

Cosmological expansion in objects bound by gravity ?

Volkmar Müller

Retired

Germany, 08428 Langenbernsdorf, Trünziger Str. 20

v.mueller@sternwarte-crimmitschau.de

Abstract

The lower limit of cosmological expansion postulated by standard cosmology is called into question for both theoretical and observable reasons. The cosmological redshift is not interpreted here as a Doppler effect and numerical increase in distance, but as an expansion of the space-time scale. All objects dominated by gravity, including small objects such as the Earth, participate in cosmological expansion contrary to standard theory. We give a number of examples for the extreme, 18-digit accordance of expansion rates of different knowledge areas with the cosmological expansion rate. This accordance means that a common cause is almost inevitable. The space expands with its content while maintaining the numerical distance, radius, rotation time and density. The Earth therefore shows no expansion, but a scale drift with a drift rate of the size of the Hubble constant. This intrinsic drift apparently also applies to distant galaxies. The expansion of galaxies assumed here makes the assumption of dark matter superfluous. The continents and our usual everyday environment are not involved in the expansion.

Keywords : Cosmological expansion - earth expansion - common rate of expansion - examples - conclusions

1. Introduction

In recent years, a number of accordances with the Hubble relation or expansion rate of the universe have been found in areas that are bound by gravitation ($72 \text{ km} \times \text{s}^{-1} \times \text{Mpc}^{-1} \cong 2.4 \times 10^{-18} \text{ s}^{-1}$). A causal connection for these was already excluded by Einstein and Straus in 1945 [1]. That was certainly a reason to ignore the counter-arguments, some of which are in areas other than astrophysics. However, there are some reasons not to do so. Therefore, some relevant terms should be highlighted here. These terms are described in more detail in Sections 2–4. Section 5 briefly describes the origin and nature of the counter-arguments and section 6 contains some of the resulting conclusions.

2. Expansion of space and relative velocity

In the spectra of extragalactic objects, redshift values occur which correspond to a recession speed greater than the speed of light (c). The cosmological speed of recession can therefore not be a relative speed in the sense of the special theory of relativity (STR) [2]. Numerical changes in the distances (relative speeds) are therefore something different from cosmological recession speeds. The numerical distance only changes if there is a relative speed. Relative speeds between the light source and the observer according to the special theory of relativity (STR) are therefore ruled out as the cause of cosmological redshift if peculiar speeds are neglected. The expansion of the universe or space is therefore **not a numerical** expansion of the increasing distances in space. This would correspond to the Doppler effect and thus a relative speed. The distances and radii expand with space, so they don't expand into space. This leads to the unusual situation that the distances increase, but not the number of distance units. The distance units are subject to the same rate of expansion as the distances. We therefore assume: Not the number of distance units, but their scaling value expands or drifts. The space expands with its objects while numerically retention distance, radius, rotation time and density. There is therefore no numerical increase in the distance or relative speed in accordance with STR by cosmological expansion! With the expansion of the space, the numerical distance does not change but the space with its units of measurement expands (Mpc, light year, AU, kilometers, seconds, etc.). The expansion of the space with the Hubble parameter means that 1 Mpc has grown approx. 72 km after one second [3]. An object at a distance of 1 Mpc does not move away with a relative speed of 72 km / s, but with this recession speed. The numerical distance of an object remains constant 1 Mpc after any time and the expansion corresponds to an increase (drift) of the unit of measurement Mpc.

3. The lower limit of cosmological expansion

The lower limit of cosmological expansion postulated by Einstein and Straus [1] can be found today by comparison (difference formation) the respective potential of a gravitational mass with the potential of cosmological expansion. A serious inaccuracy is committed here: The potential of a gravitational mass leads to a change in the relative speed of objects in the area of influence. The cosmological expansion, however, does not lead to a relative speed according to STR, but to something else. So two different phenomena are compared. Such a comparison is inadmissible ! This inadmissible comparison leads to the assumption: Gravitationally bound objects are not subject to any cosmological expansion. We consider this assumption to be incorrect. On the one hand, it is largely agreed that objects bound by gravity do not expand cosmologically, on the other hand, despite the redshift of the Virgo cluster, the so-called Virgo infall occurs [4]. In addition to this obvious contradiction, expansion effects of the size of the Hubble constant can be found repeatedly in much smaller areas. As a result of different assumptions about the lower limit of cosmological expansion, we have our own opinion:

