NOTE ON THE COMPARISONS OF GEODETIC AZIMUTHS
OF THE TUNISIAN PRIMORDIAL GEODETLIC NETWORK

Abdelmajid Ben Hadj Salem

Abstract
In this paper, we present some comparisons of geodetic azimuths obtained by GPS
by classical geodesy and by astronomical observations (Laplace’s points). The
goodetic azimuths are calculated using 3D coordinates or by plane coordinates
(Tunisian Lambert projection). The results obtained confirm that the orientation
issued by GPS is identical to astronomical orientation which confirms also the ori-
entation of the Tunisian Primordial Geodetic Network obtained by the definitive

Résumé
Dans cette note, nous présentons des comparaisons d’azimuts géodésiques obtenus
par GPS avec ceux de la géodésie classique ou par l’astronomie. Les azimuts sont
calculés par les coordonnées tridimensionnelles ou issus des coordonnées planes.
Les résultats obtenus confirment que l’orientation issue du GPS est identique à
celle obtenue par les observations astronomiques. De plus, les résultats GPS confir-
ment l’orientation du réseau géodésique primordial tunisien issu de la compensation

1 Introduction

Let $A(\varphi_0, \lambda_0, H)$ with 3D coordinates $(X_1, Y_1, Z_1)$ in a given geocentric ref-
erence frame, we consider the local geodetic frame in $A$: From $A$, we observe
the point $B$ of coordinates $(X_2, Y_2, Z_2)$. Let $(x, y, z)^T$ be the components of
the vector $AB$ in $(A, x, y, z)$. Then the geodetic azimuth of the direction
$AB$ is given by:

$$\tan(Az) = \frac{x}{y}$$  \hspace{1cm} (1)

Let:

$$DX = X_2 - X_1$$
$$DY = Y_2 - Y_1$$
$$DZ = Z_2 - Z_1$$  \hspace{1cm} (2)

Between $(DX, DY, DZ)^T$ and $(x, y, z)^T$, we have the relation :

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = R \begin{pmatrix} DX \\ DY \\ DZ \end{pmatrix}$$ \hspace{1cm} (4)
Fig. 1: Le Repère local au point A

Where $R$ is the matrix of the transformation from the geocentric reference to the local geodetic frame in $A$ and is given by:

$$R = \begin{pmatrix} -\sin\lambda_0 & \cos\lambda_0 & 0 \\ -\cos\varphi_0 \sin\lambda_0 & -\sin\varphi_0 \sin\lambda_0 & \cos\varphi_0 \\ \cos\varphi_0 \cos\lambda_0 & \cos\varphi_0 \sin\lambda_0 & \sin\varphi_0 \end{pmatrix}$$  \hspace{1cm} (5)

then:

$$x = -\sin\lambda_0 DX + \cos\lambda_0 DY$$

$$y = -\cos\varphi_0 \sin\lambda_0 DX - \sin\varphi_0 \sin\lambda_0 DY + \cos\varphi_0 DZ$$ \hspace{1cm} (6)

$$z = \cos\varphi_0 \cos\lambda_0 DX + \cos\varphi_0 \sin\lambda_0 DY + \sin\varphi_0 DZ$$

So:

$$\tan(Az) = \frac{-\sin\lambda_0 DX + \cos\lambda_0 DY}{-\cos\varphi_0 \sin\lambda_0 DX - \sin\varphi_0 \sin\lambda_0 DY + \cos\varphi_0 DZ}$$ \hspace{1cm} (7)
2 Applications: Azimuth AIN ABDOUR - H. OUED SEHILI

2.1 Calculations of the azimuths

2.1.1 From WGS84 coordinates of the OPAT - Office des Ports Aériens de Tunisie -

The geodetic points AIN ABDOUR and H. OUED SEHILI have been observed by GPS during the connection of the Tunisian airports to WGS84 in November 1997 [1]. We have:

AIN ABDOUR : \( \lambda_0 = X X^\circ Y Y'47''.38955, \varphi_0 = X X^\circ Y Y'29''.14998 \).

