Acknowledgment of non-linearity
or how to solve several conjectures

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Abstract

Several famous conjectures from Number Theory are studied. I derive a new equivalent formulation of Goldbach's strong conjecture and present an independent conjecture with some evidence for it.

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I. INFORMATION

In 2013, Harald Helfgott published a proof of Goldbach's weak conjecture [1]. As of 2018, the proof is widely accepted in the mathematics community [2], but it has not yet been published in a peer-reviewed journal. Goldbach's weak conjecture reads:

Any odd number $n > 5$ can be expressed as a sum of three prime numbers.

II. EQUIVALENT FORMULATION OF GOLDBACH'S STRONG CONJECTURE

Any odd number N can be presented as $N = M + a$, where a is an arbitrary odd number and M is an even number. Due to Goldbach's strong conjecture, $M = p_j + p_k$. Thus,

$$N = p_j + p_k + a\,. \tag{1}$$

Therefore, an equivalent formulation of Goldbach's strong conjecture reads:

Any odd number can be expressed as the sum of two primes and an arbitrary odd number.

Goldbach's weak conjecture which has been proven says that any odd number is the sum of just three prime numbers. I have inserted $a = 3$ as one of these three prime numbers as a condition into Helfgott's proof of Goldbach's weak conjecture, and the proof still holds. This fact proves Goldbach's strong conjecture in its new formulation. But in the following I present a more advanced proof.

III. PROOF OF GOLDBACH'S STRONG CONJECTURE

Due to Goldbach's weak conjecture the arbitrary odd number b is $b = p_i + p_j + p_k$. Thus, $b - p_i(b) = g(b)$ with $g = p_j(b) + p_k(b)$, where the dependence of the prime numbers p_i, p_j, p_k on b is written explicitly.

The equation is a non-linear equation for b. Therefore, b cannot be an arbitrary number, i.e., it cannot be every single odd number in the interval $7 \leq b < \infty$, e.g. 9. Because of this, Goldbach's weak conjecture cannot be the full picture of reality. Goldbach's strong conjecture "saves the day": g is not a certain function, but can be any even number. This means that any even number can be given by the sum of just two prime numbers.

Please understand me correctly: I am not disproving Dr. Helfgott's proof of Goldbach's weak conjecture. Instead, I make available Goldbach's weak conjecture. How I do this? Goldbach's weak conjecture is true, but only because Goldbach's strong conjecture is true. More is found in the "Discussion" below.

IV. NEW CONJECTURE

One can consider the set of primes as the set of pairs, namely any prime number p_j belongs to a pair of prime numbers:

$$p_j = p_k + A\,. \tag{2}$$

A numerical examination shows that for any even A in the interval $2 \leq A \leq 100$ there is at least one pair of odd prime numbers (p_j, p_k) such what $p_j = p_k + A$. For example, if $A = 8$, then we can select $8 = 11 - 3$.

A. Evidences for the new conjecture

In my opinion, evidence (often expressed as "a piece of evidence") is not a rigorous proof but just a confirmation of the theory or conjecture, like passing a confirmation test.

Due to Goldbach's weak conjecture the arbitrary odd prime number b is $b = p_i + p_j + p_k$. Thus, $b - p_i = p_j + p_k$. Because Goldbach's strong conjecture is true, $p_i \equiv a$ can be an arbitrary prime. Therefore, if $b - a$ can represent an arbitrary even number A (i.e., every even number in the interval $2 \leq A < \infty$), then Goldbach's strong conjecture is confirmed. If $b - a$ is not an arbitrary number, Goldbach's strong conjecture is not falsified.

It is natural to adopt the idea that A can be any even number in the interval $2 \leq A < \infty$ because there are infinite possibilities to fulfill Eq. (2) at least once, having the seemingly occasional distribution of an unlimited amount of prime numbers at free disposal. For example, there is only one possibility to write 4 as a sum, namely $2 + 2$, but there are very many possibilities to write 4 as the distance: $4 = 7 - 3 = 11 - 7 = 17 - 13 = \ldots$. However, we do not need many variants for 4 to be the distance, but we need only a single one.

Let us call neighboring values of A not satisfying Eq. (2) an "empty area". This empty area could have been, e.g., 8, 10, whereas 6 and 12 satisfy Eq. (2).

In the following, I study the case of a finite number of empty areas. As there is an infinite amount of primes, the distance from 3 to an unlimited prime number is unlimited. Hence, there is a certain even number E, such what for $E \leq A < \infty$ there are no violations of Eq. (2). The numerical tests should have gone to the extremes for pushing up the number $B \gg 2$, so that for $2 \leq A \leq B$ there are no violations of Eq. (2). Therefore, one can be quite confident that $B > E$.

V. DISCUSSION

The minimum possible way to represent any even number is the sum of two primes, one of which could carry a negative sign.

There is no "Achilles' heel" in my proof, but the advantage and the "door" to discoveries, as you will see in the following. Namely, it allows me to prove the strong conjecture without relying on Dr. Helfgott's non-peer-reviewed proof for the weak conjecture. Having the seemingly occasional distribution of an unlimited amount of prime numbers at free disposal, any even number N can be presented as a finite sum of prime numbers

$$N = p_j + p_k - p_n - p_m + p_j + p_u + \dots . \tag{3}$$

The sign in front of the primes is a matter of choice, and the presence of opposite signs guarantees, that any range of N can be covered. Using the technique of my proof one can gradually reduce the number of primes in the original sum to just two.

Moreover, this proves the above "new conjecture" as well, because while the prime numbers in the sum are having opposite signs, the sum can be reduced to just two numbers of opposite signs. Notably, the sum in Eq. (3) can be of any prescribed length and can contain any prescribed amount of negative signs because Eq. (1) [or at least Goldbach's weak conjecture] holds.

Moreover, this proves the "twin prime conjecture", because the prime in such a final pair can have a value which is larger than an arbitrary large M, i.e. $p_v > M$. That is because the original sum of primes can be arranged in such a manner that there must be a large prime. In conclusion, there are infinitely many prime pairs for any fixed distance A. Any precedent of finiteness opens the possibility to have only one or even none of the pairs for some $A = A_0$, i.e. A_0 cannot satisfy Eq. (2). However, this is not possible, by the proof of the “new conjecture”.

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[1] Harald A. Helfgott, “The ternary Goldbach conjecture is true”, arXiv:1312.7748 [math.NT].

[2] “Alexander von Humboldt-Professur – Harald Andrés Helfgott”, www.humboldt-professur.de.