The Concept of a Field

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Summary

This paper explores the common concept of a field and the quantization of fields. We do so by discussing the quantization of *traveling* fields using our photon model, and we also look at the quantization of fields in the context of a perpetual ring current in a superconductor. We then relate the discussion to the use of the (scalar and vector) potential in quantum physics and, finally, a brief discussion of Schrödinger's wave equation which, we argue, just models the equations of motion of charged particles in static and/or dynamic electromagnetic fields – just what Dirac was looking for. We argue that the idea that Schrödinger's equation may not be relativistically correct is based on an erroneous interpretation of the concept of the *effective* mass of an electron.

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Introduction

I do not know where to start this story, and I am also not quite sure for whom I am writing it. For people like me, perhaps: in order to *truly* understand something, we should try to explain it to others. So I should probably describe myself in order to describe the audience: I am an amateur physicist who is interested in the ontology, or *metaphysics*, of modern physics. I might also call its epistemology. Or the genealogy or archaeology of its ideas.

All these words have (slightly) different meanings but the distinctions do not matter all that much. The point is this: I write for people who want to *understand* physics in pretty much the same way as the great classical physicist Hendrik Antoon Lorentz wanted to understand physics:

> "We are representing phenomena. We try to form an image of them in our mind. Till now, we always tried to do so using the ordinary notions of space and time. These notions may be innate. They result, in any case, from our personal experience, from our daily observations. To me, these notions are clear, and I admit I am not able to have any idea about physics without those notions. The image I want to have when thinking about physical phenomena has to be clear and well defined, and it seems to me that cannot be done without these notions of a system defined in space and in time."

He said this a few months before he died. To be precise, these words are part of his reaction to the 'new theories' as presented by de Broglie, Born, Heisenberg, Schrödinger and Bohr at the occasion of the rather (in)famous 1927 Solvay Conference in Brussels. H.A. Lorentz understood electromagnetism and relativity theory as few others did – as few others do today still – and he should surely not be thought of as a classical physicist who, somehow, was stuck and could not see the new light. On the contrary: he probably understood the 'new theories' better than many of the new theorists themselves. In fact, as far as I am concerned, I think his comments or conclusions on the epistemological status of the Uncertainty Principle – which he made in the same intervention – still stand. Let me quote the original French:

> "Je pense que cette notion de probabilité [in the new theories] serait à mettre à la fin, et comme conclusion, des considérations théoriques, et non pas comme axiome *a priori*, quoique je veuille bien admettre que cette indétermination correspond aux possibilités expérimentales. Je pourrais toujours garder ma foi déterministe pour les phénomènes fondamentaux, dont je n'ai pas parlé. Est-ce qu'un esprit plus profond ne pourrait pas se rendre compte des mouvements de ces électrons. Ne pourrait-on pas garder le déterminisme en en faisant l'objet d'une croyance? *Faut-il nécessairement ériger l'indéterminisme en principe*?"

What a beautiful statement: **why should we elevate indeterminism to a philosophical principle?** Indeed, now that I have inserted some French, I may as well inject some German. The idea of a particle includes the idea of a more or less well-known position. Let us be specific and think of uncertainty in the context of position. We may not fully know the position of a particle for one or more of the following reasons:

1. The precision of our measurements may be limited: this is what Heisenberg referred to as an *Ungenauigkeit*.
2. Our measurement might disturb the position and, as such, cause the information to get lost and, as a result, introduce an uncertainty: this is what we may refer to as an *Unbestimmtheit*.
3. The uncertainty may be inherent to Nature, in which case we might prefer the term

Ungewissheit.

So what is the case? Lorentz claims it is either the first or the second – or a combination of both – and that the third proposition is a philosophical statement which we can neither prove nor disprove. I cannot see anything logical (in the theories) or practical (in the experiments) that would invalidate this point. I, therefore, intend to write a basic book on quantum physics from what I hope would be Lorentz' or Einstein's point of view.

My detractors will immediately cry wolf: Einstein lost the discussions with Bohr, did he not? I think he did not. He got tired of them. I want to try to pick up the story where Lorentz and Einstein have left it. I already jokingly rewrote two of Feynman's introductory *Lectures* on quantum mechanics. I consider this paper to be the third.[1]

Space and the vacuum

Modern physicists have multiplied concepts and do not shy away from confusing language – without much definition: spacetime oscillations, virtual particles and quantum fields are just a few of the wonderful new words. These terms are unscientific because there is no agreement on their exact meaning or because one cannot observe or measure them.

