Bell's theorem refuted via true local realism

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Abstract Bell's theorem has been described as the most profound discovery of science; one of the few essential discoveries of 20th Century physics; indecipherable to non-mathematicians. However, taking elementary analysis to be an adequate logic here, we refute Bell's theorem, correct his inequality and identify his error. Further, we do this under the principle of *true local realism*, the union of *true locality* (or *relativistic causality*: no influence propagates superluminally) and *true realism* (or *non-naive realism*: some existents change interactively). We thus lay the foundation for a more complete physical theory: one in line with Einstein's ideas and Bell's hopes. Let's see.

1. Introduction

1.1. We resolve or refute these Bellian difficulties:

> **p.5**: 'I cannot say that action at a distance (AAD is required in physics. But I can say that you cannot get way with no AAD. You cannot separate off what happens in one place and what happens in another. Somehow they have to be described and explained jointly.' **p.6**: 'The Einstein program fails, that's too bad for Einstein, but should we worry about that? So what? ... it might be that we have to learn to accept not so much AAD, but the inadequacy of no AAD.' **p.7**: 'And that is the dilemma. We are led by analysing this situation to admit that in somehow distant things are connected, or at least not disconnected. ... So the connections have to be very subtle, and I have told you all that I know about them.' **p.9**: 'It's my feeling that all this AAD and no AAD business will go the same way [as the ether]. But someone will come up with the answer, with a reasonable way of looking at these things. If we are lucky it will be to some big new development like the theory of relativity. Maybe someone will just point out that we were being rather silly, and it won't lead to a big new development. But anyway, I believe the questions will be resolved.' **p.10**: 'I think somebody will find a way of saying that [relativity and QM] are compatible. For me it's very hard to put them together, but I think somebody will put them together, and we'll just see that my imagination was too limited.' **p.12**: 'I don't know any conception of locality that works with QM. So I think we're stuck with nonlocality.' **p.13**: '... I step back from asserting that there is AAD, and I say only that you cannot get away with locality. You cannot explain things by events in their neighbourhood. But I am careful not to assert that there is AAD,' after Bell (1990).

1.2. We refute these Bellian conclusions:

> 'In a theory in which parameters are added to quantum mechanics to determine the results of individual measurements, without changing the statistical predictions, there must be a mechanism whereby the setting of one measuring device can influence the reading of another instrument, however remote. Moreover, the signal involved must propagate instantaneously, so that such a theory could not be Lorentz invariant,' Bell (1964:199).

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1.3. That is: (i) Taking the union of elementary mathematics and probability theory to be an adequate logic here, we refute Bell's famous theorem (and his related proof), and identify his error. (ii) We do this under the principle of *true local realism*, the union of *true locality* (or *relativistic causality*: ie, no influence propagates superluminally, after Einstein) and *true realism* (or *non-naive realism*: some existents change interactively, after Bohr). (iii) Now our refutation of Bell's theorem means that Bell's proof—via **B**(15); his famous inequality—is false. (iv) So in §5-Appendix, via (27)-(36), we refute **B**(15) with high-school mathematics. (v) And in §5.2, we identify and correct Bell's error.

1.4. (i) We thus lay the foundation for a more complete physical theory: ie, the extension of classical mechanics from light-speed to Planck's constant; with $0 \leq \rho(x) \equiv |\psi(x)|^2$ in an extended probability theory. (ii) We believe such a theory to be in line with Einstein's ideas and Bell's hopes. (iii) For Einstein argued (Bell 2004:86) that 'EPR correlations can be made intelligible only by completing the quantum mechanical account in a classical way:' which we do. (iv) For Bell's hopes, see §1.1.

