Nonsingular Start to Inflationary Expansion, Calculating the Cosmological Constant, and a Minimal Time Step, for Our Present Universe Using Klauder Enhanced Quantization for DE

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We start with an elementary example of a nonsingular configuration using a trivial solution to massive gravity yielding a nonzero initial time step. Then we present a history of the cosmological constant issue from Einstein's introduction—which did not work because his static universe solution to the Ricci Scalar problem and GR was unstable—to the radius of the universe being proportional to the inverse square root of the cosmological constant. We use two spacetime first integrals to isolate a nonperturbative cosmological constant solution at the surface of the start of expansion of the universe. A phenomenological solution to the cosmological constant involves scaling the radius of the present universe. Our idea is to instead solve the cosmological constant at the surface of the initial spacetime bubble, using the initially derived time step, Δt, as input for the cosmological constant. This was done in a Zeldovich4 section for dark energy; solving the initial value of the cosmological constant supports one of the models of DE and why the universe reaccelerated a billion years ago. We depend on Katherine Freese's Zeldovich4 talk of dark stars, which form supermassive black holes, to consume initially created DM, and Abhay Ashtekar's nonsingular start to the universe as part of a solution to low l values in CMBR data. We conclude with a reference to a multiverse generalization of Penrose's cyclic conformal cosmology as input to the initial nonsingular spacetime bubble.

Key words: Minimum scale factor, cosmological constant, spacetime bubble, CCC

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I. INTRODUCTION, USING A NONSINGULAR START TO THE UNIVERSE

We begin from the ideas in a HEPGC article [1] and reproduce its introduction to derive a minimum time step. We then use this minimum time to develop a simple DE models, one with an unvarying-in-the-present-era cosmological constant. We give an argument as to the existence of a minimum time step at the surface of a spacetime bubble and then discuss the failure of the initial Einstein solution for a static universe, which involves an unstable solution. That abandoned model has been in part replaced by one with the cosmological constant proportional to one over the square of the radius of an "observed" universe, which is in part unsatisfactory because it assumes, perforce that we know the boundaries of an expanding universe, which we do not. This then leads to our model comparing two first integrals at the surface of spacetime to derive an unvarying-over-time cosmological constant parameter. Then on we refer to Karen Freese's dark stars [2] for formation of super-massive black holes (SMBHs) to eat up dark matter (increasing the DE ratio) and Abhay Ashtekar's treatment of how a nonsingular start to the universe tames inhomogeneities as given by [3, Fig. 9.11, p. 257]. Ashtekar [4] claimed that the nonsingular bubble is not decisive, but I disagree and will elaborate. We conclude with an elementary discussion of how a fixed cosmological constant contributes to acceleration of expansion, with suggested tests, caveats, and concerns. The multiverse idea generalized from CCC [1] plays a role.

We start with the minimum time-step issue, which we will find is important. In [5, pp. 212–213], we have that there is a Minkowski simple model for massive gravity, leading to

$$m \cdot \partial_t (a^2 - a^3) = 0. \tag{1}$$

Using [6] and [1] and really looking at [7], we can augment the scale factor used in Eq. (1) for the surface of a presumed nonsingular start to the expansion of the universe.

$$a(t) = a_{\min} t^{\gamma} \tag{2}$$

Leading to [7] the inflaton.

$$\phi = \sqrt{\frac{\gamma}{4\pi G}} \ln \left\{ \sqrt{\frac{8\pi G V_0}{\gamma(3\gamma - 1)}} \cdot t \right\} \tag{3}$$

Eqs. (1) and (2) lead us to an interesting restraint: a minimum time step.

$$t = \left(\frac{2}{3a_{\min}} \right)^{\frac{1}{\gamma}} \tag{4}$$

Our preliminary consideration has time in Eq. (4) commensurate with Planck time and the radius of the bubble of initial spacetime matching the Planck length [8]. Why is a minimum time step so important? It underpins the nonsingular start to the universe. We will see in the formation of the solution to the cosmological constant problem. But first, we review Einstein's unstable solution for a static universe.

We go to [9, p. 144]. Their solution this is what they got for Einstein's value of the Ricci scalar, R.

$$R = g^{uv} R_{uv} = R_E = \frac{1}{\sqrt{-\Lambda}} = \frac{c}{2\sqrt{\pi G \rho}} \tag{5}$$

The density here, ρ, must be absurdly low, whereas Λ has to be 10^{-50} cm^2, as given by [10, pp. 410–411]. The fault of this is that, by an analysis of this given on [9, p. 275], any perturbation of $R = g^{uv} R_{uv} = R_E$ would lead to expanding- or contracting-universe states.