Space itself is an ambiguous term. If we empty it, we say it is the vacuum – but physicists will still endow it with physical qualities and, therefore, assume it is *not* empty at all. Let me quote Robert B. Laughlin, a Nobel Laureate in Physics, here:

> "It is ironic that Einstein's most creative work, the general theory of relativity, should boil down to conceptualizing space as a medium when his original premise [in special relativity] was that no such medium existed [..] The word 'aether' has extremely negative connotations in theoretical physics because of its past association with opposition to relativity. This is unfortunate because, stripped of these connotations, it rather nicely captures the way most physicists actually think about the vacuum."

Most – or many – physicists may effectively think like that, but I think we should reserve the term vacuum for a true vacuum: a truly empty space. We will *represent* it, in our mind, by a Cartesian 3D space: so the vacuum *corresponds* to a purely mathematical space but – in contrast to a purely mathematical space – it is real. Why? Because we can put stuff in it. It is easy to see a true vacuum does not exist: space – real space – is usually filled with light, and matter – and fields, which we will discuss in a moment.

Light is radiation – everything from low-energy radio waves to particle-like gamma rays – and matter is matter: protons and electrons, basically, and their antimatter counterparts, of course.[2] Matter-particles carry (electric) charge. Light does not: it carries energy, but no charge. We will come back to this. Let us first say a few words about fields.

[1] See: Quantum Behavior and Probability Amplitudes.

[2] The reader might miss a mention of neutrons, but we think neutrons combine a proton and an electron. We think the quark hypothesis is just what it is: a hypothesis. We think the experiments which are said to have *proved* the existence of quarks (and gluons) can be *interpreted* differently.

What fields? Electric fields, magnetic fields, and gravitational fields, of course. Fields are *real* too, and it looks like they are usually *quantized* too – just like light and matter. However, that does not necessarily mean we should imagine them as a bunch of *virtual* particles. In fact, I think we should *not*, because there is no reason to: we do not need the concept of virtual particles to make sense of the physical world.

Let us dive into it all by jotting down some formulas and, more importantly, *interpreting* them as part of what I refer to as a *realist* interpretation of quantum mechanics, which is an explanation in terms of a system defined in space and in time – as H.A. Lorentz wanted it. Here we go.

Fields and photons

In classical electromagnetic theory, light is modeled as a traveling electromagnetic field, but quantum physics tells us light consists of particles: photons. So how should we imagine them, and how are they different from the usual electromagnetic fields? Let us start with the latter. We will write an electric or magnetic field as $\boldsymbol{E}(x, y, z, t)$ and $\boldsymbol{B}(x, y, z, t)$ respectively. It may be static or dynamic. If it is static, then $\boldsymbol{E}$ and $\boldsymbol{B}$ are a function of the position $\boldsymbol{x} = (x, y, z)$ only: the time variable is irrelevant because they do not vary with time. Real-life fields are more likely to be dynamic: they become stronger or weaker, and they may appear or disappear.

Now, you may think a photon is something like a dynamic electromagnetic field, but it is quite particular. A photon is a *traveling* field, so the electromagnetic field is zero everywhere, except at the very spot where our photon happens to be. That is what makes the photon pointlike: the field vectors $\boldsymbol{E}$ and $\boldsymbol{B}$ that describe it will be zero at each and every point in time and in space *except if our photon happens to be at* $\boldsymbol{x} = (x, y, z)$ *at time t.* Of course, you also know that a photon is defined by its *wavelength* (λ), which is equal to the velocity of light (c) times the *cycle* time (T). So how does that work? What is the *physical* meaning of the wavelength? It is, quite simply, the distance over which the electric and magnetic field vectors will go through a full *cycle* of their oscillation.

