be proved mathematically by using properties of Lie algebras.

As proved in Refs. [2, 4] classical mathematics (involving the concepts of limits, infinitesimals, continuity etc.) is a special degenerate case of finite mathematics in the formal limit when the characteristic p of the ring or field in the latter goes to infinity. Therefore standard dS and AdS symmetries over the field of complex numbers can be generalized to dS and AdS symmetries over a finite ring or field of characteristic p .

Finite mathematics rejects infinities from the beginning. It starts from the ring $R_p = (0, 1, 2, \dots, p-1)$ where addition, subtraction and multiplication are performed as usual but modulo p , and p is called the characteristic of the ring. In the literature the ring R_p is usually denoted as Z/pZ . In our opinion this notation is not adequate because it may give a wrong impression that finite mathematics starts from the infinite set Z and that Z is more general than R_p . However, although Z has more elements than R_p , Z cannot be more general than R_p because Z does not

contain operations modulo a number.

One can rigorously prove [2, 4] that any operation in Z can be reproduced in R_p if p is chosen to be sufficiently large, and that is why Z can be treated as a limit of R_p when $p \rightarrow \infty$. This result looks natural from the following considerations. Since all operations in R_p are modulo p , R_p can be treated as a set $(-(p-1)/2, \dots -1, 0, 1, \dots (p-1)/2)$ if p is odd and as a set $(-p/2+1, \dots -1, 0, 1, \dots p/2)$ if p is even. In this representation, for numbers with the absolute values much less than p , the results of addition, subtraction and multiplication are the same in R_p and in Z , i.e., for such numbers it is not manifested that in R_p operations are modulo p . This example also demonstrates that in finite mathematics the concepts of positive and negative cannot be fundamental. For example, the numbers $-(p-1)/2$ and $(p+1)/2$ represent the same element of R_p because they are the same modulo p .

We use the abbreviation FQT (finite quantum theory) to denote quantum theory over the ring or field of characteristic p . By using the fact that Z is a limit of R_p when $p \rightarrow \infty$, one can prove [2] that FQT is more general (fundamental) than standard quantum theory (SQT). In view of **Definition**, this implies that, by choosing a sufficiently large value of p , FQT can reproduce any result of SQT with any desired accuracy while SQT cannot reproduce all results of FQT: it cannot reproduce results where it is important that p is finite and not infinitely large. In Ref. [2], several physical phenomena where it is important that p is finite have been considered. Some of those phenomena are mentioned in Subsec. 3.4. We also discuss that this fact is also important for understanding BAU.

The above relation between FQT and standard quantum theory is analogous to the relation between RT and NT: by choosing a sufficiently large value of c , RT can reproduce any result of NT with any desired accuracy while NT cannot reproduce all results of RT: it cannot reproduce results where it is important that c is finite and not infinitely large.

One can now consider the commutation relations (2) in spaces over a finite ring or field of characteristic p . In this way we get a generalization of standard dS and AdS quantum theories to dS and AdS quantum theories over a finite ring or field (see Refs. [2, 6] for details).

3 Properties of quantum theories based on Poincare, dS and AdS symmetries

3.1 Particles and antiparticles in Poincare invariant theories

Let p^ν be the four-momentum of a particle in Poincare invariant theory. Define $p^2 = p^\nu p_\nu$, where a sum over repeated indices is assumed. Then for usual particles $p^2 \geq 0$ while for tachyons $p^2 < 0$. The existence of tachyons is a problem, and we will consider only usual particles. Then the mass of the particle can be defined as a nonnegative number m such that $m^2 = p^2$.

Elementary particles in Poincare invariant theory are described by IRs of

the Poincare algebra by selfadjoint operators. The energy E of a particle with the momentum \mathbf{p} and mass m equals $\pm(m^2 + \mathbf{p}^2)^{1/2}$. The choice of the sign of the square root is only the matter of convention but not the matter of principle. Depending on this sign, there are IRs where energies can be only either positive or negative while the probability to have zero energy is zero. By convention, objects described by positive energy IRs are called particles, and objects described by negative energy IRs are called antiparticles.