The lower limit is neither formed by the Einstein / Straus relationship nor by comparing the potentials of gravitation

and expansion. We assume the lower limit of cosmological expansion is found by comparing the effects of gravitational potential with electromagnetic potential. If one compares the shape of planets with the shape of small planetoids or even smaller objects, it becomes evident that the shapes are shaped or dominated differently. Smaller objects are obviously less influenced by gravity. The internal bond takes place primarily by other forces (electromagnetism). We conclude that the lower expansion limit is in the transition area. With increasing mass, the shapes become more rounded and more dominated by gravity. Smaller and less massive objects show shapes that are generated by crystalline and molecular forces. The shape-forming properties are less or not dominated by gravity. For example, overhanging parts of a structure (bridges) do not obey gravity. The shape of water droplets is not created by the gravitation of their mass but by surface tension. The properties of space given by gravity are subordinate to the internal electromagnetic properties. The cosmological expansion or scale drift is a property of space. Due to this subordination, objects in our daily environment and smaller objects do not show any cosmological expansion effects. The relation (1) used below loses sense. Although the Earth is subject to cosmological scale drift (expansion), this does not apply to continents and smaller structures. Objects or phenomena bound by their own gravitation are listed in Table 1. According to standard theory, these should not show any cosmological expansion.

4. Coincidences

Zufällige Übereinstimmungen zwischen zwei verschiedenen Phänomenen in 18 (!) Zehnerpotenzen und derselben Dimension sind sehr seltene Zufälle. The occurrence of such a rare coincidence becomes much rarer when another phenomenon of the same size and dimension is added. The probability, or rather the improbability, must then be raised to the power. The rates of earthly tidal friction (earth rotation) and cosmological expansion rate are a numerical coincidence of two different phenomena. A fortuitous correspondence of this coincidence with the deceleration rate from the Pioneer anomaly is already an almost impossible coincidence. Since further phenomena (see Table 1) are added, we rule out random agreement and assume causal agreement. In other words, the phenomena mentioned have a common rate because they have a common cause! That can't be tidal friction. Causal correspondence cannot be justified in the area of the earth because, for example, even the most distant galaxies are obviously subject to this expansion rate (Table 1, [6,12]). We see the cosmological expansion as the sole cause of this common rate. Their lower limit is then in the area of gravitationally bound objects.

5. Measured values versus standard theory

The occurrence of the rate $\sim 3 \times 10^{-18} \text{ s}^{-1}$ in all areas mentioned in Table 1 from the earth's radius to the astronomical horizon is remarkable. In these two extreme cases, relative speeds are excluded as the cause of the existing rate. For the earth's radius, the exclusion is based on the results of measurements by X. Wu et al [7] and for the universe because recession speeds are greater than the speed of light at great distances ($v > c$). If there is a common cause, the relative speeds must also be rejected for intermediate values. Among other things, this applies to lunar orbit and solar orbit in the galaxy. If there are no relative speeds as a result of cosmological expansion, the corresponding distances are numerically constant despite this expansion. The cosmological expansion manifests itself as the speed of recession or scale drift. For the distance of the earth's radius it is approx. 0.05 cm / a [5]. Depending on the type of measurement, the lunar orbit results in approx. 3 - 4 cm / a [8, 9, 10]. The distance of the solar orbit in the galaxy results in a recession speed of approx. $4 \times 10^{15} \text{ km / revolution}$ or $18 \times 10^6 \text{ km / a}$ and for the Andromeda nebula these are approx. 50 km / s. However, the recession speed of the Andromeda Nebula is overlaid by a larger, opposite relative speed. The values listed are obtained by using the rate $\alpha = 2.5 \times 10^{-18} \text{ s}^{-1}$ and relation (1). The values obtained from this agree with the measured values (if measurable).

$$\alpha = \Delta r / (t \times r) \quad (1)$$

($\alpha = \text{expansion rate} \equiv \text{recession rate}$, $\Delta r = \text{distance difference or recession value}$, $t = \text{period}$, $r = \text{distance}$)

The value $\alpha \approx 3 \times 10^{-18} \text{ s}^{-1}$ is found several times in the solar system, but also in other systems bound by gravity. α should not be confused with the cosmological scaling factor α .

