On How to *(Properly)* Measure a Circle

(*Without the Need/Inclining* for "*Approximation*")

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Abstract

This investigation is a product of the ongoing scientific inquiry ' whence human suffering? ', the same encountering a critical need to call into serious question the long-standing π "approximation" methodolgy (ie. of exhaustion) employed by (and ever since) Archimedes (late, c. 287 – c. 212 BCE).

To begin, the author draws attention to an important inquiry: ' does π ever *naturally emerge* as a *product* of a *square? '* If so, it must be *measureably so* such to *negate* any/all need/inclining for "approximation" methodology(s) employing the use of multiple straight-edged polygons. Now consider the quadratic:

$$x^2 - x - 1 = 0$$

and find it to have positive solution $x = (1+\sqrt{5})/2$ *which*, as the reader may recognize, is the so-called *golden ratio* (hence: Φ). By expressing Φ in/on a *base* of 2π (for general applicability to *rotational motion*):

$$\Phi = (\boldsymbol{\pi}+\pi\sqrt{5})/2\pi = \mathbf{1}.618...$$

and then *squaring*:

$$\Phi^2 = (\mathbf{3}\boldsymbol{\pi}+\pi\sqrt{5})/2\pi = \mathbf{2}.618...$$

we find a numerator difference (ie. a matter) of a *discrete* **2π**:

$$\Phi^2 - \Phi = \mathbf{2}\boldsymbol{\pi}/2\pi$$

and so we have an answer to the previous inquiry: 2π *discretely* emerges as a *natural product* of a *square* (if/when on a base of *itself*).

Concerning Φ: there are non-trivial (universally unique) properties it possesses as *intrinsic* - it is the only positive number (*irrational,* no less) whose *reciprocal* is *precisely* one *less* than itself:

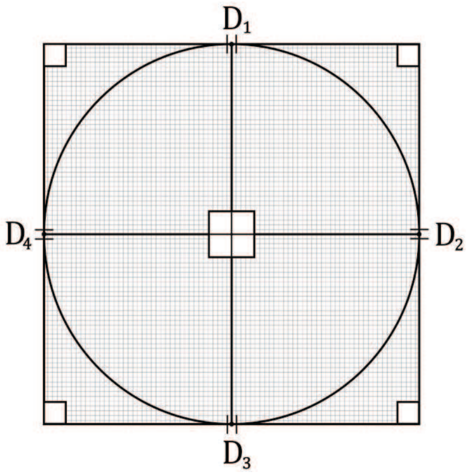
$$\Phi = (1+\sqrt{5})/2 = 1.618...$$
$$\mathbf{1}/\Phi = (\Phi - \mathbf{1}) = \mathbf{0}.618...$$

and (as we previously encountered) Φ is the only positive number whose own square is *precisely* one *greater* than itself:

$$\Phi = (1+\sqrt{5})/2 = 1.618...$$
$$\Phi^2 = (\Phi + \mathbf{1}) = \mathbf{2}.618...$$

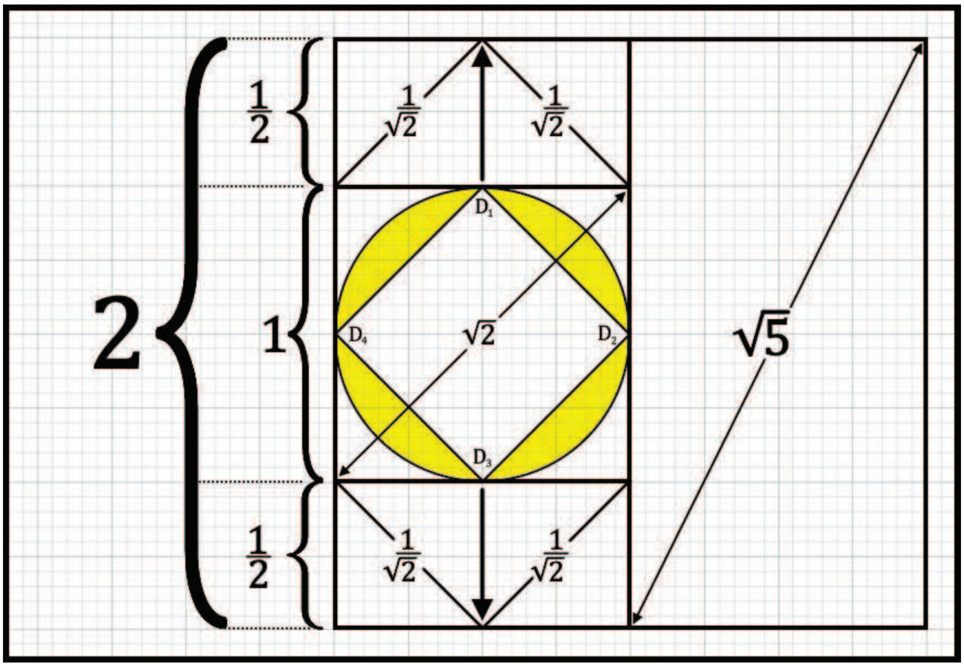
If π is a natural product of a square, we must be able to utilize the geometry implied by Φ such to *precisely* measure this *emergent* π and, importantly: do so *without the need/inclining for"approximation"*.