When we consider a system consisting of particles and antiparticles then the energy sign of both, particles and antiparticles should be the same. Indeed, consider, for example a system of two particles with the same mass m and let the momenta \mathbf{p}_1 and \mathbf{p}_2 be such that the total momentum $\mathbf{p}_1 + \mathbf{p}_2$ equals zero. Then, if the energy of particle 1 is positive, and the energy of particle 2 is negative then the total four-momentum of the system would be zero what contradicts experimental data. By convention, the energy sign of all particles and antiparticles in question is chosen to be positive. For this purpose, the procedure of second quantization is defined such that after the second quantization the energies of antiparticles become positive. Then the mass of any particle is the minimum value of its energy in the case when the momentum equals zero.

One of the key principles of quantum theory is the superposition principle: if ψ_1 and ψ_2 are possible states of a system then $a\psi_1 + b\psi_2$, where a and b are some coefficients, also is a possible state. However, this principle is not universal because superselection rules prohibit certain types of superpositions. For example, superpositions of states with different electric charges are prohibited, and a wave function ψ of a system cannot be a superposition $\psi = a\psi_1 + b\psi_2$ where ψ_1 refers to a particle and ψ_2 refers to an antiparticle.

Suppose now that we have two particles such that particle 1 has the mass m_1 , spin s_1 and is characterized by some additional quantum numbers (e.g., electric charge, baryon quantum number etc.), and particle 2 has the mass m_2 , spin $s_2 = s_1$ and all additional quantum numbers characterizing particle 2 equal the corresponding additional quantum numbers for particle 1 with the opposite sign. A question arises when particle 2 can be treated as an antiparticle for particle 1. Is it necessary that m_1 should be exactly equal to m_2 or m_1 and m_2 can slightly differ each other? In particular, can we guarantee that the mass of the positron exactly equals the mass of the electron, the mass of the proton exactly equals the mass of the antiproton etc.?

If particle 2 (for some reasons) is treated as an antiparticle for particle 1, and the particles are considered only on the level of IRs, then the relation between m_1 and m_2 is fully arbitrary. However, in local quantum field theory (QFT), IRs for a particle and its antiparticle are combined together in the framework of a local field. A non-quantized quantum field $\psi(x)$, where x is a point in Minkowski space, combines together two IRs with positive and negative energies. The IR with the positive energy is associated with a particle and the IR with the negative energy is associated with the corresponding antiparticle. From mathematical point of view, a local quantum field is described by a reducible representation induced not from the little algebra IRs are induced from but from the Lorenz algebra. The local fields depend on x because

the factor space of the Poincare group over the Lorentz group is Minkowski space. However, there is no physical operator corresponding to x , i.e., x is not measurable. Since the fields describe nonunitary representations, their probabilistic interpretation is problematic. As shown by Pauli [7], in the case of fields with an integer spin there is no invariant subspace where the spectrum of the charge operator has a definite sign while in the case of fields with a half-integer spin there is no invariant subspace where the spectrum of the energy operator has a definite sign. It is also known that the description of the electron in the external field by the Dirac spinor is not accurate (e.g. it does not take into account the Lamb shift).

A secondly quantized field $\psi(x)$ is an operator in the Fock space and therefore the contribution of each particle is explicitly taken into account. Each particle in the field can be described by its own coordinates (in the approximation when the position operator exists - see e.g., Ref. [2]). In view of this fact, the following natural question arises: why do we need an extra coordinate x which does not belong to any particle? This coordinate does not have a physical meaning and is simply a parameter arising from the second quantization of the non-quantized field $\psi(x)$.

In QFT the Lagrangian density depends on local quantized fields and the four-vector x in them is associated with a point in Minkowski space. However, x does not have a physical meaning and is only the formal integration parameter which is used in the intermediate stage. The goal of the theory is to construct the S -matrix and, when the theory is already constructed, one can forget about Minkowski space because no physical quantity depends on x . This is in the spirit of the Heisenberg S -matrix program according to which in relativistic quantum theory it is possible to describe only transitions of states from the infinite past when $t \rightarrow -\infty$ to the distant future when $t \rightarrow +\infty$. The fact that the S -matrix is the operator in momentum space does not exclude a possibility that in some situations it is possible to have a space-time description with some accuracy but not with absolute accuracy [2].