5.1. Earth radius

With relation (1) one gets a recession value of (Δr) 0.05 cm per year (t) and earth radius [r]. X.Wu et al [7] obtained the same value 0.05 cm / a when measuring a possible earth expansion using ITRF 2008+ Grace + OMCT + ECCO. However, the result does not refer to a relative velocity between the center of the earth and surface, but on a scale drift or drift of origin with the same distance and period of time. The value found corresponds to a drift rate of $2.5 \times 10^{-18} \text{ s}^{-1}$. The scale drift corresponds to an expansion of the scale values. If the drift is continuous, then smaller drift values are obtained for smaller distances and periods and larger drift values for larger distances and periods.

5.2. Inner core of the Earth

S.K. Runcorn gives a growth of 243 km / 10^9 years for the inner core of the Earth [16]. With relation (1), this corresponds to an expansion rate or drift rate of $\sim 3 \times 10^{-18} \text{ s}^{-1}$.

5.3. Lunar distance

The distance to the moon is about 60 times larger than the earth's radius. The drift value or recession value is 60 times greater than to be expected for the earth's radius. The scale drift is then approx. 3 cm per year and orbital radius of the moon. With relation (1) and the same rate a ($2.5 \times 10^{-18} \text{ s}^{-1}$) one obtains the same recession value or drift value $\Delta r = 3.0 \text{ cm / a}$. This agrees with values measured using solar eclipses [8]. Measurements with LLR give the larger value $3.82 \pm 0.07 \text{ cm / a}$ ($3.15 \times 10^{-18} \text{ s}^{-1}$). However, it is not excluded here that a relative speed and the recession speed or scale drift complement one another. For the Andromeda Nebula, a relative speed must also be added. A relative speed for the Moon could result from the tidal friction (see paragraph 6.).

5.4. Pioneer anomaly

The Pioneer anomaly describes an abnormal deceleration of the Pioneer X and XI spacecraft. The delay value is $8.74 \times 10^{-8} \text{ cm/s}^2$ [15]. It results from a frequency shift of the radio signals assuming the Doppler effect. If you divide the deceleration value by the speed of light, you get a deceleration rate of $2.91 \times 10^{-18} \text{ s}^{-1}$. This value corresponds exactly to the rotation delay of the Earth and corresponds approximately to the Hubble parameter (!). Shortly after the Pioneer anomaly was discovered, cosmological expansion was believed to be a possible cause. This possibility was rejected by cosmology [22]. The reasoning for this: Cosmological expansion only leads to redshift and only outside of gravitationally bound systems. We contradict these assumptions in sections 2 and 3. If, contrary to standard theory, the universe is also expanding in the solar system, the units of length (meters, light years, etc.) at the time the probes were launched were smaller than they are today. If the speed of light was numerically constant, the second used to be shorter. The time scale correspond to the course of the UT time. This deviates secularly from the SI time at a rate of $2.91 \times 10^{-18} \text{ s}^{-1}$. As a result of the expanded units of length, the currently measured distance is numerically smaller than expected. This corresponds to the opposite of expansion and manifests itself as a blue shift.

5.5. Size evolution of galaxies

The effective radius of large galaxies decreases with increasing distance and the inner density increases. There are a number of studies on this subject, for example in [6,12,21]. By P.v.Dokkum et al [6] galaxies are described which are at a distance $z \sim 2.2$ ($\sim 10.7 \times 10^9 \text{ Ly}$). We see these galaxies as they were after 20% of today's world age. The radii are approx. 0.9 kpc, i.e. 20% of the radius that is common today for galaxies of the same type and mass. Such galaxies do not exist in today's near universe. Any other explanation than expansion does not seem possible. Let us assume that today's galaxies began at this size and density. In this case, the expansion rate results from the difference in radius per radius and expansion time (1).

$$\alpha = \frac{\Delta r}{r \times t} = \frac{5 - 1}{5 \times 10.7 \times 10^9 \times 31.56 \times 10^6 \text{ s}} = \frac{4}{1.688 \times 10^{18} \text{ s}} = 2.37 \times 10^{-18} \text{ s}^{-1} \quad (1)$$

$\alpha =$ required expansion rate, $\Delta r =$ difference between today's radius (5) and emission radius (1), $r =$ adequate today's radius (5), $t =$ distance in light travel time (SI-s).