Prior to this endeavor, the author implores the reader to *suspend* (if even temporarily) any/all hitherto taken-to-be-true notions concerning π: both *quantitative* and *qualitative*.

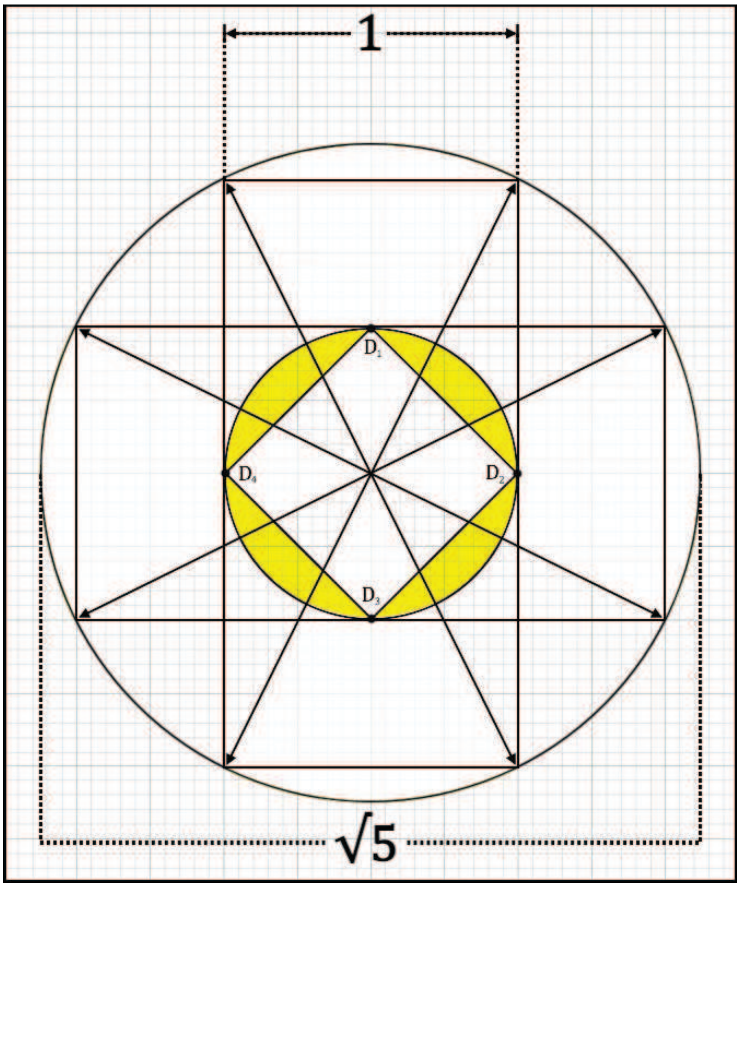


The square is composed of *four* equal sides whose interior angles are *four* right angles. The circle is composed of *four* symmetrical quarters whose axial radii *also* compose *four* right angles. By way of inscribing a circle of diameter d = 1 (*equiv.*: r = 1/2) inside the unit square s = 1, we find *four axially* situated points (D_{1-4} shown above) dividing the circumference of the circle into *four equal quarters* (each c/4 wherein c = π). These *four* critical points both *simultaneously* and *geometrically* correlate the r = 1/2 *circle* with the unit *square* s = 1. *Further,* these same points compose the square whose side lengths are equal to the *reciprocal* of $\sqrt{2}$ viz. s = $1/\sqrt{2}$, noting:

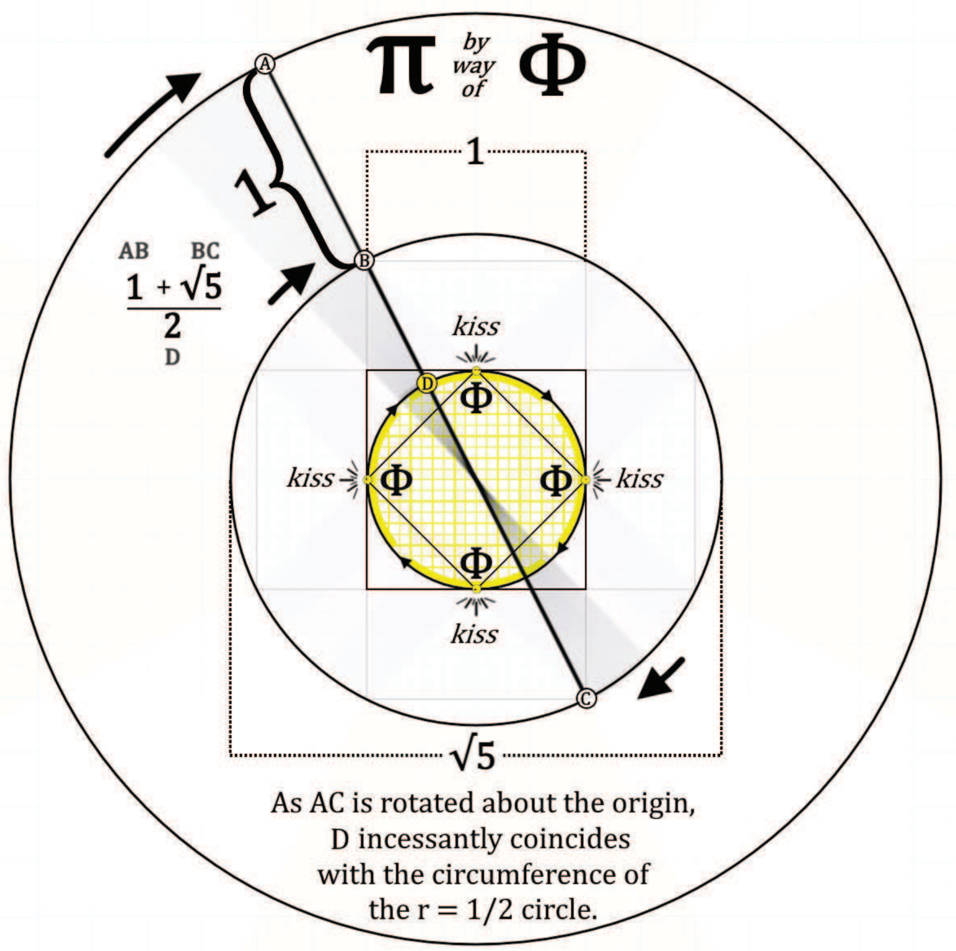
$$1/\sqrt{2} = \sqrt{2}/2$$



By extending any two opposing sides of the unit square s = 1, we obtain the remaining constituents of Φ: √5 (as the diagonal of the resulting 2x1 rectangle) and (a division by) 2. This extension of the unit square can be performed on *both sides* wherein the 8 vertices of both 2x1 rectangles can be used to compose another *larger* circle whose diameter is *equal* to any √5 diagonal:



By extending the √5 diameter circle in *all directions* by one (1) *discrete* unit, we find the *real* geometric basis underlying the circumference of the r = 1/2 circle (such to measure):