In QFT the fact that $m_1 = m_2$ follows from the CPT theorem which is a consequence of locality since *we construct* local covariant fields from a particle and its antiparticle with equal masses. A question arises what happens if locality is only an approximation: in that case the equality of masses is exact or approximate? Consider a simple model when electromagnetic and weak interactions are absent. Then the fact that the proton and the neutron have equal masses has nothing to do with locality; it is only a consequence of the fact that the proton and the neutron belong to the same isotopic multiplet. In other words, they are simply different states of the same object—the nucleon.

Since the concept of locality is not formulated in terms of selfadjoint operators, this concept does not have a clear physical meaning, and this fact has been pointed out even in known textbooks (see e.g. Ref. [8]). Therefore, without additional assumptions (e.g., locality), such theories cannot conclude whether the masses of particles and corresponding antiparticles must be exactly equal to each other. Note also that in Poincare invariant quantum theories there can exist elementary particles for which all additional quantum numbers are zero. Such particles are called neutral because they coincide with their antiparticles.

3.2 Particles and antiparticles in dS invariant theories

The descriptions of elementary particles in the dS and AdS cases are considerably different. In the former case all the operators M^{0a} ($a = 1, 2, 3, 4$) are on equal footing. Therefore, M^{04} can be treated as the Poincare analog of the energy only in the approximation when R is rather large. In the general case, the sign of M^{04} cannot be used for the classification of IRs.

In his book [10] Mensky describes the implementation of dS IRs when the representation space is the three-dimensional unit sphere in the four-dimensional space. In this implementation, there exists one-to-one relation between the northern hemisphere and the upper Lorentz hyperboloid with positive Poincare energies, and one-to-one relation between the southern hemisphere and the lower Lorentz hyperboloid with negative Poincare energies, while points on the equator correspond to infinite Poincare energies. However, the operators of IRs are not singular in the vicinity of the equator and, since the equator has measure zero, the properties of wave functions on the equator are not important.

Since the number of states in dS IRs is twice as big as the number of states in IRs of the Poincare algebras, one might think that each IR of the dS algebra describes a particle and its antiparticle simultaneously. However, a detailed analysis in Refs. [2, 4, 9] shows that states described by dS IRs cannot be characterized as particles or antiparticles in the usual meaning.

For example, let us call states with the support of their wave functions on the northern hemisphere as particles and states with the support on the southern hemisphere as their antiparticles. Then states which are superpositions of a particle and its antiparticle obviously belong to the representation space under consideration, i.e., they are not prohibited. However, as noted in the preceding subsection, this contradicts the superselection rule that the wave function cannot be a superposition of states with opposite electric charges, baryon and lepton quantum numbers etc. Therefore, in the dS case there are no superselection rules which prohibit superpositions of states with opposite electric charges, baryon quantum numbers etc. In addition, in this case it is not possible to define the notion of neutral particles.

As noted in Sec. 2, dS symmetry is more general than Poincare one, and the latter can be treated as a special degenerate case of the former in the formal limit $R \rightarrow \infty$. This means that, with any desired accuracy, any phenomenon described in the framework of Poincare symmetry can be also described in the framework of dS symmetry if R is chosen to be sufficiently large, but there also exist phenomena for explanation of which it is important that R is finite and not infinitely large (see Subsec. 3.4).

As shown in Refs. [2, 9], dS symmetry is broken in the formal limit $R \rightarrow \infty$ because one IR of the dS algebra splits into two IRs of the Poincare algebra with positive and negative energies and with equal masses. Therefore, the fact that experimentally the masses of particles and their corresponding antiparticles are equal to each other, can be explained as a consequence of the fact that observable properties of elementary particles can be described not by exact Poincare symmetry but by dS

symmetry with a very large (but finite) value of R . In contrast to QFT, for combining a particle and its antiparticle into one object, there is no need to assume locality and involve local field functions because a particle and its antiparticle already belong to the same IR of the dS algebra (compare with the above remark about the isotopic symmetry in the proton-neutron system).

The fact that dS symmetry is higher than Poincare one is clear even from the fact that, in the framework of the latter symmetry, it is not possible to describe states which are superpositions of states on the upper and lower hemispheres. Therefore, breaking the IR into two independent IRs defined on the northern and southern hemispheres obviously breaks the initial symmetry of the problem. This fact is in agreement with the Dyson observation (mentioned above) that dS group is more symmetric than Poincare one.