This expansion rate corresponds to a Hubble constant of 73.2 (km / s) / Mpc. The objects are objects bound by gravity. They expand according to the cosmological expansion, what the standard cosmology contradicts. I. Trujillo notes something similar when he writes: "Consequently, the very dense nature of our objects at high z could reflect the much denser condition of the Universe at the time of their formation" [21]. We see that these examined objects show the same effect and expansion rate as is observed today as a scale drift on the earth (see above: earth radius).

6. Conclusions

The multiple occurrence of the cosmic recession rate $2 - 3 \times 10^{-18} \text{ s}^{-1}$ in gravitationally bound systems suggests that the expansion of space is also present in these objects in full size. Section 2 shows that the expansion or recession is not a relative speed in the sense of the STR. So there is no numerical change in distance due to the occurrence of this rate. The work of X Wu [7] and NASA confirms in the case of the earth that the said rate is caused by scale drift and not by a relative speed between the center of the earth and the surface. The recession rates listed in Table 1 suggest that there is a common cause for the listed phenomena. Expansion or recession would therefore have a scale drift as the cause. This applies to both space and time. The day length (LoD) grows at approximately the same rate and therefore the same cause. An increase in LoD due to tidal friction is assumed here in addition to the scale drift. The tidal friction theory requires a significantly faster increase in LoD (2.3 ms / cy) than the observation allows (1.6-1.8 ms / cy) [23]. This indicates shortcomings in the theory of tidal friction. In Table 1, the LLR measurements with $\alpha \sim (3.15 \pm 0.06) \times 10^{-18} \text{ s}^{-1}$ result in the highest value for scale drift or recession near the Earth. We therefore assume that the difference to the Hubble constant is caused by tidal friction. Only this share would then correspond to a relative speed between the Earth and the Moon. The numerical distance of the Moon in the early days is then only slightly smaller than it is today. A destruction of the Moon by the Roche boundary of the Earth is therefore not to be assumed.

With the assumption that cosmic recession is not a relative speed according to STR and is present in gravitationally bound objects, further, following phenomena should occur or be present, e.g.:

- 1 According to paragraph 2, there is no relative velocity due to the cosmological recession. Orbiting objects thus maintain their numerical distance to the center of gravity during this recession. With the numerically constant orbital radius, the orbital velocity also remains numerically constant despite the recession. The result is a level of the orbital velocity that is flat towards the outside. The flat course of the orbital velocities in galaxies [14] does not require any dark matter. Modified Newtonian Dynamics (MOND) are also not required.
- 2 The moon is moving away from the earth at approximately the same rate of recession as according to Hubble's constant. We conclude that the measured recession is not primarily a relative speed and that the distance is numerically approximately constant.
- 3 Galaxies expand while maintaining the numerical radius. Radii and distances used to be smaller due to cosmic expansion, but numerically constant.
- 4 The continents and our everyday environment do not participate in the expansion or scale drift. These objects as well as e.g. small moons and planetoids are obviously dominated and shaped by electromagnetic forces and not by gravity (Section 3 and [5]).
- 5 The lower limit of the cosmic expansion is not removed. However, it is located at a much smaller distance than according to standard theory. This makes this value interesting for space travel (Pioneer anomaly), geophysics (LoD), time determination (leap seconds) and other areas.
- 6 In the sense of the SRT there is no relative speed of the galaxies mentioned in 5.5 (peculiar speed neglected). The distance has remained numerically constant since the emission of the radiation now received. The number of the original distance units is therefore the same as the current number. However, the scale value of the distance units is today 5 times the original units due to scale drift. After emission, the light initially only covered 1/5 of the distance covered today in the same time. The light, which comes from a distance of 10.7×10^9 Ly in today's scale, has been on the move for a time that results from relation (2).

$$\Sigma = 0,5n (x_1 + x_n) \quad (2)$$

(Σ = sum of the average distance units, n = number of current distance units, x_1 = scale value of the first unit after the emission (= 1), x_n = scale value of the unit in the observation (= 5)).

The light reaching us was not 10.7×10^9 years but 32.1×10^9 years on the way. This means that the radiation is registered weaker than expected. According to paragraph 5.5, the angle at which the galaxies appear is smaller. When using the distance module, the greater distance leads to a changed absolute size (M). Since the factors M and m (absolute and apparent brightness) are very likely constant, the Hubble constant at the time of emission was smaller than it is today. An unknown, dark energy must have accelerated the expansion of the universe according to standard theory. Please note, however: The Hubble constant was smaller and the distance units (e.g. Mpc) were also smaller due to expansion or scale drift. The ratio of Hubble constant / Mpc thus remains constant.