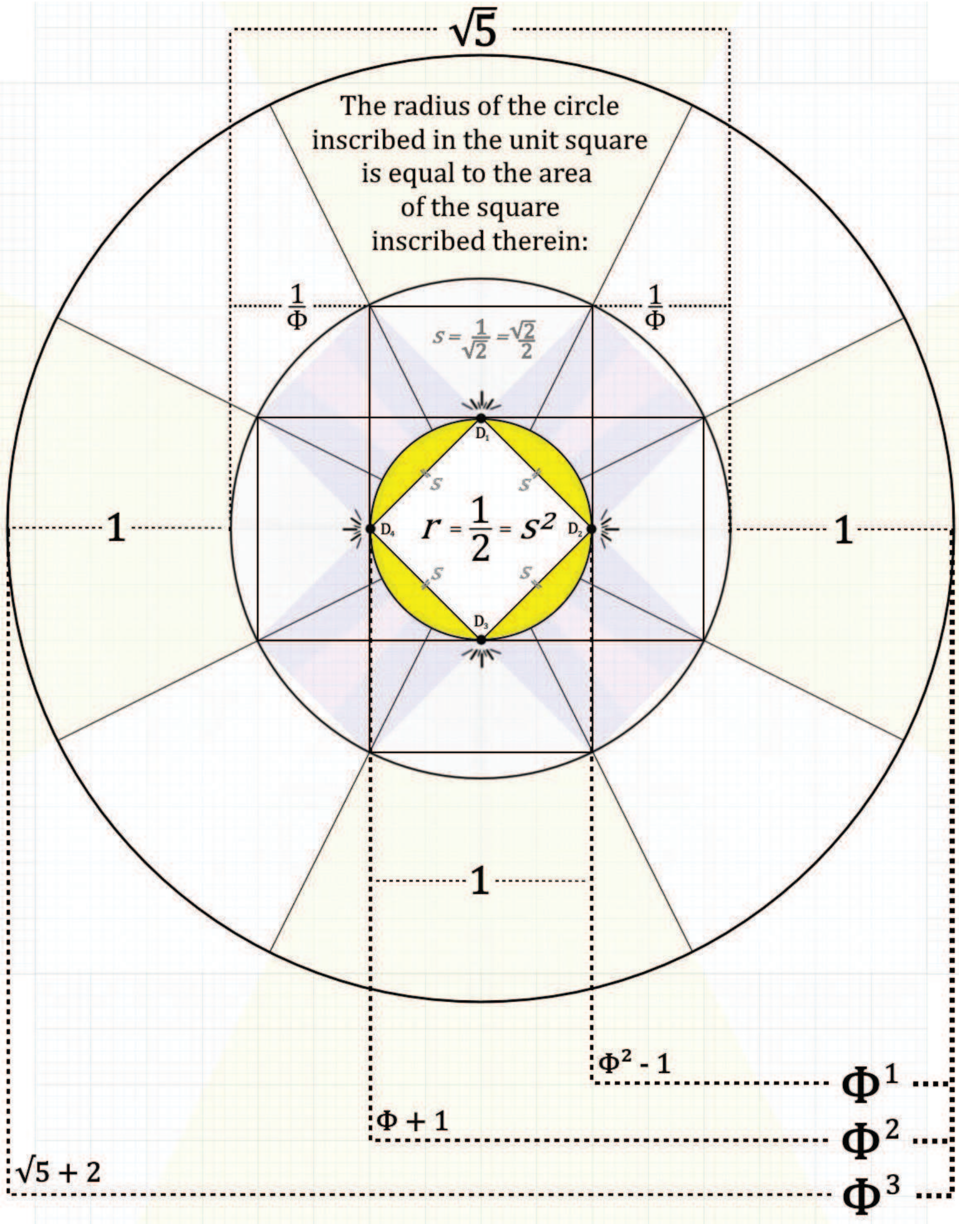
Upon one (1) full rotation (2π), D (= Φ) *incessantly coincides* with the *full circumference* of the r = 1/2 circle while "kissing" each of the *four* sides of the unit square *equidistantly*. The real geometric square underlying this relation can be obtained arithmetically via:

$$\frac{\sqrt{4\left(\frac{1+\sqrt{5}}{2}\right)}}{2} = \frac{\sqrt{2\left(1+\sqrt{5}\right)}}{2} = \sqrt{\Phi}$$

wherein the irrational √Φ has an underlying magnitude(s) of ±1.27201964... and whose own *reciprocal* (renormalizing to 1) is:

$$\frac{2}{\sqrt{2\left(1+\sqrt{5}\right)}} \times \frac{\sqrt{2\left(1+\sqrt{5}\right)}}{2} = 1$$