When $R \rightarrow \infty$, standard concepts of particle-antiparticle, electric charge and baryon and lepton quantum numbers are restored, i.e., in this limit superpositions of particle and antiparticle states and states with positive and negative additive quantum numbers become prohibited according to the superselection rules. Therefore, those concepts arise as a result of symmetry breaking, i.e., they are not universal.

3.3 Particles and antiparticles in AdS invariant theories

In theories where the symmetry algebra is the AdS algebra, the structure of IRs is known (see e.g. Refs. [2, 11]). The operator M^{04} is the AdS analog of the energy operator. Let W be the Casimir operator $W = \frac{1}{2} \sum M^{ab} M_{ab}$ where a sum over repeated indices is assumed. As follows from the Schur lemma, the operator W has only one eigenvalue in every IR. By analogy with Poincare invariant theory, we will not consider AdS tachyons and then one can define the AdS mass μ such that $\mu \geq 0$ and μ^2 is the eigenvalue of the operator W .

As noted in Sec. 2, the procedure of contraction from the AdS algebra to the Poincare one is defined such that if R is a parameter with the dimension *length* then $M^{\nu 4} = R P^\nu$. This procedure has a physical meaning only if R is rather large. In that case the AdS mass μ and the Poincare mass m are related as $\mu = Rm$, and the relation between the AdS and Poincare energies is analogous. Since AdS symmetry is more general (fundamental) than Poincare one then μ is more general (fundamental) than m . In contrast to the Poincare masses and energies, the AdS masses and energies are dimensionless. From cosmological considerations (see Subsec. 3.4), the value of R is usually accepted to be of the order of $10^{26}m$. Then the AdS masses of the electron, the Earth and the Sun are of the order of 10^{39} , 10^{93} and 10^{99} , respectively. The fact that even the AdS mass of the electron is so large might be an indication that the electron is not a true elementary particle. In addition, the present accepted upper level for the photon mass is $10^{-17}ev$. This value seems to be an extremely tiny quantity. However, the corresponding AdS mass is of the order of 10^{16} , and so, even the mass which is treated as extremely small in Poincare invariant theory might be very large in AdS invariant theory.

In the AdS case there are IRs with positive and negative energies, and

they belong to the discrete series [2, 11]. Therefore, one can define particles and antiparticles. If μ_1 is the AdS mass for a positive energy IR, then the energy spectrum contains the eigenvalues $\mu_1, \mu_1+1, \mu_1+2, \dots, \infty$, and, if μ_2 is the AdS mass for a negative energy IR, then the energy spectrum contains the eigenvalues $-\infty, \dots, -\mu_2-2, -\mu_2-1, -\mu_2$. Therefore, the situation is pretty much analogous to that in Poincare invariant theories, and there is no way to conclude whether the mass of a particle equals the mass of the corresponding antiparticle.

As noted in Sec. 2, FQT is more general (fundamental) than SQT, and in FQT it is also possible to define the concepts of dS and AdS symmetries. As discussed in Ref. [2], in FQT the dS and AdS cases are physically equivalent. The description of the energy spectrum in standard IRs of the AdS algebra has been given above. We will now explain why in FQT the spectrum is different, and in FQT the situation is similar to that in standard dS case but not standard AdS one because IRs in FQT contain both, positive and negative energies. Let us note first that, while in SQT the quantity μ can be an arbitrary real number, in FQT μ is an element of R_p . As noted above, if p is odd then R_p contains the elements $-(p-1)/2, \dots, -1, 0, 1, \dots, (p-1)/2$ and the case when p is even is analogous. For definiteness, we consider the case when p is odd.

By analogy with the construction of positive energy IRs in SQT, in FQT we start the construction from the rest state, where the AdS energy is positive and equals μ . Then we act on this state by raising operators and gradually get states with higher and higher energies, i.e., $\mu+1, \mu+2, \dots$. However, in contrast to the situation in SQT, we cannot obtain infinitely large numbers. When we reach the state with the energy $(p-1)/2$, the next state has the energy $(p-1)/2+1=(p+1)/2$ and, since the operations are modulo p , this value also can be denoted as $-(p-1)/2$ i.e., it may be called negative. When this procedure is continued, one gets the energies $-(p-1)/2+1=-(p-3)/2, -(p-3)/2+1=-(p-5)/2, \dots$ and, as shown in Ref. [2], the procedure finishes when the energy $-\mu$ is reached.