That is all there is to it: nothing more, nothing less. Let me be very explicit about this – because I want to make sure we start off on the right foot. The wavelength is a *linear* distance, of course. To be precise, it is the distance Δs between two points (x_1, y_1, z_1) and (x_2, y_2, z_2) where the $\boldsymbol{E}$ and $\boldsymbol{B}$ vectors have the same value. The photon will need some time Δt to travel between these two points, and these intervals in time and space are related through the (constant) velocity of the wave, which is also the velocity of the pointlike photon. That velocity is, of course, the speed of light, and the time interval is the cycle time T = $1/f$. We, therefore, get the mentioned $\lambda = c \cdot \mathrm{T}$ equation:

$$c = \frac{\Delta s}{\Delta t} = \frac{\lambda}{\mathrm{T}}$$

We can now relate this to the Planck-Einstein relation $\mathrm{E} = h \cdot f = \hbar \cdot \omega$, which – as you should know – is the most important equation in all of quantum physics. In fact, it is the only one we need to add to Maxwell's equations. The Planck-Einstein relation pops up in many very different contexts. In this case, it relates the frequency f and the cycle time T to the photon energy (E) through Planck's constant (h):

$$\mathrm{E} = h \cdot f = \hbar \cdot \omega \Leftrightarrow \mathrm{E} \cdot \mathrm{T} = h$$

Think of the photon as *packing* not only the energy E but also an amount of *physical action* that is equal to $h \approx 6.626 \times 10^{-34}$ N·m·s.[3] You may not be very familiar with the concept of *physical action* (do not worry: many academic physicists do not understand it very *intuitively* either). Here again, the original German term for it – a *Wirkung* – captures the imagination much better. Let us explore the meaning of Planck's *quantum of* (physical) *action* by further developing our photon model.

Physical action can *express* itself in two ways: as some energy over some time (E·T) or – alternatively – as some momentum over some distance (p·λ). Indeed, we know that the (pushing) momentum of a photon[4] will be equal to p = E/*c*. We can, therefore, write the Planck-Einstein relation for the photon in two equivalent ways:

$$\mathrm{E} \cdot \mathrm{T} = \frac{E}{c} \cdot c\mathrm{T} = h \Leftrightarrow \mathrm{p} \cdot \lambda = h$$

We could jot down many more relations, but we should not be too long here – because it is just an introductory chapter, really. We said the photon *packs* an energy that is given by its frequency (or its wavelength or cycle time: $c = \lambda f = \lambda/\mathrm{T}$) through the Planck-Einstein relation ($\mathrm{E} = h \cdot f$). We also said it packs an amount of *physical action* that is equal to *h*. So how should we think of that? Let us connect a few more dots here.

We said the Planck-Einstein relation pops up everywhere in quantum physics. It does not only apply to a photon, for example – but also to electron orbitals: the Planck-Einstein tells us electron orbitals are separated by an amount of *physical action* that is equal to $h = 2\pi \cdot \hbar$.[5] Hence, when an electron jumps from one level to the next – say from the second to the first – then the atom will lose one unit of *h*. As mentioned above, that is the amount of physical action which our photon will have to pack. In addition, it will also have to pack the related energy, which is given by the Rydberg formula:

$$\mathrm{E}_{n_2} - \mathrm{E}_{n_1} = -\frac{1}{{n_2}^2}\mathrm{E}_R + \frac{1}{{n_1}^2}\mathrm{E}_R = \left(\frac{1}{{n_1}^2} - \frac{1}{{n_2}^2}\right) \cdot \mathrm{E}_R = \left(\frac{1}{{n_1}^2} - \frac{1}{{n_2}^2}\right) \cdot \frac{\alpha^2 \mathrm{m}c^2}{2}$$

To focus our thinking, let us consider the transition from the second to the first level, for which the $1/{n_1}^2 - 1/{n_1}^2$ factor is equal 0.75. Hence, the photon energy should be equal to $(0.75) \cdot \mathrm{E}_R \approx 10.2$ eV. Now, if the total action is equal to *h*, then the cycle time T can be calculated as:

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[3] You may think of Planck's constant as a very tiny value, but then you should also remember the force unit (newton), distance unit (meter), and the second (time unit) are *very* large units at the subatomic scale: all is relative. The physical dimension of *h* is the same as that of angular momentum, and angular momentum comes in units of *h* at the subatomic scale, so you should think of the size of *h* as being appropriate for the scale at which it is being used.