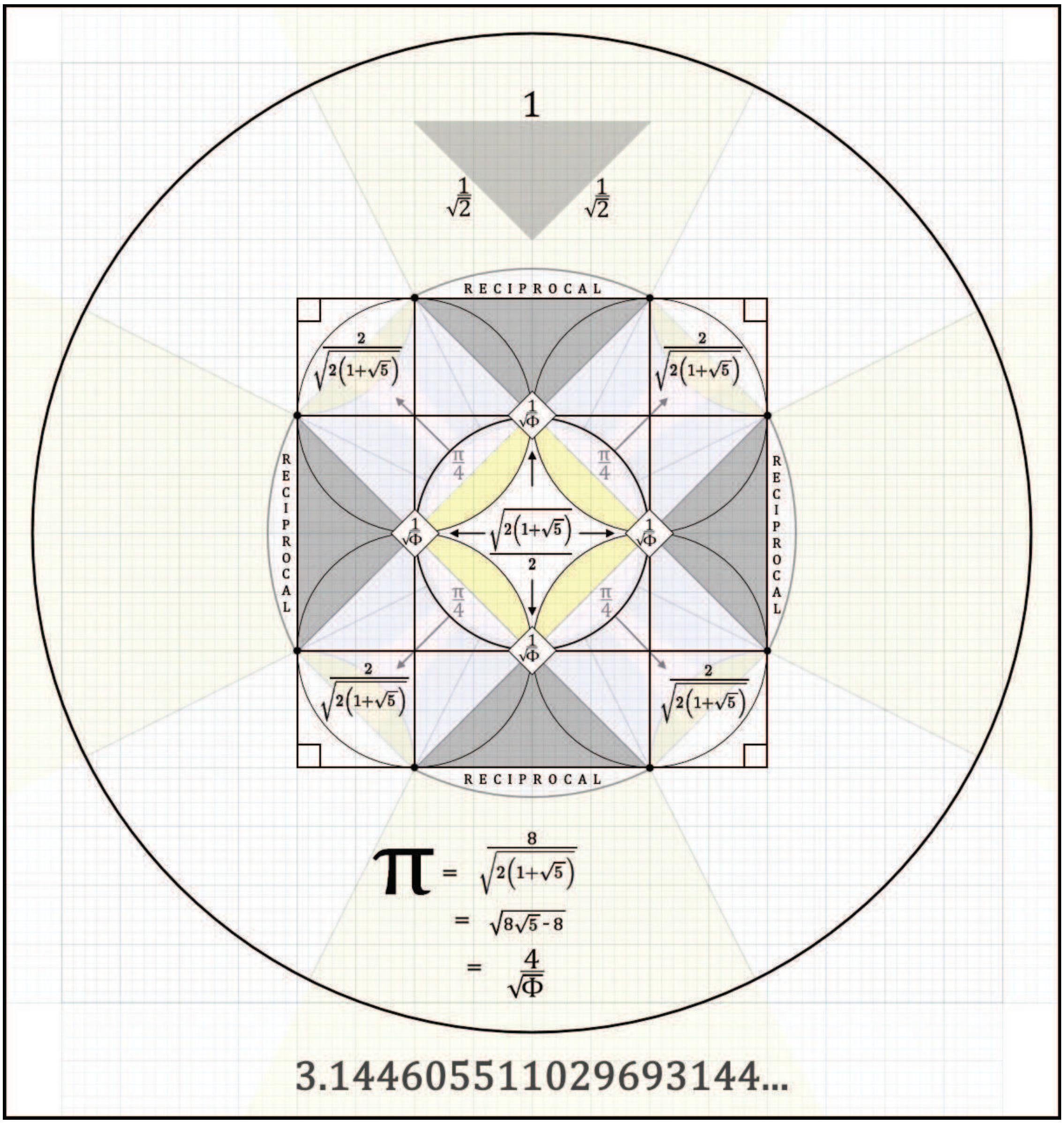
If/when plotting the first three powers of Φ (as they relate to the geometry we are working with):



the square of the golden ratio can be seen to geometrically coincide with a *real* diameter (2r) of a *real* circle in *real* relation to a *real* square(s) of *equal area* - the emphasis on *real* being as (in) contrast to "transcendental". A *real* circumference of a *real* circle (ie. π) can *not* possibly be "transcendental" if possessing a *real geometric radius*. The area of the inscribed square (whose vertices are D_{1-4} as shown) is equal to the *radius* of the circle *viz.* r = 1/2 = s^2.

