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Beal Conjecture Proved Very Simply

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Abstract

The author proves directly the original Beal conjecture (and not the equivalent conjecture) that if $A^x + B^y = C^z$, where A, B, C, x, y, z are positive integers and $x, y, z > 2$, then A, B and C have a common prime factor. One will let r, s and t be prime factors of A, B and C .respectively, where D, E and F are positive integers, such that $A = Dr$, $B = Es$, $C = Ft$. Then, the equation $A^x + B^y = C^z$ becomes $D^x r^x + E^y s^y = F^z t^z$. The proof would be complete after proving that $r^x = t^x$ and $s^y = t^y$, which would imply that $r = s = t$. The proofs of the above equalities would also be complete after showing that the ratio, $\frac{r^x}{t^x} = 1$ and the ratio, $\frac{s^y}{t^y} = 1$. The main principle for obtaining relationships between the prime factors on the left side of the equation and the prime factor on the right side of the equation is that the power of each prime factor on the left side of the equation equals the same power of the prime factor on the right side of the equation. High school students can learn and prove this conjecture as a bonus question on a final class exam.

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Option 1
Introduction

One will let r, s and t be prime factors of A, B and C, respectively, where D, E and F are positive integers, such that $A = Dr$, $B = Es$, $C = Ft$. Then, the equation, $A^x + B^y = C^z$ becomes $D^x r^x + E^y s^y = F^z t^z$. The proof would be complete after showing that $r = s = t$.
At a glance, and from the experience gained in solving exponential and logarithmic equations, one can identify the powers involved with respect to the prime factors, r, s, t, as r^x, s^y, and t^z. Thinking like a tenth grader, one would like to have equalities involving r^x, t^x, s^y, t^y. One will therefore, let $t^z = t^x t^{z-x}$ to introduce t^x, and $t^z = t^y t^{z-y}$ to introduce t^y, The possible equalities between the prime factors on the left side of the Beal equation and the prime factor on the right side of the equation are $r^x = t^x$, $r^x = t^z$, $s^y = t^y$ and $s^y = t^z$. Of these possibilities, only $r^x = t^x$ and $s^y = t^y$, on inspection, would lead to the conclusion, $r = t$ $s = t$, and $r = s = t$. Therefore, one conjectures the equalities, $r^x = t^x$ and $s^y = t^y$. These conjectures will be proved in the Beal conjecture proof. To prove these two equalities, one will show that the ratio, $(r^x/t^x) = 1$ and the ratio, $(s^y/t^y) = 1$. Two main steps are involved in the proof. In the first step, one will determine how r and t are related, and in the second step, one will determine how s and t are related.

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Option 2

Beal Conjecture Proved Very Simply

Given: $A^x + B^y = C^z$, A,B,C,x,y,z are positive integers and $x,y,z > 2$.
Required: To prove that A, B and C have a common prime factor.
Plan: Let r, s and t be prime factors of A, B and C .respectively, where D, E and F are positive integers, such that $A = Dr$, $B = Es$, $C = Ft$. Then, the equation $A^x + B^y = C^z$ becomes $D^x r^x + E^y s^y = F^z t^z$. The proof would be complete after showing that $r = s = t$. The conjectured equalities, $r^x = t^x$ and $s^y = t^y$, which would imply that $r = s = t$, will be proved by showing that $(r^x/t^x) = 1$ and .$(s^y/t^y) = 1$. In the first step of the proof, one will determine how r and t are related, and in the second step, one will determine how s and t are related.

Proof

Step 1: The conjectured equality, $r^x = t^x$ would be true if and only if $(r^x/t^x) = 1$.

$D^x r^x + E^y s^y = F^z t^z$ (1)

$\dfrac{D^x r^x + E^y s^y}{F^z t^z} = 1$ (Dividing both sides by $F^z t^z$) (2)

Because of the equality , $r^x = t^x$, a t^x factor is needed on the right side of equation (1)

$D^x r^x + E^y s^y = t^x t^{z-x} F^z$ (In eq.1, let $t^z = t^x t^{z-x}$)

$D^x r^x + E^y s^y = r^x t^{z-x} F^z$ (Replace t^x by r^x)

$D^x r^x + E^y s^y = r^x t^{-x} F^z t^z$ (Splitting t^{z-x})

$D^x r^x + E^y s^y = \dfrac{r^x}{t^x} F^z t^z$ (Positive exponents only)

$\dfrac{D^x r^x + E^y s^y}{F^z t^z} = \dfrac{r^x}{t^x}$ (solving for $\dfrac{r^x}{t^x}$)

$1 = \dfrac{r^x}{t^x}$ (From (2), $\dfrac{D^x r^x + E^y s^y}{F^z t^z} = 1$)

If $\dfrac{r^x}{t^x} = 1$,

$r^x = t^x$ $\left(t^x \dfrac{r^x}{t^x} = t^x\right)$

If $r^x = t^x$, $r = t$.