So what do we have left? In phenomenology, we have the idea of a linkage to the radius of the universe for Λ. [11, pp. 192–193] suggests it could be L if

$$\Lambda \equiv \frac{3}{L^2}. \tag{6}$$

The question we then have to ask is, then, What *is* the radius of the universe? Is the universe a sphere? What other shape could it be?

Now for the general relativity first integral [1]. We use the Padmanabhan first integral [7] with the third entry of Eq. (1) having a Ricci scalar defined via [9] and usually the curvature, $\mathcal{N}$, set extremely small [1]:

$$S_1 = \frac{1}{2\kappa} \int \sqrt{-g} \cdot \mathrm{d}^4 x \cdot (\mathcal{R} - 2\Lambda) \tag{7}$$

with $-g = -\det g_{uv}$ and $\mathcal{R} = 6 \cdot \left(\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 + \frac{\mathcal{N}}{a^2} \right)$. Also, the variation of $\delta g_{tt} \approx a_{\min}^2 \phi$ as given by [1] will have an inflaton, ϕ, given by [1] Leading to [1, 7] to the inflaton which is combined into other procedures for a solution to the cosmological constant problem. Here, $a_{\min}$ is a minimum value of the scale factor [1].

Next for the idea from Klauder. We are going to go to [12, p. 78], which mentions a restricted quantum action principle: S_2. We write a 1–1 equivalence as in [1, 12]:

$$S_2 = \int_0^T \mathrm{d}t \cdot [p(t)\dot{q}(t) - H_N(p(t), q(t))] \approx S_1 = \frac{1}{2\kappa} \int \sqrt{-g} \cdot \mathrm{d}^4 x \cdot (\mathcal{R} - 2\Lambda) \tag{8}$$

Our assumption is that Λ is constant. Hence we approximate from [1]:

$$\frac{p_0^2}{2} = \frac{p_0^2(N)}{2} + N; \text{ for } 0 < N \leq \infty, \text{ and } q = q_0 \pm p_0 t. \tag{9}$$

$$V_n(x) = 0;\ \text{for } 0 < x < 1.\ \text{and } V_n(x) = N;\ \text{otherwise.} \tag{10}$$

$$H_N(p(t), q(t)) = \frac{p_0^2}{2} + \frac{(\hbar \cdot \pi)^2}{2} + N;\ \text{for } 0 < N \leq \infty. \tag{11}$$

Our innovation is equating $q = q_0 \pm p_0 t \sim \phi$ and assuming small time step values [1]:

$$\Lambda \approx \frac{-\left[\frac{V_0}{3\gamma-1} + 2N + \frac{\gamma\cdot(3\gamma-1)}{8\pi G\cdot\tilde{t}^2}\right]}{\frac{1}{\kappa}\int\sqrt{-g}\cdot \mathrm{d}^3x} + \left(6\cdot\left(\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2\right)\right)\Bigg|_{t=\tilde{t}} \tag{12}$$

We have a candidate balance of contributions to Λ at the start of the universe. Assuming DE is Λ, how do we account for a possible reduction of DM's role in the early universe?

Black holes of 10^8 solar masses formed early on in the centers of spiral galaxies from DM stars [2]. The classic DE/DM balance probably had more DM then than today, with the dark stars perhaps shrinking the amount of DM and boosting the total budget of DE in the cosmos.

We do not exactly know what DE is, but we have a clue to its role in the present universe. DM may have been considerably more prevalent than today, if the SMBHs from the galaxies' centers are from giant DM stars.

