A Kitchen Sink Measurement of g

Ron Larham, December 5th 2020

Abstract

In this paper I look at the experimental determination of the acceleration due to gravity g and the flow rate from my kitchen tap as suggested in a comment to a puzzle in "The Chicken From Minsk" (TCFM) [2] using only equipment already on hand. This differs from the method proposed in TCFM in that it uses a method of taking the measurements that is practical rather than seemingly dismissing the problem of taking measurements with a wave of the hand.

Introduction

I set challenge problems on the Math Help Forum [1]. While browsing my collection of puzzle books I found an interesting problem in "The Chicken From Minsk" (TCFM) [2] on page 46, "More Practical Advice from Your Plumber (How to Measure Gravity With a Ruler and a Bucket)". The problem itself requires a moderately simple application of conservation of mass and energy and the odd simplifying assumption to a question about the steady smooth flow from a tap. But what piqued my interest was the throw away remarks implying it was easy to perform the measurements required, which seemed not totally plausible as a five gallon container was mentioned for measurement of the flow rate, and calipers to measure the width of the water stream from the tap (faucet).

If I was to make the required measurements the main requirement was that they should not involve procuring any equipment that I did not already have access to. This is essentially the same constraint as that on my measurement of the speed of the domino effect [3].

Problem as set

There are two problems in the section of TCFM that raises the first is a scene setter:

1. *A Plumbing Problem*
A faucet has been left slightly open for some time, and a gentle stream of water flows downward. Why does the stream become thinner as it gets further away from the faucet?

The answer to this is that the flow is accelerating as it falls away from the tap under gravity. Then the conservation of mass requires that the stream becomes thinner.

2. *More Practical Advice from Your Plumber*
(How to Measure Gravity with a Ruler and a Bucket)
Consider the stream of water in the previous problem, becoming thinner as it emerges from the tap. Can you find the volumetric flow rate (e.g. gallons per minute or litres per second) and the velocity of the stream using only a ruler? (If you insist on greater precision, use a caliper.) Can you use this technique to measure the acceleration of gravity g?

A reworded version of this has been set as a challenge problem on the MathHelpForum [4]

Solution(s) to 2.
There are a number of approaches to solving this, at least two of which I have used, but I think the method in the answers section of TCFM (and the second of the ones that I used) is probably the best. This uses the conservation of mass (water being an compressible fluid this is equivalent to conservation of volume occupied by a given mass of liquid) and energy.
The volumetric flow across a horizontal plane cutting the flow at height x is:

$$Q = v_x \times \pi r_x^2 \tag{1}$$

where Q is the volumetric flow rate and is constant for a particular tap setting (I will be using units of m^3/s), and r_x is the radius of the flow at x.
The KE of a volume/mass element when it falls from height x_1 to x_2 is:

$$KE(x_2) = \frac{1}{2}mv_{x_2}^2 = \frac{1}{2}mv_{x_1}^2 + mg \times (x_1 - x_2)$$

so:

$$\frac{1}{2}v_{x_2}^2 = \frac{1}{2}v_{x_1}^2 + g \times (x_1 - x_2) \tag{2}$$

Now using (1) we have: $v_{x_2} = v_{x_1} \times (r_{x_1}^2/r_{x_2}^2)$, which may be substituted into (2):

$$v_{x_1}^2 \left(\frac{r_{x_1}^4}{r_{x_2}^4} - 1 \right) = 2 \times g \times (x_1 - x_2) \tag{3}$$

So if we know g, equation (3) allows us to find v_{x_1} and hence putting this back into (1) Q using measured $x_1,\ x_2,\ r_{x_1},\ r_{x_2}$. Alternatively if we also measure Q we can find v_{x+1} and so g.

Experiment
The experimental methods for measuring the diameter of the flow and the flow rate hinted at in TCFM seem impractical to use in a kitchen setting.