1.5. So, en route to converting the above Bellian issues to mathematical expressions: (i) Let β denote Bohm's experiment in Bell (1964); let **B**(.) denote Bell's equation (.) therein. (ii) Let $A^{\pm}$ and $B^{\pm}$ be the causally-independent same-instance results in **B**(1), pairwise correlated via λ and functions A, B: so Bell's *vital assumption* [below **B**(1)] is equivalent to our *relativistic causality*. (iii) Then, reserving P for probabilities, let's replace Bell's expectation $P(\vec{a},\vec{b})$ in **B**(2) with its identity $E(a,b\,|\,\beta)$. (iv) So, from **B**(1), **B**(2), RHS **B**(3) and the line below it (with Λ denoting the space of λ), here's Bell's 1964 theorem ($\mathbf{BT}_1$) in our notation: followed by Bell's 1975 variant; in our terms, $\mathbf{BT}_2$:

$$\mathbf{BT}_1\text{: } E(a,b\,|\,\beta) = \textstyle\int_\Lambda d\lambda\, \rho(\lambda)A(a,\lambda)B(b,\lambda) \neq -a\cdot b \text{ [??]:} \tag{1}$$

$$\text{with } \lambda \in \Lambda : A(a,\lambda) = A^{\pm} = \pm 1,\ B(b,\lambda) = B^{\pm} = \pm 1,\ A(a,\lambda)B(b,\lambda) = \pm 1. \tag{2}$$

$$\mathbf{BT}_2\text{: } E(a,b\,|\,\beta) \neq \textstyle\int_\Lambda d\lambda\, \rho(\lambda)A(a,\lambda)B(b,\lambda) \neq -a\cdot b = E(a,b\,|\,\beta) \text{ [??]. For, after} \tag{3}$$

Bell (1975:3): "with these *local* forms $A(a,\lambda), B(b,\lambda)$, it is *not* possible to find functions A and B and a probability distribution ρ which give the correlation $E(a,b\,|\,\beta) = -a\cdot b$." (4)

Now, below **B**(3) and as we prefer, Bell allows that λ may fall into two sets (say $\lambda^+$and λ^-), with $A^{\pm}$ dependent on λ^+ and $B^{\pm}$ on λ^-. So, refuting (1) and (3) as we'll now show, (5)

$$\text{here's our unifying claim: } E(a,b\,|\,\beta) = \textstyle\int_\Lambda d\lambda\, \rho(\lambda)A(a,\lambda^+)A(b,\lambda^-) = -a\cdot b. \tag{6}$$

2. Analysis

2.1. (i) Seeking nature's laws, we include every β-*relevant* element of physical reality[2] in our analysis (mathematics, text, schematics). (ii) And each is denoted by *a unique physically-significant symbol*: eg, see $\lambda^+$and λ^- in (5). (iii) So, with explanatory notes to follow at §2.2, here's β in our terms:

$$\pm 1 = A^\pm = \delta_a^\pm(.) \leftarrow p(\lambda^+) \leftarrow [S_\beta] \rightarrow p(\lambda^-) \rightarrow \delta_b^\pm(.) = B^\mp = \mp 1; \text{ or} \tag{7}$$

$$\pm 1 = A^\pm = [p(a^\pm) \leftarrow \Phi_a^\pm] \leftarrow p(\lambda^+) \leftarrow [S_\beta] \rightarrow p(\lambda^-) \rightarrow \left[\Phi_b^\pm \rightarrow p(b^\mp)\right] = B^\mp = \mp. \tag{8}$$

$$\lfloor \quad \text{Alice's local domain} \quad \rfloor \qquad \lfloor \text{Source} \rfloor \qquad \lfloor \quad \text{Bob's local domain} \quad \rfloor \tag{9}$$

$$\text{Thus, via } \delta_a^\pm\text{: } p(\lambda^+) \rightarrow [\Phi_a^\pm \rightarrow p(a^\pm)] = A^\pm = \pm 1\text{: or, in short, } \delta_a^\pm(\lambda^+) = \pm 1\text{; etc.} \tag{10}$$

nb: $\delta_a^\pm(\lambda^+) = A(a,\lambda) = \pm 1$ and $\delta_b^\pm(\lambda^-) = B(b,\lambda) = \mp 1$ each satisfy this definition of a (11)

function: a transformative process over two sets that associates to every element λ of the (12)

first set exactly one element of the second set $\{\delta_\bullet^\pm(.)\,|\,(.) \in \Lambda\} = \{1, +1\}$; $\bullet = a$ or b: (13)

$$\text{ie, } A(a,\lambda) = \delta_a^\pm(\lambda^+) = A^\pm = \pm 1, B(b,\lambda) = \delta_b^\pm(\lambda^-) = B^\mp = \mp 1, \lambda^\pm = \pm\lambda, \lambda^+ + \lambda^- = 0. \tag{14}$$