- 7 If we observe objects whose distance is close to the age of the world, the time of flight at a numerically constant speed of light is close to ∞ .

We should note that there are two systems of measurement available for space and time: One is based on gravitation / inertia (e.g. LoD) and the other on electromagnetism (SI system). Both systems are not synchronous.

For the difference between the rates of expansion $2.3 \times 10^{-18} \text{s}^{-1}$ and $2.9 \times 10^{-18} \text{s}^{-1}$ no explanation has been found. However, the latter value was only found at a cosmologically small distance.

Based on the arguments presented here, we make the following assumptions:

Objects that are bound by their own gravity are subject, contrary to standard theory, to cosmological expansion (scale drift). This does not apply to other objects. The first-mentioned objects include massive objects such as fixed stars, pulsars, earth, and galaxy clusters. Other objects include, for example, smaller, irregular planetoids (<200 km), continents and objects in our environment.

Hilgenberg, Carey, Scalera and many others are assuming the Earth is expanding, but not the continents. It seems that there is a justification. X.Wu et al found [7]: The number of units of measurement remains almost constant, but their size expands along with the radius of the Earth.

Table 1

1	2	3	4	5
Row	Example	Rate ($\times 10^{-18} \text{s}^{-1}$)	Possible other causes	References*
1	Expansion-rate of the universe	2.33 ± 0.26	-	[3]
2	Expansion of galaxies	2.37	?	[6, 12]
3	Pioneer anomaly	2.91 ± 0.44	Thermal radiation pressure	[15]
4	Expansion of moon's orbit (LLR-Technology)	3.15 ± 0.06	Tidal friction	[9]
5	Expansion of moon's orbit (Eclipse)	2.32	Tidal friction	[8, 10]
6	Delay of the Earth's rotation	2.93	Tidal friction	[11]
7	Distribution of rotational delay of pulsars	2.7 ± 0.4	?	[18]
8	Surface relation oceans/continents	3.0	?	[5]
9	5-dimensional field theory	3.6	Scale effect	[19]
10	Polar diameter of Earth	2.5 ± 0.95	Post glacial uplift	[17]
11	Inner Earth's core	3.0	Growth by phase-conversion	[16]
12	Origin drift CM ITRF2008+GRACE+OMCT	2.5 ± 1.0	Networksparseness	[7]
13	Expansion of galaxies	2.33 ± 0.26	DM in Bose–Einstein condensate	[13]

* Values contained in column 3 were calculated by the author. Calculation basis is information from column 5.