[4] For an easily accessible treatment and visualization in space, as well as the calculation of the formula itself, see: *Feynman's Lectures*, Vol. I, Chapter 34, section 9.

[5] The model of the atom here is the Bohr model. It does *not* take incorporate the finer structure of electron orbitals and energy states. That finer structure is explained by differences in magnetic energies due to the *spin* (angular momentum) of the electron. We will come back to this. Also note we take the most general of cases: a photon being emitted or absorbed by an atom. Photons can also be (briefly) absorbed and emitted by free electrons in an excited state: this usually involves a change in wavelength and an equivalent change in the kinetic energy of the electron (Compton scattering).

$$\mathrm{E}\cdot\mathrm{T} = h \Leftrightarrow \mathrm{T} = \frac{h}{\mathrm{E}} \approx \frac{4.135\times10^{-15}\mathrm{eV}\cdot\mathrm{s}}{10.2\ \mathrm{eV}} \approx 0.4\times10^{-15}\ \mathrm{s}$$

This corresponds to a wave train with a length of (3×10^8 m/s)·(0.4×10^{-15} s) = 122 nm. It is, in fact, the wavelength of the light (λ = *c*/*f* = *c*·T = h·*c*/E) that we would associate with this photon energy.

What if an electron transitions from a higher level? The reasoning remains the same. However, the photon will still pack *one* unit of *h* only. That is why we call it a spin-1 particle: its angular momentum is one unit of *h*. We will let you think this through for yourself as an exercise.

Now, if you think all of the above is rather trivial, then that is good: just consider it as a warm-up for the math that follows. If not, then it is also good: it then means it was useful to take you through this. At this point, we are tempted to make some remarks in regard to the *spin* of a photon, but we will leave that for later. Indeed, as we are still introducing stuff, it is more useful at this point to contrast this photon model with the idea of (non-traveling) electric or magnetic fields.

The quantization of fields

Matter-particles – electrons, for example – are, obviously, discrete. In the previous section, we also showed the particles of light – photons – are discrete or pointlike as well, albeit in a rather special way: when a photon hits the detector, *all* of the energy is absorbed. Hence, one obtains the energy from integrating over the whole wavelength.[6] What if we have fields but no light – no radiation? Does such situation exist?

It surely does. A good example is the magnetic field that is trapped by a superconducting ring. There is no heat there: no *thermal* motion of electrons, nuclei or atoms or molecules as a whole and, therefore, no (heat) *radiation*. Also, the perpetual currents in a superconductor behave just like electrons in some electron orbital in an atom: they do *not* radiate their energy out. In fact, that is why superconductivity is said to be a quantum-mechanical phenomenon which we can effectively observe at the *macroscopic* level. Hence, we have a magnetic field but no radiation[7] and, since 1961 (the experiments by Deaver and Fairbank in the US and, independently, by Doll and Nabauer in Germany[8]), we know this field is quantized. To be precise, the product of the charge (q) and the magnetic flux (**Φ**), which is the product of the magnetic field *B* and the area of the loop *S*, – will always be an integer (*n*) times *h*[9]:

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[6] It may take the reader a while to combine the ideas of a photon being pointlike and having a linear wavelength at the same time, but there is no contradiction.

[7] The critical reader will immediately note we do need photons to *observe* what is going on (think of [one of the many videos showing magnetic levitation here]). That is, obviously, true, but these photons – this light – goes in and out and does not affect the perpetual current because of the Meissner effect: when a metal becomes a superconductor, it will expel all fields. Other remarks may be made, but we will limit ourselves here to a rather rudimentary description of the fundamentals.

[8] It may be noted that the *theoretical* prediction (quantization of the flux trapped by a superconducting ring) had been made by F. London in 1950 already, so the mentioned physicists knew what they were looking for.

[9] Φ = **B**·**S** is a *vector* (dot) product but – because of the set-up – reduces to an ordinary scalar product: Φ = ***B***·***S*** = |**B**||**S**|cosθ = *B*·*S*. As usual, it is always instructive to check the physical dimensions: the magnetic field is expressed in N/C times s/m, while the surface area is expressed in m^2. Hence, [q·*B*·*S*] = C·(N/C)·(s/m)·m^2 = N·m·s, which is effectively the physical dimension of Planck's quantum of action.

$$q \cdot \mathbf{\Phi} = q \cdot B \cdot S = n \cdot h$$

This quantization does *not* imply that we should assume that the magnetic field itself must, somehow, consist of (discrete) *field quanta*. Not at all, really. The magnetic field is just what it is: a finite quantized magnetic field. There is absolutely no need whatsoever to think of virtual particles or other nonsense here.