2.2. In (7): (i) Source $[S_\beta]$ emits particle-pairs $p(\lambda^+)$ and $p(\lambda^-)$; their properties (.) pairwise-correlated via the same-instance conservation of total angular-momentum: ie, $\lambda^+ + \lambda^- = 0$, pairwise. (ii) $\delta_\bullet^\pm(.)$, identical detectors, provide identical functions over different arguments: eg, $p(\lambda^+)$ interacts with detector $\delta_a^\pm(.)$, a dichotomic (2-channel) polarizer-analyzer with principal-axis a and output channels $a^\pm = \pm a$; etc. (iii) Note: the inversion $\mp$ is a reminder that, in the same instance [akin to **B**(13)]: $\delta_a^\pm(\lambda^+)\delta_a^\pm(\lambda^-) = -1$.

2.3. (i) In (8) we expose $\delta_a^\pm(.)$'s inner workings: $p(\lambda^+)$, interacting with polarizer $\Phi_a^\pm$, is transformed to $p(a^\pm)$. (ii) Here, under $p(.)$, $a^\pm$ is the particle's post-interaction spin-axis. (iii) The analyzer [not shown] equates $a^\pm$ to the result $A^\pm = \pm 1$; eg, via straight-forward capitalization of $a^\pm$ to $A^\pm$, and the allocation of ± 1. (iv) Or (if you prefer), via an internal analyzer-function, $a{\cdot}a^\pm = \pm 1$; etc.

2.4. (i) In some expressions we show a particle $p(.)$ heading toward ($\rightarrow$) an interaction with a macroscopic device: $\delta_\bullet^\pm(.)$ or $\Phi_\bullet^\pm$. (ii) In other expressions, *equivalently*, we show a macroscopic device operating on a particle property: eg, in short form, $\delta_a^\pm(\lambda^+) = A^\pm = \pm 1$, as in (11); etc.

[2] We prefer (but here resist) the term *reable* (from *Latin res*, a thing); for we include latent properties as well. 'In a complete theory there is an element corresponding to each element of reality. A sufficient condition for the reality of a physical quantity is the possibility of predicting it with certainty, without disturbing the system,' EPR (1935:777).

3. Refutation

3.1. (i) On the elements of $\delta_a^{\pm}$'s domain, let $\overset{\delta_a^{\pm}}{\sim}$ denote the equivalence relation *has the same output under* $\delta_a^{\pm}$; etc. (ii) So $\delta_a^{\pm}(\lambda^+)$ detects the equivalence class, $[p(a^+)]$ or $[p(a^-)]$, of each pristine (pre-test) $p(\lambda^+)$. (iii) Thus, *via their pairwise correlation*, a latent property of pristine (pre-test, undisturbed) $p(\lambda^-)$ is revealed: a property confirmable via a test, under $\delta_a^{\pm}(\lambda^-)$, in Bob's domain. (iv) So, in the i-th instance, given Alice's result A_i^+; and so for any A^+ result in Alice's domain:

$$\text{In Alice's domain, } p(\lambda_i^+) \overset{\delta_a^{\pm}}{\sim} p(a^+)\text{: so in Bob's domain, } p(\lambda_i^-) \overset{\delta_a^{\pm}}{\sim} p(a^-)\text{. Thus:} \tag{15}$$

$$P(B^+|A^+) = P(p(\lambda^-) \overset{\delta_b^{\pm}}{\sim} p(b^+)\,|\,p(\lambda^-) \overset{\delta_a^{\pm}}{\sim} p(a^-)) = \cos^2\tfrac{1}{2}(b^+,a^-) = \sin^2\tfrac{1}{2}(a,b); \tag{16}$$