We began by correlating the four right angles of the square to the four axial radii of the circle, the latter dividing π into four symmetrical quarters (each π/4). We observed the four associated axial points to *simultaneously* correlate the square s = 1 with the circle r = 1/2 and found them to be vertices of the square s^2 = 1/2. We also found how the real circumference of the r = 1/2 circle *naturally emerges* by way of *rotational motion* utilizing the *real* geometry implied by Φ.

We may now obtain the exact circumference of the r = 1/2 circle by observing the nature of the relationship *between* √Φ and **π/4**:

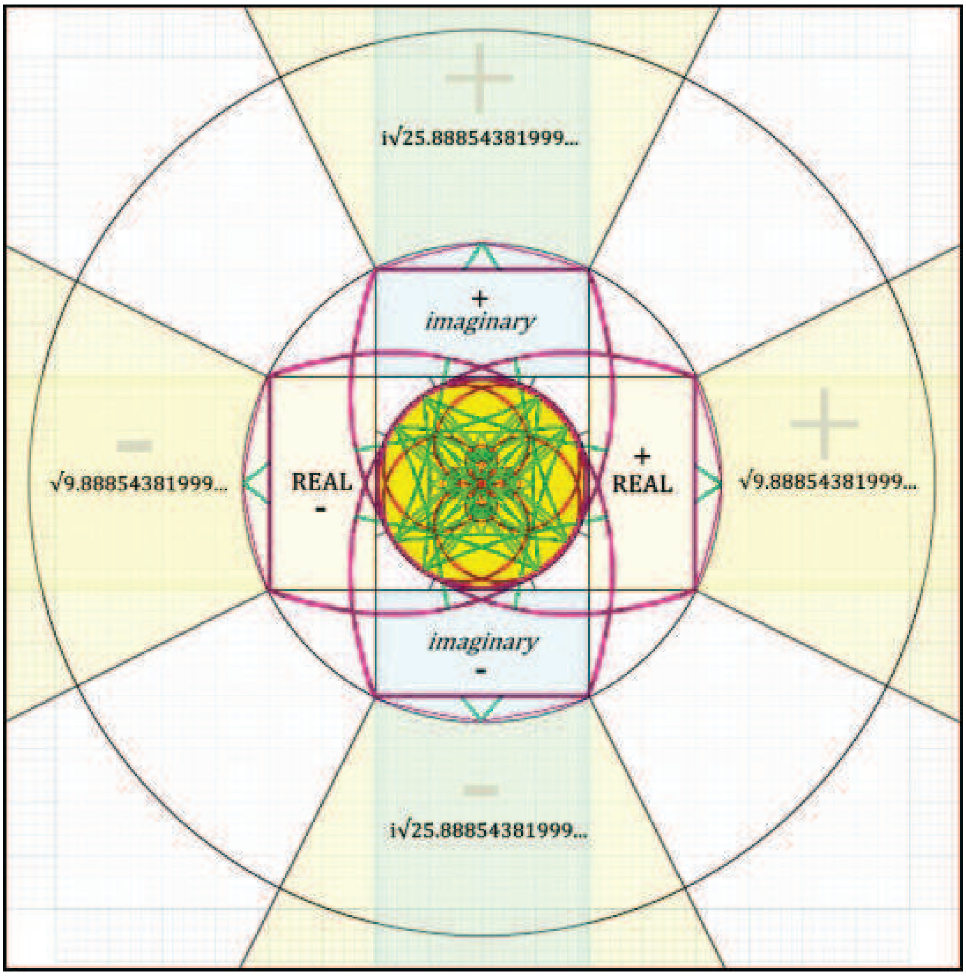


π ≠ 3.14159265358979...
(*human approximation error*)

Line and curve are resolutely
***reciprocally related*:**
$1/\sqrt{\Phi} = \pi/4$
*"...from Φ's own **root** is derived π..."*