$(\log r^x = \log t^x; x\log r = x\log t; \log r = \log t;\ r = t)$

Step 2: The conjectured equality, $s^y = t^y$ would be true if and only if $(s^y/t^y) = 1$.

$D^x r^x + E^y s^y = F^z t^z$

Because of the equality, $s^y = t^y$, a t^y factor is needed on the right side of equation (1)

$D^x r^x + E^y s^y = t^y t^{z-y} F^z$ (Let $t^z = t^y t^{z-y}$)

$D^x r^x + E^y s^y = s^y t^{z-y} F^z$ (Replace t^y by s^y)

$D^x r^x + E^y s^y = s^y t^{-y} F^z t^z$ (Splitting t^{z-y})

$D^x r^x + E^y s^y = \dfrac{s^y}{t^y} F^z t^z$ (Positive exponents)

$\dfrac{D^x r^x + E^y s^y}{F^z t^z} = \dfrac{s^y}{t^y}$ (solving for $\dfrac{s^y}{t^y}$)

$1 = \dfrac{s^y}{t^y}$ (From (2) in Step 1, $\dfrac{D^x r^x + E^y s^y}{F^z t^z} = 1$)

If $\dfrac{s^y}{t^y} = 1$,

$s^y = t^y$ $\left(t^y \dfrac{s^y}{t^y} = t^y\right)$

If $s^y = t^y$, $s = t$.

$(\log s^y = \log t^y;\ y\log s = y\log t; = \log s = \log t;$
$= s = t)$

Step 3: It has been shown in Step 1 that $r = t$, and in Step 2 that $s = t$; therefore, $r = s = t$.

Since $A = Dr$, $B = Es$, $C = Ft$ and $r = s = t$, A, B and C have a common prime factor, and the proof is complete.

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Discussion

The proof was centered on proving the equalities $r^x = t^x$ and $s^y = t^y$ which imply that $r = s = t$, where r, s and t are prime factors of A, B and C respectively. In the previous papers viXra:2001.0694, viXra:2012.0041), the conjecture of these equalities was based on the properties of factored numerical Beal equations. In the present paper, no factoring was involved.
Fermat's Last Theorem can be proved by modifying the above proof as follows: For the hypothesis, let $x, y, z = n > 2$, $r \neq s \neq t$ and prove by contradiction (see viXra:2003.0303).

Illustration of the equalities $r^x = t^x$ and $s^y = t^y$ of factored Beal equations

For the factorization with respect to r^x: $r^x = t^x$

$$D^x r^x + E^y s^y = F^z t^z$$

$$\underbrace{r^x}_{K}[\underbrace{D^x + E^y s^y \bullet r^{-x}}_{L}] = \underbrace{t^x}_{M}\underbrace{t^{z-x} F^z}_{P} \qquad (K = M)$$

Example 1:

$$33^5 + 66^5 = 33^6$$

$$11^5 \bullet 3^5 + 11^5 \bullet 2^5 \bullet 3^5 = 11^6 \bullet 3^6$$

$$11^5(3^5 + 2^5 \bullet 3^5) = 11^5 \bullet 11 \bullet 3^6$$

$$\underbrace{11^5}_{k}(\underbrace{3^5 + 2^5 \bullet 3^5}_{L}) = \underbrace{11^5}_{M} \bullet \underbrace{11 \bullet 3^6}_{P}$$

For the factorization with respect to s^y: $s^y = t^y$

$$D^x r^x + E^y s^y = F^z t^z$$

$$\underbrace{s^y}_{K}[\underbrace{E^y + D^x r^x \bullet s^{-y}}_{L}] = \underbrace{t^y}_{M}\underbrace{t^{z-y} F^z}_{P} \quad (K = M)$$

Example 2:

$$34^5 + 51^4 = 85^4$$

$$17^5 \bullet 2^5 + 17^4 \bullet 3^4 = 17^4 \bullet 5^4$$

$$17^4(17 \bullet 2^5 + 3^4) = 17^4 \bullet 5^4$$

$$\underbrace{17^4}_{k}(\underbrace{17 \bullet 2^5 + 3^4}_{L}) = \underbrace{17^4}_{M} \bullet \underbrace{5^4}_{P}$$

Conclusion

The original Beal conjecture (and **not** the equivalent conjecture) has been proved in this paper. The proof was based on the two equalities, $r^x = t^x$ and $s^y = t^y$. which were conjectured .. From these. equalities, it was concluded that $r = s = t$, establishing the truthfulness of the Beal conjecture. High school students can learn and prove this conjecture as a bonus question on a final class exam.

PS: Other proofs of Beal Conjecture by the author are at viXra:2001.0694, viXra:1702.0331; viXra:1609.0383; viXra:1609.0157; viXra:2012.0041

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