Therefore, in contrast to the situation in SQT, in FQT, IRs are finite-dimensional (and even finite since the ring R_p and its complex extension R_p+iR_p are finite). By analogy with the dS case in SQT, one can call the states with the energies $\mu, \mu+1, \mu+2, \dots, \infty$ particles and states with the energies $-\infty, \dots, -\mu-2, -\mu-1, -\mu$ antiparticles. Therefore, in FQT the mass of a particle automatically equals the mass of the corresponding antiparticle. This is an example when FQT can solve a problem which standard quantum AdS theory cannot. By analogy with the situation in the dS case, for combining a particle and its antiparticle together, there is no need to involve additional coordinate fields because a particle and its antiparticle are already combined in the same IR.

Then, since states which are superpositions of particles and antiparticles belong to the representation space, we conclude by analogy with the situation in Subsec. 3.2, that in FQT there are no superselection rules which prohibit superpositions of states with opposite electric charges, baryon quantum numbers etc. Moreover, the representation operators of the enveloping algebra can perform transformations *particle* \leftrightarrow *antiparticle*.

As shown in Ref. [2], in the formal limit $p \rightarrow \infty$, one IR in FQT splits into two standard IRs of the AdS algebra with positive and negative energies. Therefore, in this limit the concept of particle-antiparticle and the superselection rules have the usual meaning. In turn, in situations when one can define the quantity R such that the contraction to the Poincare algebra works with a high accuracy, one can describe particles and antiparticles in the framework of Poincare symmetry.

Even from the fact that in standard quantum theory, there are no superpositions of states belonging to a particle and its antiparticle, it is clear that symmetry described by one IR in FQT is higher than symmetry described by two IRs obtained from one IR in FQT in the formal limit $p \rightarrow \infty$. Therefore standard concepts of particle-antiparticle and superselection rules arise as a result of symmetry breaking, i.e., they are not universal.

3.4 Applications

The above discussion indicates that FQT will be based on principles which in several important aspects considerably differ from the principles of SQT. Therefore, probably, the construction of FQT will be very difficult. In this situation it is important to find phenomena which can be treated in favor of principles of the new quantum theory. In Ref. [2] we discussed such phenomena, and some of them are mentioned in this subsection.

Consider first the problem of cosmological acceleration. This effect is usually interpreted as a manifestation of dark energy because it is assumed that quantum theory describing this phenomenon is based on Poincare symmetry. However, as shown in Refs. [2, 3, 4, 9], the quantity R in semiclassical approximation coincides with the radius of dS space. In General Relativity (GR), Λ is a formal parameter called the cosmological constant. However, in the formula for the cosmological acceleration given by GR, Λ is the standard curvature of the dS space given by $\Lambda = 3/R^2$, and this quantity is extracted from the data on cosmological acceleration. The effect of the universe expansion clearly shows that the radius of the world is increasing over time. Therefore, Λ cannot be a time-independent constant. The assumption that it is can be only a good approximation when we consider periods of times which are much less than cosmological times. The comparison of the formula for the cosmological acceleration with experimental data gives that now R is of the order of $10^{26}m$. Since this value is very large then, at present, the cosmological constant is small, and Poincare symmetry works with a very high accuracy.

The formula for the cosmological acceleration derived in our approach coincides with the formula obtained in GR. However, in contrast to the result of GR, our result has been obtained without using geometry of dS space (its metric and connection) but only by considering quantum mechanics of the two-body system in quantum dS theory. We believe that our result is more important than the result of GR because GR is a pure classical theory, and any classical result should be a consequence of quantum theory in semiclassical approximation. Our explanation does not need dark energy or other artificial notions (see the discussion in Ref. [2] for

more details). As a consequence, the fact that $\Lambda > 0$ can be treated as an indication that, on quantum level, dS symmetry is more pertinent than AdS one.