I have tried measuring the width of the flow using calipers and find it very difficult to get a sensible measurement while maintaining a known distance from a suitable vertical reference. A more practical approach is to photograph the stream with a ruler in the the photo at the same distance from the camera as the stream, to provide a scale reference (figure 1). This is not without its own problems, the main one being deciding where the edge of the stream is.

Measuring the flow rate by weighing the water captured in a suitable container in a given time is plausible as a method for this. But the use of a five gallon container in the kitchen is impractical for a number of reasons, one being that I would need to procure a suitable set of scales to measure the weight accurately, and a second being the problems associated with handling the full container. Instead I opted for a smaller container and shorter filling times, with the measurement, filling and weighing, repeated multiple times and averaged (this also had the advantage of giving an estimate of the standard error of the mean mass of the water flow over the timing interval. Also this measurement was repeated with several timing intervals to allow an estimate of the systematic error (personal equation) in the timing to be estimated.

Results

The photograph of the flow was transferred into the image processing package GIMP [5] and measurements of the flow width at two heights and the difference in the heights taken in pixels. Also the scale was also measured to give the number of pixels per cm, and the errors in the measurements estimated. This data is presented in Table 1.

	measured	σ	corrctd	σ(corrctd)
r_1	22.5px	1px	8.2mm	0.2mm
r_2	11px	1px	4.0mm	0.2mm
Δx	1085px	2px	19.7mm	0.4mm

Table 1: Flow Data (rounded to 1 decimal place)

Using the corrected values from Table 1 in equations (1) and (3) and $g = 9.81$ gives an estimated flow rate of $\sim 0.0000254\ m^3/s$

Table 2 shows the water flow measurements, when converted to m^3/s and corrected for my personal timing error we find that the flow rate $Q \sim 26.5\ 10^{-6}$ m^3/s with standard error $\sim 0.2\ 10^{-6}$ m^3/s

	5 sec.	15 sec.
weight 1	117 gm	390 gm
weight 2	122 gm	404 gm
weight 3	121 gm	386 gm
weight 4	119 gm	374 gm
weight 5	127 gm	381 gm
mean	121.2 gm	387.0 gm
Std Dev	3.77 gm	11.23 gm
Srd Err	1.69 gm	5.02 gm

Table 2: Tap flow timings

Using the measured flow rate and values for r_1, r_2 and Δx I can now estimate g as ~ 10.6 m/s^2

Error Analysis

The estimates above for g and Q are point estimates but the measurements that went into them have their own errors distributions. Now I could find approximations for the errors in the estimates for using the errors for r_1, r_2, Δx and Q , but that involves a fussy analysis, I would prefer to use a Monte-Carlo approach generating a sample of the measurements contaminated with errors and looking at the resulting distributions of estimates of g and Q.

Using samples sizes of 10000 using Monte-Carlo I find that the mean estimate of Q is 25.6 m^3/s, with standard deviation 3.0 m^3/s. Similarly doing the same for the estimate for g gives a mean estimate of 11.0m/s^2 with a standard deviation of 2.7m/s^2.

The estimates are close to the measured value of Q and the known value of g, but the distribution of the estimates is wide. We might also conclude that this is not a good method of measuring g, due to width of the error distribution.

It might be interesting to see if the measurement errors can be improved to the point that we can test the if simple assumptions used in the analysis hold up with closer scrutiny. Though this might be difficult in my kitchen and with the flow width measurement technique I have been using.

Figure 1: Photo of the stream from my kitchen tap with the end of a metre stick at the same distance as the stream from the camera, for scale

References

1. The Math Help Forum, https://mathhelpforum.com/.

2. Chernyak Y. B., Rose R. M., The Chicken From Minsk, Phœnix, 1997.

3. Larham, R., Demonic Dominoes, ArXiv.org, 2013, https://arxiv.org/abs/1302.3311.

4. Larham, R., Kitchen Sink Measurement of g?, https://mathhelpforum.com/threads/kitchen-sink-measurement-of-g.287760/.

5. GIMP image manipulation program, https://www.gimp.org/.

Ron Larham
generalsynopsis@gmail.com