$$P(B^-|A^+) = P(p(\lambda^-) \overset{\delta_b^{\pm}}{\sim} p(b^-)\,|\,p(\lambda^-) \overset{\delta_a^{\pm}}{\sim} p(a^-)) = \cos^2\tfrac{1}{2}(b^-,a^-) = \cos^2\tfrac{1}{2}(a,b); \tag{17}$$

ie, under relativistic causality, and heuristically via reverse-engineering (Watson 1998), we equate the probability LHS (16) in Bob's domain *to properties in that domain*. LHS (17) similarly. (18)

3.2. So we now proceed, via LHS (6), to refute (1) and (3); ie, we refute $\mathbf{BT}_1$ and $\mathbf{BT}_2$:

$$\begin{aligned}
E(a,b|\beta) &= \int_\Lambda d\lambda\, \rho(\lambda) A(a,\lambda^+)A(b,\lambda^-) = \int_\Lambda d\lambda\, \rho(\lambda)\delta_a^{\pm}(\lambda^+)\delta_b^{\pm}(\lambda^-) && (19)\\
&= \int_\Lambda d\lambda\, \rho(\lambda)[(\delta_a^{\pm}(\lambda^+)=1)(\delta_b^{\pm}(\lambda^-)=1) - (\delta_a^{\pm}(\lambda^+)=1)(\delta_b^{\pm}(\lambda^-)=-1) \\
&\quad -(\delta_a^{\pm}(\lambda^+)=-1)(\delta_b^{\pm}(\lambda^-)=1) + (\delta_a^{\pm}(\lambda^+)=-1)(\delta_b^{\pm}(\lambda^-)=-1)]. && (20)\\
&= P(A^+B^+) - P(A^+B^-) - P(A^-B^+) + P(A^-B^-) : \text{the weighted-sum of same-inst-}\\
&\quad \text{ance results } [A^+B^+, A^+B^-, A^-B^+, A^-B^-] \text{ that deliver each } AB \text{ result } (\pm 1). && (21)\\
&= P(A^+)P(B^+|A^+) + P(A^-)P(B^-|A^-) - P(A^+)P(B^-|A^+) - P(A^-)P(B^+|A^-) :\\
&\quad \text{via the product rule for the paired (same-instance) results correlated as in (2).} && (22)\\
&= \tfrac{1}{2}\left[P(B^+|A^+) + P(B^-|A^-) - P(B^-|A^+) - P(B^+|A^-)\right] : \text{for, with}\\
&\quad \lambda \text{ a random latent variable, the marginal probabilities } \left[\text{like } P(A^+)\right] = \tfrac{1}{2}. && (23)\\
&= \tfrac{1}{2}\left[\sin^2\tfrac{1}{2}(a,b) + \sin^2\tfrac{1}{2}(a,b) - \cos^2\tfrac{1}{2}(a,b) - \cos^2\tfrac{1}{2}(a,b)\right] : && (24)\\
&\quad \text{ie, in full accord with true local realism, we equate the probabilities in (23)}\\
&\quad \text{to the } \beta\text{-based laws, (16)-(17), that apply in Bob's domain.} && (25)\\
&= -\cos(a,b) = -a\cdot b. \text{ So (1) and (3) are refuted. QED.} && (26)
\end{aligned}$$

4. Conclusions

4.1. (i) Bell's theorem: the most profound discovery of science? One of the few essential discoveries of 20th Century physics? Indecipherable to non-mathematicians? (ii) In refuting Bell's theorem and identifying his errors via elementary analysis, we conclude: those claims, and many others like them, are false. (iii) For, in (16)-(17) and under true local realism, we provide the first of a family of laws that refute Bell's theorem in other settings.