A review of nonsingular cosmologies, the Friedman Equation, and how we use Eq. (4). A given in our work is that, within the nucleation of spacetime, time, even as given in Eq. (5) simply does not exist. We can use a nonsingular Friedman equation [14] to get

$$H^2 = \frac{8\pi}{3M_P^2}\cdot\left(\rho - \frac{\rho^2}{2|\sigma|}\right). \tag{13}$$

Here, we have that ρ is a spacetime density function and σ is related to the tension of a spacetime bubble on the order of a Planck radius. We also use the dark energy model in [13] for energy density, if $\mathcal{R}$ is the Ricci scalar [10]. We have also the discussion given in [13] and set κ as spacetime curvature.

$$\rho_{\mathrm{DE}} = \frac{3\tilde{\alpha}}{8\pi}\cdot\left(\dot{H} + 2H^2 + \frac{\kappa}{a^2}\right) = -\frac{\tilde{\alpha}}{8\pi}\cdot\mathcal{R} \tag{14}$$

In terms of the bubble of spacetime before inflation, we submit that time does not really exist and that then we will be considering a rewrite Eq. (14 as having, effectively, $\dot{H} = 0$ and $\rho_{\mathrm{DE}} = \rho$. If we apply Eq. (13), we have $\rho_{\mathrm{DE}} = \rho$ at just about the surface of the bubble:

$$\rho^2 - \rho\cdot(2|\sigma|)\left(1 - \left(\frac{3\tilde{\alpha}}{8\pi}\right)^{-1}\cdot\left(\frac{16\pi}{3M_P^2}\right)^{-1}\right) - \left(\frac{16\pi}{3M_P^2}\right)^{-1}\cdot(2|\sigma|)\cdot\frac{\kappa}{a^2} = 0. \tag{15}$$

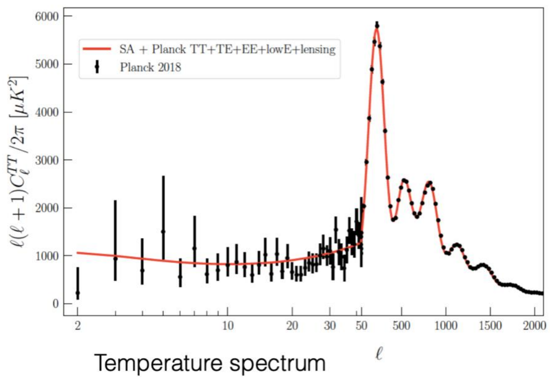

FIG. 1: Abhay Ashtekar in Zeldovich4 on September 7, 2020, [4].

Unlike the Wheeler De Witt formulation, with zero net energy commensurate with no time evolution of the wave function of the universe [15], with variants proposed in [16], we will be involved in using an input to the initial physical system [1]. This allows us to say something about the formulation of a time step at the surface of the spacetime bubble according to the minimum uncertainty principle. At the surface of the bubble of about one Planck length in radius, we would have then a minimum time step $\Delta E \Delta t \equiv \hbar$ [17]. Then, if the initial volume is the cube of a Planck length, we have, if the $\tilde{\alpha}$ value were small,

$$\Delta t = \frac{\frac{l^3_{\text{Planck}}\hbar}{|\sigma|}}{\left(1 - \left(\frac{3\tilde{\alpha}}{8\pi}\right)^{-1} \cdot \left(\frac{16\pi}{3M_P^2}\right)^{-1}\right) \pm \sqrt{\left(1 - \left(\frac{3\tilde{\alpha}}{8\pi}\right)^{-1} \cdot \left(\frac{16\pi}{3M_P^2}\right)^{-1}\right)^2 + \left(\frac{16\pi}{3M_P^2}\right)^{-1} \cdot (2|\sigma|) \cdot \frac{8\kappa}{a^2}}}. \tag{16}$$

A limiting solution is if the curvature term is small, with, among other approximations, the bubble tension term dropping out. But this is not a general solution.

$$\Delta t \approx 2 l^3_{\text{Planck}} \hbar \frac{+\left(\frac{3\tilde{\alpha}}{8\pi}\right)^{-1} \cdot \left(\frac{16\pi}{3M_P^2}\right)}{\frac{8\kappa}{a^2}} \tag{17}$$

In either case, we do our calculations to determine the frequency of a signal from this event, as well as the GW strength and possible polarization states. This would have to be contrasted with Eq. (4) for bounding values for the input into Eq. (16). The value of time picked, either Eq. (4) or Eq. (14) or equating them would approximate the first values of the scale factors given in Eq. (12). In Figure 1, we copy what was part of anisotropic fits to the E and B polarization [4]. The crux is the claim that Loop Quantum Gravity could solve CMBR data glitches [4].

We can smooth out the error bars using LQG with the Starobinsky Potential [4]:

$$V(\phi) = \frac{3M^2}{32\pi} \cdot \left(1 - \mathrm{e}^{-\sqrt{\frac{16\pi G}{3}\phi^2}}\right). \tag{18}$$

We are suggesting that the nonsingular start to the universe, not just LQG, may play a role in resolving anisotropic conditions as given in Figure 1 in low-l-value contingencies. This issue has raised many comments in recent Rencontres De Moriond Cosmology seminars.