Now that we are here, we should note another thing: the equation above makes it clear the field cannot be separated from the circulating charge, which – because electrons form Cooper pairs in superconductors – is *twice* the electron charge. This explains why the basic flux unit is defined as:

$$\phi_0 = \frac{h}{2q_e}$$

Finally, we should make another simple but interesting calculation here. We have a circular current. Hence, this current loop will have a magnetic moment (μ) equal to the product of the current and the surface area of the loop: $\mu = I \cdot \pi \cdot a^2 = I \cdot S$. Of course, the current in such loop is equal to the charge times the frequency of the orbit ($I = q \cdot f$), so we can re-write the magnetic moment as $\mu = q \cdot f \cdot S$. Now, we also know the (potential) magnetic *energy* is calculated as the product of the magnetic moment and the magnetic field: $U_{mag} = \mu \cdot B$. We can, therefore, show that the Planck-Einstein relation is valid here again. Indeed, the (magnetic) energy is an integer multiple of Planck's constant times the frequency of the current:

$$E = U_{mag} = \mu \cdot B = q \cdot f \cdot S \cdot \frac{n \cdot h}{q \cdot S} = n \cdot h \cdot f$$

While the formula is very straightforward, its interpretation is much less so. First, we should note that we should take the directions of the magnetic moment and the magnetic field into account. The energy is, therefore, a vector dot product: $U_{mag} = -\boldsymbol{\mu} \cdot \mathbf{B}$. In fact, the magnetic moment and the magnetic field are aligned here, and this vector product should, therefore, be zero. Also, the $U_{mag} = -\boldsymbol{\mu} \cdot \mathbf{B}$ formula is normally used in the context of an *external* magnetic field, so that is a very different magnetic field than our **B** here. We are, therefore, probably calculating some kind of *potential* rather than actual energy here.[10]

Secondly, it would probably be useful to relate the frequency to the *velocity* of the charge so as to possibly further simplify or interpret this result. Unfortunately, quantum physics does not busy itself with this question, so we have to leave it open for the time being.[11]

A final question concerns the spatial distribution of the magnetic field. As can be seen from the illustration below (taken from Feynman's *Lectures*, III-21-7) the magnetic field is quite local – but the question is: how local, *exactly*?

[10] We should probably try to analyze the set-up as an oscillation in two dimensions. We developed such model for electrons, which we think of as tiny perpetual currents too. We will come back to this later.

[11] We may refer the reader to a recent paper of ours (Matter-waves, amplitudes and signals) for an initial exploration of this issue.

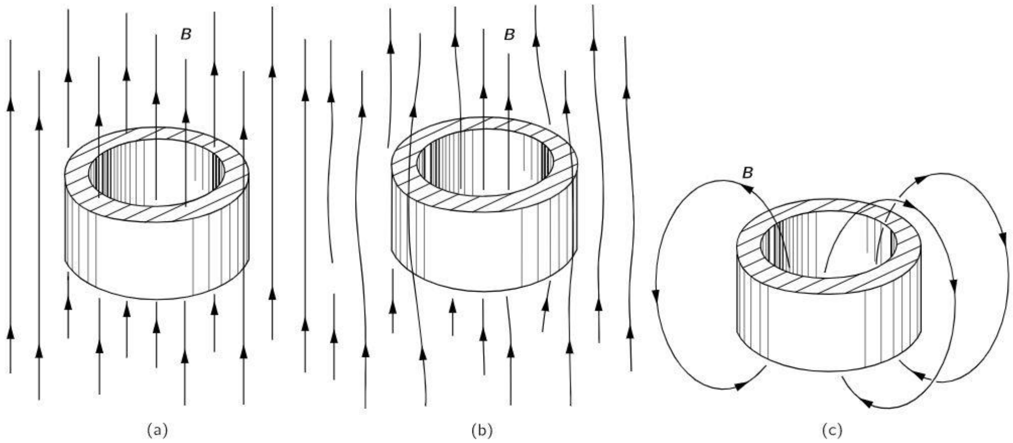

Figure 1: A ring in a magnetic field: (a) in the normal state; (b) in the superconducting state (Meissner effect pushes external fields out; (c) after the external field is removed.