Now we consider effects where it is important that the characteristic p in FQT is finite, and therefore they cannot be described in the framework of SQT. One of such effects is the famous Dirac vacuum energy problem. The vacuum energy should be zero, but in SQT the sum for this energy diverges. In Sec. 8.8 of Ref. [2], we take the standard expression for this sum, explicitly calculate this sum in finite mathematics without any assumptions, and, since all the calculations are modulo p , we get zero as it should be.

Another example is as follows. SQT cannot describe gravity because the theory is nonrenormalizable. But in our approach, the universal law of gravitation can be derived as a consequence of FQT in semiclassical approximation [2]. In this case the gravitational constant G depends on p as $1/\ln(p)$. By comparing the result with the experimental value, one gets that $\ln(p)$ is of the order of 10^{80} or more, and therefore p is a huge number of the order of $\exp(10^{80})$ or more. One might think that since p is so huge then in practice p can be treated as an infinite number. However, since G depends on p as $1/\ln(p)$, and $\ln(p)$ is "only" of the order of 10^{80} , gravity is observable. In the formal limit $p \rightarrow \infty$, G becomes zero and gravity disappears. Therefore, in our approach, gravity is a consequence of finiteness of nature.

4 Explanation of baryon asymmetry of the universe

The problem of the baryon asymmetry of the universe (BAU) is a long-standing problem of particle theory and quantum cosmology described in a vast literature (see e.g., Ref. [12] and references therein). According to modern quantum theories, the baryon number is a conserved quantum number, and, according to modern cosmological theories, the universe has been created with equal numbers of baryons and antibaryons. Then a problem arises why there is an imbalance in baryonic matter and antibaryonic matter in the observable universe.

As noted by Sakharov in 1967, one of his three necessary "Sakharov conditions" for the asymmetry is baryon number non-conservation. This condition was investigated in Grand Unified Theories, and extensive experiments on the search of the proton decay have been performed. However, the result of all those experiments was negative, i.e., no proton instability has been found.

As noted in Subsec. 3.4, the usual choice for R is $R \approx 10^{26}m$. The fact that this value is very large shows that currently Poincare symmetry is satisfied with a very high accuracy. Therefore, the above concepts work with a very high accuracy. However, in early stages of the universe the value of R was much less than now. As explained in the preceding sections, at such conditions the meaning of the above concepts differs from standard meaning. In particular, conservation of electric charge and baryon quantum number does not take place. Therefore, the BAU problem does

not arise because the statement that the universe has been created with equal numbers of baryons and antibaryons does not have a physical meaning.

The BAU problem can also be explained proceeding from the fact that, as proved in Ref. [2] and explained above, FQT is the most general quantum theory. Then the fact that currently the above concepts work with a very high accuracy indicates that the value of p is very large. As noted in Subsec. 3.4, currently p is at least of the order of $\exp(10^{80})$.

As noted in Ref. [2] the fact that finite mathematics is more general (fundamental) than standard one is clear even from the philosophy of verificationism and the philosophy of quantum theory. Every computing device can perform mathematical operations only modulo some number p which are defined by the number of bits this device can operate with. It is reasonable to believe that finite mathematics describing physics in our universe is characterized by a characteristic p which depends on the current state of the universe, i.e., the universe can be treated as a computer. Therefore, it is reasonable to believe that the number p is different at different stages of the universe.

At the present stage of the universe the number p is huge but, as argued in Ref. [2], in earlier stages this number was much less than now. However, when p increases then for a greater and greater number of particles, standard concepts of particle-antiparticle, electric charge and the baryon number work with a greater and greater accuracy. Finally, at the present stage of the universe standard concepts work with an extremely high accuracy. Since the particle theory in FQT is not yet developed, the existing theory cannot predict what the relation between the numbers of baryons and antibaryons should be at present. However, there are no reasons to think that those numbers should be the same.

For example, suppose that those numbers have arisen as a result of pure random circumstances. Then, the probabilities of different numbers can be described by the Poisson distribution. For illustration, consider the case when one throws a coin N times and calculates the numbers of tails and heads. The probability of each event is $1/2$, and therefore, when N is very large, one can expect that approximately $N/2$ events will be heads and approximately $N/2$ events will be tails. However, it is extremely improbable that the corresponding numbers will be exactly $N/2$ and $N/2$. Indeed, the root-mean-square-deviation is proportional to $N^{1/2}$. If, for example, $N = 1000$ then the most probable number of the deviation is approximately 32, and therefore the most probable difference between the numbers of heads and tails is 64. The real difference can considerably differ from 64 but it is extremely improbable that there will be exactly 500 heads and 500 tails.