We extend Penrose's suggestion of cyclic universes [18], black hole evaporation, and the embedded structure our universe. This multiverse embeds BHs and may resolve what appears to be an impossible dichotomy. Working from [1], there are no fewer than N universes undergoing Penrose "infinite expansion" [18] contained in a megauniverse structure. Each of the N universes has black-hole evaporation, with the Hawking radiation from decaying black holes. If each of the N universes is defined by a partition function, $\{\Theta_i\}_{i\equiv N}^{i\equiv 1}$, then an information ensemble of mixed minimum information is about 10^7–10^8 bits of information per partition function in the set $\{\Theta_i\}_{i\equiv N}^{i\equiv 1}|_{\text{before}}$. Minimum information is conserved between a set of partition functions per universe [1]:

$$\{\Theta_i\}_{i\equiv N}^{i\equiv 1}\Big|_{\text{before}} \equiv \{\Theta_i\}_{i\equiv N}^{i\equiv 1}\Big|_{\text{after}}. \tag{19}$$

However, nonuniqueness of information in each partition function $\{\Theta_i\}_{i\equiv N}^{i\equiv 1}$ and Hawking radiation from the black holes is collated via a strange attractor collection in the mega universe structure to form a new big bang for each of the N universes represented by.$\{\Theta_i\}_{i\equiv N}^{i\equiv 1}$. Verification of this mega structure compression and expansion of information with nonuniqueness of information placed in each of the N universes favors ergodic mixing treatments of initial values for each of N universes expanding from a singularity beginning. The n_f value, use $S_{\text{entropy}} \sim n_f$ [19]. To tie in this energy expression, as in Eq. (20), look at the formation of a nontrivial gravitational measure as a new big bang for each of the N universes as by $n(E_i)$ the density of states at a given energy E_i for partition function [1, 20]

$$\{\Theta_i\}_{i\equiv N}^{i\equiv 1} \propto \left\{\int_0^\infty \mathrm{d}E_i n(E_i) \cdot \mathrm{e}^{-E_i}\right\}_{i\equiv 1}^{i\equiv N}. \tag{20}$$

Each E_i is identified with Eq. (9–11), with the iteration for N universes [1, 18]. By asserting the following claim to the universe, as a mixed state, with black holes playing a major part, using Ergodic mixing, to a degree [1, 21]

$$\frac{1}{N} \cdot \sum_{j=1}^{N} \Theta_j\big|_{j \text{ before nucleation}} \xrightarrow[\text{vacuum nucleation transfer}]{} \Theta_i\big|_{i \text{ fixed after nucleation}} \tag{21}$$

For N universes, with each $\Theta_j|_{j \text{ before nucleation}}$ for $j = 1$ to N being the partition function of each universe just before the blend into the RHS of Eq. (16) for our present universe. Also, each of the independent universes given by $\Theta_j|_{j \text{ before nucleation}}$ is constructed by the absorption of one to ten million black holes taking in energy [1]. Furthermore, the main point is in terms of general ergodic mixing [1, 21].

Claim 2

$$\Theta_j|_{j \text{ before nucleation}} \approx \sum_{k=1}^{\max} \tilde{\Theta}_k \Big|_{k \text{ blackholes } j\text{th universe}} \tag{22}$$

In Claims 1 and 2 we derive how a multidimensional representation of black-hole physics enables continual mixing of spacetime [1, 21] largely to avoid the anthropic principle with a preferred set of initial conditions. Claim 2 is to use what is known as CCC. First, have a big bang (initial expansion) for the universe. for say redshift $z = 10$, ten billion years ago, SMBH formation starts. Matter-energy is vacuumed up by the SMBHs [1]. Penrose's main methodology in Eq. (23) evaluates a change in the metric g_{ab} by a conformal mapping $\widetilde{\Omega}$ [1]:

$$\widehat{g}_{ab} = \widehat{\Omega}^2 g_{ab}. \tag{23}$$

Penrose suggested using [1]

$$\widehat{\Omega} \underset{ccc}{\longrightarrow} \widehat{\Omega}^{-1}. \tag{24}$$

Cosmic black holes have been the main mechanism for the recycling apparent in Eq. (16) with the caveat that $\hbar$ is kept constant from cycle to cycle as represented by $\hbar_{\text{old cosmology}} = \hbar_{\text{present cosmology}}$. The invariance of Planck's $\hbar$ combined with Eq. (24) clearly indicates a uniform mass to a graviton, per cycle, as far as heavy gravity, provided the caveat holds [1].