References

- [1] A.Einstein, and E.G.Straus, „The Influence of the Expansion of Space on the Gravitation Fields Surrounding the Individual Stars“, Rev. Mod. Phys.17(1945),120-124
- [2] Davis T. and Lineweaver C.[Online] " Expanding Confusion: common misconceptions of cosmological horizons and the superluminal expansion of the universe " <https://arxiv.org/pdf/astro-ph/0310808.pdf> (accessed 2020 Sept21.)
- [3] W.L.Freedman, B.F.Madore, B.Gibson, L.Ferrarese, D.D.Kelson, S.Sakai, J.R.Mould, R.C.Jr.Kennicutt, H.C.Ford, J.Graham, J.Huchra, S.Hughes, G.D.Illingworth, L.Macri, P.B.Stetson „Final Results from the Hubble Space Telescope Key Project to Measure the Hubble Constant“. Astrophysical Journal. Band 553 (2001) p. 47
- [4] I. D. Karachentsev and O. G. Nasonova,[Online] „The observed infall of galaxies towards the Virgo cluster“, (Dated: Feb 9, 2010) <http://mnras.oxfordjournals.org/content/405/2/1075.full> (accessed 2020 Sept.21)
- [5] V.Müller,[Online]"Earth expansion is an expansion of the universe at a small distance" <https://www.researchgate.net/publication/322233784>, DOI:10.13140/RG.2.2.14974.33609(accessed2018June25
- [6] P.v Dokkum, M.Franx, M.Kriek [Online] „A high stellar velocity dispersion for a compact massive galaxy at $z=2.2$ ". (2009) <http://arxiv.org/abs/0906.2778v1> (accessed 2020 Sept.21)
- [7] X.Wu, X.Collilieux, Z. Altamimi, B.L.A.Vermeersen, R.S.Gross, I.Fukumori,“Accuracy of the International Terrestrial Reference Frame origin and Earth expansion“,Geophysical Research Letters,v 38,(2011)L13304.5 p
- [8] L.Riofrio [Online] "Calculation of lunar orbit anomaly",Planetary Science 2012 1:1 <https://planetary-science.springeropen.com/articles/10.1186/2191-2521-1-1> (accessed 2020 Sept.21)
- [9] J.O.Dickey, P.L.Bender, J.E.Faller, X.X.Newhall, R.L.Ricklefs, J.G.Ries, P.J.Shelus, C.Veillet, A.L.Whipple, J.Wiant, J.G.Williams, F.Yoder „Lunar laser ranging:A continuing legacy of the Apollo program“. Science,265,(1994)pp 482
- [10] G.E. Williams (2000) [Online]"Geological constraints on the Precambrian history of Earth's rotation and the Moon's orbit", Rev. Geophys.,38(1),37–59, doi:10.1029/1999RG900016. https://www.eoas.ubc.ca/~mjl/line/453website/eosc453/E_prints/1999RG900016.pdf (accessed Aug 30 2018)
- [11] N.A.Bär [Online],„Die Akzeleration“, (Dated:Apr.2013) <http://www.nabkal.de/akzel.html> (accessed 2020Sep.21)
- [12] A.v d Wel, B P.Holden, A W. Zirm, M Franx, A Rettura, G D. Illingworth and H C.Ford."Recent Structural Evolution of Early-Type Galaxies: Size Growth from $z = 1$ to $z = 0$ ", ApJ,Band 688,Nr1
- [13] J-W Lee (2009) [Online]"Are galaxies extending?"<https://arxiv.org/pdf/0805.2877.pdf> (accessed 2020 Feb 28)
- [14] Y.Sofue and V.Rubin „Rotation curves of spiral galaxies“ Annu. Rev. Astron. Astrophys. 2001. 39: 137-174
- [15] J.D.Anderson, P.A.Laing, E.L.Lau, A.S.Liu, M.M.Nieto, S.G.Turyshv. [Online],„Study of the anomalous acceleration of Pioneer 10 and 11“. (Dated:11 April 2002) https://arxiv.org/PS_cache/gr-qc/pdf/0104/0104064v5.pdf (accessed Feb12,2014)
- [16] S.K.Runcorn, "Towards a theory of continental drift." Nature, 193, 311–314 (1962). "Convection currents in the Earth's mantle." Nature, 195, 1248–1249 (1962)
- [17] H.Ruder, M.Schneider, M.Soffel, „Geodäsie und Physik“,Physikalische Blätter Nr 46(1990,No.2.) S.41-46
- [18] Müller V. [Online], 15. Pulsare <http://zeitexpansion.de/#p> (accessed Feb 04, 2020)
- [19] E.Schmutzer, „ Approximate global treatment of the expansion of the cosmic objects induced by the cosmological expansion“, Astron. Nachr., 321, p. 227 -233.(2000)
- [20] Müller V. [Online] 2. Die Rotationsverzögerung der Erde und die variable Weltzeit <http://zeitexpansion.de/#c> (accessed Feb 04, 2020)
- [21] Trujillo I., Conselice C., Bundy K., M.C.Cooper, C.P.Eisenhardt P and R.S.Ellis [Online]"Strong size evolution of the most massive galaxies since $z \sim 2$ " Mon.Not.R. Astron.Soc.382,109–120 (2007). <https://academic.oup.com/mnras/article/382/1/109/983576>(accessed Feb 04,2020
- [22] Dittus H. & Lämmerzahl C.[Online] "Die Pioneer-Anomalie " Physik Journal 30 5 (2006) Nr. 1 S. 25-31 https://www.zarm.uni-bremen.de/uploads/tx_sibibtex/2006_LaemmerzahlDittus.pdf (accessed Feb 04, 2020)
- [23] Stephenson F. R.,Morrison L. V.and Hohenkerk C. Y.2016 Measurement of the Earth's rotation: 720 BC to AD 2015Proc. R. Soc. A.47220160404 [Online] <https://royalsocietypublishing.org/doi/10.1098/rspa.2016.0404> (accessed 2020 Aug 04.)