In regard to our last question – the spatial extent of such magnetic fields – I cannot help thinking of yet another of H.A. Lorentz's remarks – one he made at the occasion of the 1921 Solvay Conference, while discussing the ring current model of an electron:

> "The idea of a rotating ring [in French: *anneau tournant*] has a great advantage when trying to explain some issues [in the theory of an electron]: **it would not emit any electromagnetic radiation. It would only produce a magnetic field in the immediate space that surrounds it.** [...]" (H.A. Lorentz, 1921 Solvay Conference, boldface and italics added)

Indeed, the analysis of a superconducting ring is quite similar to the analysis of the (likely) structure of an electron! In any case, we cannot dwell on this here. The point is this: there is no need to think that the field, while quantized in (flux) units equal to $h/2q_e$, should, somehow, consist of discrete units (field quanta or virtual particles or whatever you would want to think of) *in space*.

So why, then, did quantum physicists go this way after WWII? Why did they develop these weird quantum field theories?

We honestly have no idea. We wrote about Yukawa's contribution to quantum physics last year[12], and we do not think we should add anything to that. As for now, you should just note the following:

1. Fields are real: they are *relative*, of course – as relative as energy or momentum – but they are equally real.
2. There is no reason to assume fields are discrete. On the contrary, they seem to be continuous. As such, they effectively *fill* the vacuum.
3. Fields are not to be confused with light-like particles – photons and, in our particular interpretation of quantum physics[13] – neutrinos. We may think of them as *traveling* fields,

[12] See: The Nature of Yukawa's Nuclear Force and Charge, followed by: Who Needs Yukawa's Wave Equation? (June 2019). We may or may not (probably not) explain the gist of these papers in this book.

[13] See our Principles of Quantum Physics (June 2020).

as long as we make sure we do not confuse them with the concept of static and/or dynamic fields as used in most analyses.

Fields and potentials

We said fields are relative but real. They are relative – Einstein's relativity theory applies to them – but as real as concepts that are not relative, such as the speed of light, the quantum of action, and the elementary charge, which we *measure* the same regardless of the reference frame. Combining relative and absolute concepts with mathematical constants and shapes, we get a set of equations which gives us a *representation* of reality. This set of equations, too, does not depend on your reference frame: the facts are the facts.