Then we obtain: \( DX = 2740.786 \) m; \( DY = -16245.609 \) m; \( DZ = -534.888 \) m and :

\[
Azg_1 = 267^\circ 48'30''.7058 \tag{8}
\]

2.1.2 From WGS84 coordinates of the Tunisian-Polish GPS Campaign

The geodetic points AIN ABDOUR et H. OUED SEHILI have been observed during the GPS campaign 19 June - 3 July, 1996 by the OTC and the Space Research Centre of the Polish Academy of Sciences [2], [3] and we have the following results : AIN ABDOUR \((\varphi_0, \lambda_0)\) and :

\[
\begin{align*}
DX &= 2740.789 \text{ m} \\
DY &= -16245.616 \text{ m} \\
DZ &= -534.883 \text{ m}
\end{align*}
\tag{9}
\]

Then the geodetic azimuth:

\[
Azg_2 = 267^\circ 48'30''.7474 \tag{10}
\]

2.1.3 From the Carthage34 coordinates

We have the following elements:
- at the geodetic point AIN ABDOUR:

\[
x_1(Lambert - sud) = 363044.79 \text{ m}, \quad y_1(Lambert - sud) = 407020.09 \text{ m}
\]

\[
\varphi_0 = X X^\circ Y Y'23''.0712, \quad \lambda_0 = X^\circ Y Y'45''.8138
\]

- at the geodetic point H. OUED SEHILI:

\[
x_2(Lambert - sud) = 346570.13 \text{ m}, \quad y_2(Lambert - sud) = 406623.60 \text{ m}
\]

so :

\[
\begin{align*}
DX &= 2741.505 \text{ m} \\
DY &= -16245.353 \text{ m} \\
DZ &= -535.155 \text{ m}
\end{align*}
\tag{11}
\]
Using the formula (7), we obtain:

\[ Azg_3 = 267^\circ 48'21''709 \]  

(12)

From the relation between the geodetic azimuth and the bearing angle:

\[ G = Az - \gamma + Dv \]  

(13)

With:
- \( G \) the bearing angle of the direction,
- \( Az \) the geodetic azimuth of the direction,
- \( \gamma \) the bearing angle of the meridian given by \((\lambda - \lambda_0').sin\varphi'_0\) with \(\varphi'_0 = 33^\circ 18' = +40\) gr the geodetic latitude of the origin parallel of the Lambert-sud map projection and \(\lambda_0' = 9^\circ 54' = 11\) gr East of Greenwich the longitude of the origin meridian of the same projection,
- \( Dv \) the correction of the chord of the side AIN ABDOUR - H.OUED SEHILI.

Numerically, we obtain:

\[ G = 268^\circ 37'16''.8536 \]
\[ Dv = -0^\circ 00'04''.4566 \]
\[ \gamma = -0^\circ 48'59''.5704 \]

so:

\[ Azg_4 = 267^\circ 48'21''.7399 \]  

(14)

### 2.1.4 From the coordinates issued of the IGN84 adjustment

We have the following elements:
- at the geodetic point AIN ABDOUR:

\[ x'_1(\text{Lambert – sud}) = 363\,039.86\,m, \quad y'_1(\text{Lambert – sud}) = 407\,024.97\,m \]

\[ \varphi_0 = X \, X \, Y \, Y' \, 23''.2274, \quad \lambda_0 = X \, \circ \, Y \, Y' \, 45''.6181 \]

- at the geodetic point H.OUED SEHILI:

\[ x'_2(\text{Lambert – sud}) = 346\,565.09\,m, \quad y'_2(\text{Lamb. sud}) = 406\,629.15\,m \]

Then, we obtain:

\[ DX = 2741.138\,m \]
\[ DY = -16245.515\,m \]  

(15)
\[ DZ = -534.610\,m \]
Using the formula (7), we obtain:

\[ Azg_5 = 267^\circ 48'30".0055 \] (16)