One big takeaway from all this is that the cosmological constant is rooted right into the value of a massive graviton. That is [22],

$$m_g = \frac{\hbar \cdot \sqrt{\Lambda}}{c}. \tag{25}$$

Where does this leave us? (i) We have that Eq. (25) leaves a solid link as to the interlinkages between massive gravitons and the calculated cosmological constant. (ii) Do we have evidence of a multiuniverse contribution as to the formation of an initial universe expansion? This needs to be confirmed experimentally. (iii) As outlined, this can lead to confirming or denying the truth of what Corda raised in his article about a minimal alteration of GR [23, 24]. (iv) Our methodology, if confirmed and developed, will lead to among other things, a nonperturbative, but different procedure for bounding the cosmological constant, and by extension DE. Since DE is, in its simplest iteration, the cosmological constant, we have that in terms of the transition from the interior bubble of the spacetime start to inflation to traditional spacetime inflation the following to explore, with dS the surface of an event horizon between feeding into the spacetime bubble, and cosmic inflation.

$$c\rho_{dS} = \left.\frac{R_{00} - g_{00}R}{8\pi}\right|_{dS} \Leftrightarrow \left.\frac{-3\frac{\ddot{a}}{a} - g_{00}6\Lambda}{8\pi}\right|_{dS} \approx \left.\frac{-3\frac{\dot{a}}{a} + g_{00}6\left(\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 + \frac{2\kappa}{a^2}\right)}{8\pi}\right|_{dS}. \tag{26}$$

(v) See this final take away as to what the cosmological constant is equivalent to. (vi) To obtain maximum results, we will be stating that the following will be assumed to be equivalent. [1], i.e. the term comes from using expression of [25, 26]

$$\sqrt{g_{tt}}\left(\frac{g_{tt}\dot{V}_3^2}{V_3(t)} + k_2 V_3^{\frac{1}{3}}(t)\right) \sim \frac{1}{2\kappa}\int\sqrt{-g}\cdot \mathrm{d}^3x\cdot\left(6\cdot\left(\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2\right)\right) \tag{27}$$

$$\sqrt{g_{tt}}(\lambda V_3(t)) \sim \frac{1}{2\kappa}\int\sqrt{-g}\cdot \mathrm{d}^3x\cdot(2\Lambda) \text{ leading to } \lambda \sim \frac{1}{\kappa}\sqrt{\frac{-g}{\delta g_u \approx a_{\min}^2\phi}}\cdot\Lambda. \tag{28}$$

Our assumption is that the Lagrangian multiplier is roughly equivalent to a mass which is about 10^4 times the mass of a Planck sized black hole mass, i.e. that we have Black holes initially produced which are of say 10^2 times the Planck mass. In Corda's recent work, [27] we have that n is the quantum number n put in where Planck mass is normalized to 1, so then, if there are 10^2 black holes of mass 10^2 times Planck mass $M \approx 10^2 m_{\text{Planck}} = 100$. Here we make the following simplification of a so called Horizon volume Eq. (23) to read as. After normalizing Planck mass as being 1, then by [28] we state

$$\Delta V_{n-1\to n}|_{\text{Total}} \approx 10^2 \times \Delta V_{n-1\to n} \approx 10^3 \times 16\pi\cdot\left(\frac{n}{4\times 10^4}\right)\cdot\left|\sqrt{1-\left(\frac{n-1}{n}\right)}\right|. \tag{29}$$

Our supposition is that there are 10^2 mini black holes, and a mass of 10^2 times Planck mass, per each black hole, so that we. find n for quantum number. So if we assume [28] conditions. Then we have an entropy count, on r.h.s. as

$$16\pi\cdot\left(\frac{n}{4\times 10^4}\right)\cdot\left|\sqrt{1-\left(\frac{n-1}{n}\right)}\right| \approx 10^6. \tag{30}$$

That is. that say 1000 times Planck length, we have the beginning of say creation of 100 mini black holes, each of mass about 100 times Planck mass which would put a huge restriction upon the admissible quantum value, n.

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