4.2. (i) Moreover, we do this under the principle of *true local realism*, the union of *true locality* (or *relativistic causality*: no influence propagates superluminally) and *true realism* (or *non-naive realism*: some existents change interactively). (ii) We therefore conclude: the Bellian difficulties in §1.1 are resolved or refuted, and Bell's conclusions in §1.2 are false and refuted.

4.3. (i) Further, as foreshadowed in §1.4, in making 'EPR correlations intelligible by completing the quantum mechanical account in a classical way,' we conclude that we have here the foundation for a more complete physical theory. (ii) By which we mean the extension of classical mechanics from light-speed to Planck's constant; with $0 \leq \rho(x) \equiv |\psi(x)|^2$ in an extended probability theory.

4.4. To see this wrt Planck's constant we note: (i) With true local realism established under β, and comparing β's results with similar experiments using photons, we find Planck's constant hidden in our data. (ii) That is: the factor of spin $s = \frac{1}{2}$ in results like (16)-(17) may be compared to the related (hidden) factor of $s = 1$ in photonic experiments; with s in units of $\hbar$, Planck's reduced constant.

4.5. Then, wrt to an extended probability theory: $0 \leq \rho(x) \equiv |\psi(x)|^2$, so common in quantum mechanics, is available classically via the Riesz-Fejér theorem; see Fröhner (1998).

4.6. To be continued.

5. Appendix

5.1. Bell's inequality (**BI**) [ie, **B**(15)] is refuted here: since Bell uses it as proof of his theorem.

$$\textbf{BI:}\ |E(a,b) - E(a,c)| - 1 \leq E(b,c)\text{: ie, }\mathbf{B}(15)\text{ in our notation,} \tag{27}$$

$$\text{where } -1 \leq E(a,b) \leq 1,\ -1 \leq E(a,c) \leq 1,\ -1 \leq E(b,c) \leq 1.\text{ However:} \tag{28}$$

$$E(a,b)[1 + E(a,c)] \leq 1 + E(a,c);\text{ for, if } V \leq 1\text{, and } 0 \leq W\text{, then } VW \leq W. \tag{29}$$

$$\therefore E(a,b) - E(a,c) - 1 \leq -E(a,b)E(a,c).\text{ And thus, similarly:} \tag{30}$$

$$E(a,c) - E(a,b) - 1 \leq -E(a,b)E(a,c). \text{ Hence our irrefutable} \tag{31}$$

$$\text{counter-inequality, } \mathbf{WI:}\ |E(a,b) - E(a,c)| - 1 \leq -E(a,b)E(a,c). \tag{32}$$

$$\text{So, with test-settings } 0 < (a,c) < \pi;\ (a,b) = (b,c) = \frac{(a,c)}{2} = \frac{x}{2}:\ \text{and, via (26),} \tag{33}$$

$$\text{using test-functions } E(a,b) = E(b,c) = -\cos\left(\frac{x}{2}\right),\ E(a,c) = -\cos(x):\ \text{please} \tag{34}$$

$$\text{copy and test this next expression in WolframAlpha}^{\circledR}\text{; free-online, see References.} \tag{35}$$

$$\text{plot}|cos(x) - cos(x/2)| - 1\&\& - cos(x/2)\&\& - cos(x/2)cos(x), 0 \leq x \leq \pi \tag{36}$$

5.2. Thus, under (33)-(34): (i) for $0 < x < \pi$: **BI** (27) is everywhere false and our (32) is everywhere true. (ii) Further, let Bell's unnumbered relations above **B**(15) be **B**(14a)-**B**(14c). (iii) Then Bell's error is his move from *true* **B**(14a) to *false* **B**(14b): for **B**(14b) leads to *false* **B**(15); ie, (27). (iv) In other words, given the common LHS in (27) and (32): Bell's error equates his false $E(b,c)$ in (27) to our irrefutable $-E(a,b)E(a,c)$ in true (32). (v) So Bell's equality holds only at $x = 0$ and $x = \pi$: for then Bell's $-\cos\left(\frac{x}{2}\right) = -\cos\left(\frac{x}{2}\right)\cos(x)$.

6. References

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