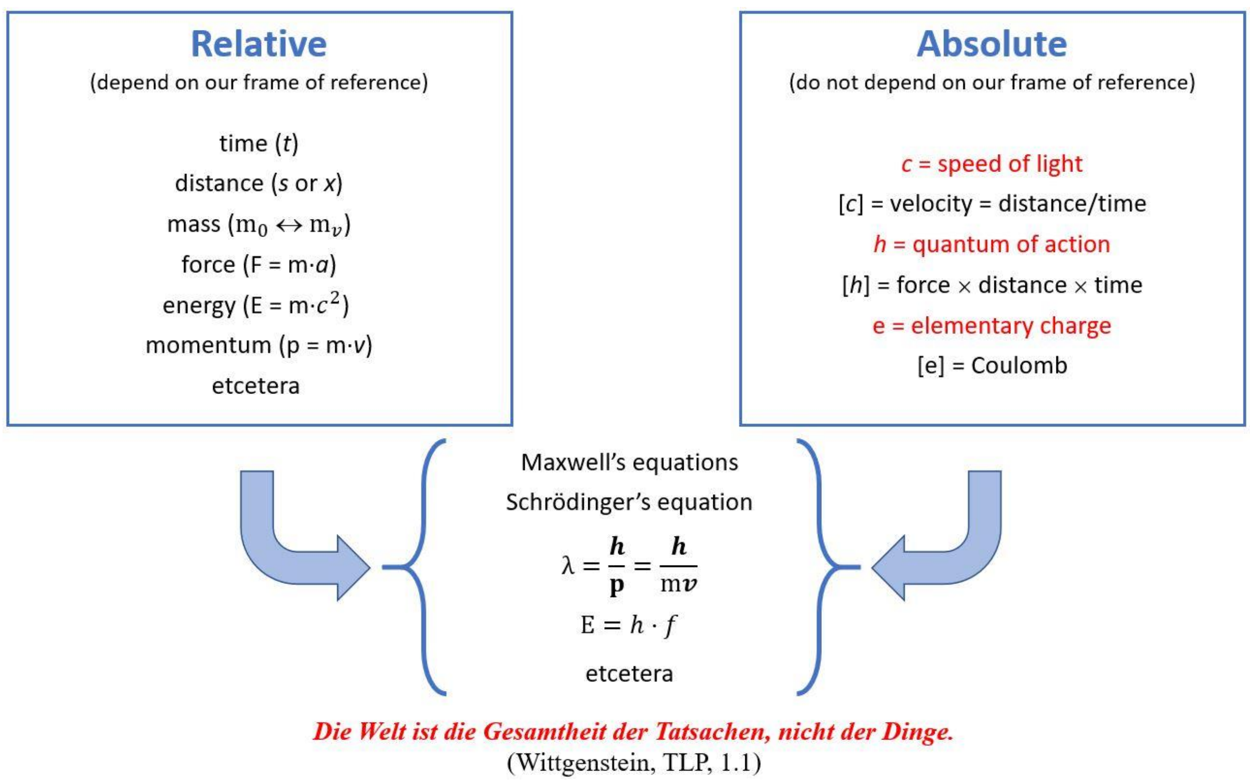

Figure 2: Relative versus absolute concepts: reality is represented by equations

Now, the ***E*** and ***B*** vectors have three components each: E_x, E_y, E_z and B_x, B_y, B_z respectively, each of which is a function – obviously – of $\boldsymbol{x}$ = (x, y, z) and t, in our reference frame, obviously (there is no need to switch to some other reference frame right now). ***E*** and ***B*** appear in Maxwell's equations, and we could keep analyzing things in terms of ***E*** and ***B*** but, as you know, we can *simplify* the math because ***E*** and ***B*** behave pretty regularly mathematically. We can, therefore, apply a few theorems (Gauss and Stokes) and conventions[14] and rewrite Maxwell's equations not in terms of ***E*** and ***B*** but in terms of the

[14] You will probably have heard about *gauge* theories – the need to *choose* a gauge when developing your field theory. In this particularly simple context, it amounts to the following. The whole derivation starts by writing ***B*** as the curl of some other vector field: $B = \nabla\times\boldsymbol{A}$. Now, any $\boldsymbol{A}' = \boldsymbol{A} + \nabla\psi$ (where ψ is any scalar field) will do, so we need to make some *choice*. This combines with some other choice we need to make when rewriting ***E*** in terms of a Φ (scalar potential) and an ***A*** (vector potential). We refer to Feynman's from [Feynman's *Lectures*], II-18-6 for the full

scalar potential Φ and the *vector* potential $\boldsymbol{A} = (A_x, A_y, A_z)$. So we have four variables depending on $\boldsymbol{x}$ and t now instead of six. Now, Φ and $\boldsymbol{A}$ can be combined using four-vector notation – $A_\mu = (\Phi, \boldsymbol{A}) = (\Phi, A_x, A_y, A_z)$ – and then all of the physics in Maxwell's equations can be written in one single equation:

$$\Box^2 A_\mu = \frac{j_\mu}{\epsilon_\mu}$$

As Feynman notes, the beauty of this equation is that it shows the invariance of electrodynamics under a Lorentz transformation, because four-vector dot products are invariant: they have the same value in every reference frame.[15] To put it simply: this represents the *reality* of electromagnetic fields, regardless of how we *measure* them. Now, four variables are not always easier to work with than six, but quantum physicists prefer the four, so it is the scalar and vector potentials Φ and $\boldsymbol{A}$ that are used in, say, Schrödinger's equation for a particle with charge q moving in some electromagnetic field. Now that we are here, we may as well write it down[16]:

