On a nonlinear oscillator equation of Lienard type

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Abstract

We propose in this paper a nonlinear oscillator equation of Lienard type which contains the Ermakov-Pinney equation as special case. A major result is that the exact and explicit general periodic solution may be easily calculated.

Keywords: Lienard equation, Ermakov-Pinney equation, general periodic solution, trigonometric function

Theory

Let us consider [1]

\[ \ddot{x} + \frac{1}{2} (\alpha - q)x^{\alpha - q - 1} + \frac{qb}{2} x^{-q - 1} = 0 \]  

obtained from the first order differential equation [1]

\[ \dot{x}^2 x^q + a x^\alpha = b \]  

where the dot over a symbol means differentiation with respect to time \( t \).

Substituting \( \alpha = q + 2 \), and \( b = \frac{a(q + 2)}{4} \), into (1), yields as equation

\[ \ddot{x} + ax + \frac{aq(q + 2)}{8} x^{-q - 1} = 0 \]  

where \( a \), \( b \), \( \alpha \) and \( q \) are arbitrary parameters.

The equation (3) is the proposed nonlinear oscillator where \( q > -2 \). For \( q = 2 \), the equation (3) gives the well-known Ermakov-Pinney equation

\[ \ddot{x} + ax + \frac{a}{x^3} = 0 \]  

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The use of (2) leads to

\[ \int \frac{\frac{q}{2} x^2 dx}{\sqrt{1 - \frac{4}{q + 2} x^{q+2}}} = \sqrt{b} (t + K) \]  

(5)

where \( K \) is an arbitrary constant, from which one may secure the exact and explicit general solution of (3) as

\[ x(t) = \left( \frac{\sqrt{q + 2}}{2} \sin \left( \frac{q + 2}{2} \sqrt{a} (t + K) \right) \right)^{\frac{2}{q+2}} \]  

(6)

Reference