From the equation (13), we obtain:

\[ G = 268^\circ 37'25".2704 \]
\[ Dv = -0^\circ 00'04".4566 \]
\[ \gamma = -0^\circ 48'59".6778 \]

and:

\[ Azg_6 = 267^\circ 48'30".0491 \] (17)

2.1.5 From the astronomical azimuth of AIN ABDOUR - SELDJA

From the astronomical azimuth of AIN ABDOUR - SELDJA observed during the astronomical campaign in 1982, we calculate the geodetic azimuth from Laplace equation. Having the observed horizontal angle \( \alpha = SELDJA - AIN ABDOUR - H. OUEDSEHILI \), we obtain \( Azg \) the geodetic azimuth of the direction AIN ABDOUR - H. OUED SEHILI.

Using the generalized Laplace equation:

\[ Azg = Aza + (\lambda - \Lambda).\sin\varphi + ((\lambda - \Lambda).\cos\varphi.\cos Aza + (\varphi - \Phi).\sin Aza).\cotgz \] (18)

Where \( \Phi \) and \( \Lambda \) are respectively the astronomical latitude and longitude, \( z \) the zenith angle. We calculate \( Azg \) of the direction AIN ABDOUR - SELDJA.

In the point AIN ABDOUR, we have:

\[ \Phi = XX^\circ YY'13".8982, \quad \varphi = XX^\circ YY'23".0712 \]
\[ \Lambda = XX^\circ YY'41".8767, \quad \lambda = XX^\circ YY'45".8138 \]
\[ Aza = 337^\circ 09'08".6956, \quad \text{Zenith angle} \ z = 87^\circ 28'42".8484. \]

Then the geodetic azimuth of \( AIN ABDOUR - SELDJA = Azg = 337^\circ 09'10".0887 \), but \( \alpha = 69^\circ 20'40".9632 \). Then:

\[ Azg_7 = Azg - \alpha \Rightarrow Azg_7 = 267^\circ 48'29".9242 \] (19)

2.2 Comparisons of the azimuths

We summarize below the obtained geodetic azimuths:

- for the azimuths by GPS, \( Az_{gps} = (Azg_1 + Azg_2)/2 = 267^\circ 48'30".727, \)
3 Conclusions

- for the azimuths in Carthage34 frame, $A_{\text{Cart}} = (Azg_3 + Azg_4)/2 = 267^\circ 48'21''.725$,

- for the azimuths from the IGN84 adjustment, $A_{\text{Ign}} = (Azg_5 + Azg_6)/2 = 267^\circ 48'30''.026$,

- and the azimuth issued from the astronomy, $Azg_a = Azg_7 = 267^\circ 48'29''.924$.

We obtain these differences:

$$A_{\text{gps}} - A_{\text{Cart}} = 09''.0007$$  \hspace{1cm} (20)

$$A_{\text{gps}} - A_{\text{Ign}} = 00''.6998$$ \hspace{1cm} (21)

$$A_{\text{gps}} - Azg_a = 00''.8035$$ \hspace{1cm} (22)

$$A_{\text{Ign}} - A_{\text{Cart}} = 08''.0088$$ \hspace{1cm} (23)

We concluded that:

* the orientation issued by GPS is conformal to those obtained by astronomical observations (equation 22),

* the adjustment of the IGN has put in evidence the disorientation of Carthage34 frame (equation 23),

* the orientation obtained by the adjustment of the IGN84 is similarity to those of the GPS (equation 21),

* the GPS observations put in evidence the disorientation of Carthage34 frame (equation 20).

3 Conclusions

This paper allowed to do some comparisons of the geodetic azimuths obtained by different methods of a direction of the Tunisian Primordial Geodetic Network and to put in evidence the disorientation of the geodetic frame Carthage34 and the contribution of the GPS techniques for the orientation of the geodetic networks.

References


3 Conclusions

Rapport des Calculs, Space Research Center, Polish Academy of Sciences, Warsaw, Poland.