$$-\frac{\hbar}{i}\frac{\partial\psi}{\partial \mathrm{t}} = \mathrm{H}\psi = \frac{1}{2\mathrm{m}}\left(\frac{\hbar}{i}\boldsymbol{\nabla} - \mathrm{q}\boldsymbol{A}\right)\cdot\left(\frac{\hbar}{i}\boldsymbol{\nabla} - \mathrm{q}\boldsymbol{A}\right)\psi + \mathrm{q}\boldsymbol{A}\psi$$

$$\Leftrightarrow i\hbar\frac{\partial\psi}{\partial \mathrm{t}} = -\frac{1}{2\mathrm{m}}(i\hbar\boldsymbol{\nabla} + \mathrm{q}\boldsymbol{A})^2\psi + \mathrm{q}\boldsymbol{A}\psi$$

$$\Leftrightarrow \mathrm{H} = i\hbar\frac{\partial}{\partial \mathrm{t}} = -\frac{1}{2\mathrm{m}}(i\hbar\boldsymbol{\nabla} + \mathrm{q}\boldsymbol{A})^2 + \mathrm{q}\boldsymbol{A}$$

Physicists – Feynman included – will tell you this equation is non-relativistic and excludes the idea of spin because the 1/2 factor comes out of some weird interpretation of the *effective* mass of an electron[17], which has nothing to do with it: the solutions to Schrödinger's equation for electron orbitals are actually solutions for orbitals with two electrons, so that is what explains it: Schrödinger's equation for an atom – with the electrostatic potential and the 1/2 factor works perfectly fine because it models electron orbitals for two electrons. Those two electrons have opposite spin, and the mechanism that makes superconducting electrons (Cooper pairs) or electron pairs in atomic/molecular orbitals move in pairs is exactly the same: by aligning spin, they can lower the energy of the *ensemble*. The Pauli

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development, but the point is this: we will choose a *gauge* for electromagnetics by choosing $\boldsymbol{\nabla}\cdot\boldsymbol{A}$ to be equal to $\boldsymbol{\nabla}\cdot\boldsymbol{A} = -\frac{1}{c^2}\frac{\partial\phi}{\partial t}$. It is referred to as the Lorenz gauge – without *t* in the name because the man who thought of this is not H.A. Lorentz from the Netherlands, but the Danish physicist Ludvig Lorenz. I am grateful to a reader who pointed this out to me because the *t* is usually there – in the original 1963 print edition of the *Lectures*, for example!

[15] The reasoning should be somewhat more subtle here – the dot product is *not* very obvious here! – but the reader should be able to easily fill the gap in the explanation.

[16] See: Feynman's Lectures, III-21-1. The transformation (1/*i* equals –*i*, and we also separate the operators from the wavefunction ψ) and the writing of it as an equality of operators is ours. Please do check for any mistake that may or may not have been made in the formulas.

[17] See my paper on matter-waves, signals and amplitudes.

exclusion principle might and should, therefore, probably be rephrased in terms in terms of the pairing mechanism of elementary particles with opposite spin.[18]

Schrödinger's equation: what does it model, *exactly*?

This is a question which is easy and difficult to answer at the same time! Easy, because we obviously get Dirac's 'equations of motion' for charged particles out of them! So the solutions to Schrödinger's equation (not Dirac's[19]) gives us electron orbits – for paired-up electrons with opposite spin. Schrödinger's equation works when modeling a hydrogen atom (atomic orbitals), and it also works in the context of perpetual currents in superconductors. It should work generally, but more work is needed to show it does.

We also need to do some more thinking on *how and why exactly* it does what it does. We made a start with that[20], but we readily admit we are still operating more on the basis of intuition – it cannot be anything else, can it? – than on the basis of firm mathematical proof. But, yes, equations of motion of charged particles in static and/or dynamic electromagnetic fields. No mystery. No hocus-pocus.

Jean Louis Van Belle, 19 October 2020

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[18] As for wavefunction (and, therefore, wave equations) not incorporating the concept of spin, see: Euler's Wavefunction and the Double Life of –1.

[19] Dirac's equation for free electron does not work, as he admits himself: it gives us 'run-away electrons' (Dirac's terms for the dissipating wave packets). We elaborate on the why and how here in our paper on de Broglie's matter-wave: concept and issues.

[20] See: A geometric interpretation of Schrödinger's wave equation.