Since we do not know what happens when p is not anomalously large, we cannot exclude a possibility that the numbers of baryons and antibaryons depend not only on pure random circumstances. For example, if something analogous to spontaneous symmetry breaking happens then the numbers of baryons and antibaryons at the present stage of the universe can be considerably different. Probably the future development of FQT will make it possible to estimate realistic differences between the numbers. However, let us stress again that even for a scenario when the numbers of

baryons and antibaryons in the universe are fully random, it is extremely improbable that those numbers will be the same.

5 Discussion

It has been noted in Subsec. 3.2 that in quantum theory based on classical mathematics, standard concepts of particle-antiparticle, the electric charge and the baryon quantum number arise as a result of symmetry breaking at $R \rightarrow \infty$ when dS invariant quantum theory is replaced by Poincare invariant quantum theory. It has been also noted in Subsec. 3.3 that those standard concepts arise as a result of symmetry breaking at $p \rightarrow \infty$ when FQT is replaced by standard quantum theory. Therefore those standard concepts are not universal.

The present fundamental particle theories are based on Poincare invariant QFT, and, as noted in Subsec. 3.1, for solving the problem why a particle and its antiparticle have equal masses, those theories involve local quantized field $\psi(x)$ where x does not belong to any particle and is simply a parameter arising from the second quantization of a non-quantized field. So, the physical meaning of x is not clear. Although QFT has many successes, it also has problems because, as noted, for example, in the textbook [8], $\psi(x)$ is an operatorial distribution, and the product of distributions at the same point is not a well defined mathematical operation.

As explained in Subsecs. 3.2 and 3.3, in standard quantum theory based on dS symmetry and in FQT, the masses of a particle and the corresponding antiparticle are automatically equal, and this is achieved without introducing local quantized fields. However, in those theories the concepts of particle-antiparticle and additive quantum numbers differ from standard ones because one IR combines together a particle and its antiparticle. The construction of such theories is one of the most fundamental (if not the most fundamental) problems of quantum theory.

At present this construction has not been yet developed and therefore at the present stage of quantum theory it is not possible to describe physics in earlier stages of the universe when the values of R and p were much less than now. *In particular, this implies that existing cosmological theories describing very early stages of the universe are not reliable.* We can only say that, because at these stages standard concepts of particle-antiparticle, the electric charge and the baryon quantum number do not have standard meaning, the statement that the numbers of baryons and antibaryons at those stages are the same is not substantiated. *Therefore, the BAU problem does not arise because there is no reason to expect that the numbers of baryons and antibaryons at the present stage of the universe are the same.*

Let us now discuss the following point. As noted above, standard concept of electric charge also arises as a result of symmetry breaking when, either as a result of taking the limit $R \rightarrow \infty$ in dS SQT, this theory becomes Poincare invariant SQT, or, as a result of taking the limit $p \rightarrow \infty$ in FQT, this theory becomes SQT based on AdS symmetry. Therefore, standard concept of the electric charge also cannot be universal, as well as the concept of the baryon number. Therefore, the problem of the

electric charge can be discussed in full analogy with the above consideration of the BAU problem. According to the present cosmological models, the universe has been created as electrically neutral, and then, in view of the electric charge conservation, it should be electrically neutral at the present stage. *However, in full analogy with the above consideration, one can conclude that the present state of the universe is not electrically neutral.* The usual statement is that the total electrical charge of stars is typically positive because electrons can escape from thermonuclear reactions inside stars, but the total electric charge of the universe is zero.

Acknowledgements

The idea to write this paper has arisen as a result of my numerous discussions with Vladimir Karmanov and his constructive criticism. He also has pointed out to me that one of the possibilities for explaining the BAU problem is to involve the Poisson distribution. I am also grateful to anonymous referees for important remarks. They helped me understand how the article can be perceived by physicists.

Conflict of Interest: The author declares that he has no conflicts of interest.

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