The author wishes to impart that Archimedes' *"approximation"* methodology *catastrophically* misses an *entire constituency* of the circle (albeit small, *non-trivially* so). A *real*, symmetrical 1000*mm* diameter circle will *certainly* have a *real* circumference *greater* than ~3141.6*mm* (ie. the latter is *too short*). Should this ever become a source of dispute, the author suggests a simple experiment such to resolve: *actually measure a real 1000mm diameter circle*, and should it discretely measure (any) *more* than c = 3141.6*mm*, the same would *resolutely* demonstrate the *deficiency* of a "transcendental" π of 3.14159... as $4/\sqrt{\Phi}$ is a *real root* of an *integral function*:

$$f(x) = x^4 + 16x^2 - 256$$



It is the opinion of the author that the very notion π is somehow "transcendental" (let alone "proven" to be so) is *absolutely absurd.* A *real circle* is composed of a *real radius* relating four discretely *real* loci. While the "approximated" number of 3.14159... is *indeed* "transcendental," it is so for a simple reason: it is *not* π, but an "approximation" of π *deficient* from the thousandth decimal place. Because Φ is geometric, π follows, as from the *root* of the former do we *derive* the latter *naturally* by way of *reciprocity viz.* $1/\sqrt{\Phi} = \pi/4$.

As for the so-called golden ratio: the author suggests stripping it of any/all exotic and/or esoteric notions, and rather to focus on the *real* underlying mechanics (ie. the *practicality* of the relation). The Φ ratio uniquely possess a self-similarity (fractal) property, thus the presence of it should be readily observable in DNA/atomic fine structures incl. initial excited states of atoms (such as hydrogen).

The geometric union of Φ and π is reflected in/as the above integral function: the real/imaginary roots reflect a *discrete* rational integer *difference* of '16'. The real element is imperatively fixed to the integral ratio of 1/2 as this constitutes the *real, scalar* constituency of a *real radius,* the same 1/2 to be found in/of:

$1/2 + \sqrt{5}/2 =$

("*real" terminating rational*)

+

("*imaginary" non-terminating irrational*)

In other words: all *real* circumferences of all *real* circles *resolutely* possess a *real, scalable* base of 1/2 (such to scale *from)* and *only* the golden ratio permits/employs such a *universal scalability*.

Thus as it concerns the outstanding Riemann Hypothesis problem; in particular, the underlying non-trivial question:

"for which s does ζ(s) = 0?"

the *problem* (ie. question) is outstanding due to the catastrophically culprit "approximation" (ie. *deficieny*) of π. In short: Euler's famous solution to the Basel problem deriving a ζ(2) involves a sin(x)/x relationship, thus implies (radians in terms of) a π of 3.14159...

While the solution fits a *mathematically constructed* "reality" upon a "transcendental" π of 3.14159... the *real unrecognized problem* is the *real, physical universe* does *not* adhere to such an "approximated" (let alone "transcendental") π. For this reason, the hypothesis *itself* is not (only*)* a *problem*, but *in reality* a *symptom* of a much *deeper underlying problem (*hitherto measurable over a span of *at least* ~2200 years): a *deficient* π as due to a *deficient* "approximation" methodology.

The underlying magnitude of such a *blunder* (of millenia) compels the author to sympathetically hypothesize: the Riemann Hypothesis problem shall *not* be solved until humanity *consciously acknowledges* the underlying "approximation" *deficiency* in/of a π of 3.14159...

Finally, as for the concerned inquiry ' *whence human suffering?* ', though the real underlying root lies beyond the limited scope of this *investigation*, for the purposes of the *latter alone* (suffice it to say): as a *natural consequence* of a general failure(s) to incessantly <u>challenge basic underlying assumptions</u> (*incl.* substance of *"beliefs"*), human beings *suffer* knowing not *how to (properly) measure a circle*, as:

π ≠ 3.14159...

$$\pi/4 = 1/\sqrt{\Phi}$$
$$\pi = 4/\sqrt{\Phi}$$
$$\pi^2 = 16/\Phi$$
$$\mathbf{16 =} \Phi\pi^2$$
$$(e = MC^2)$$
$$1 = \Phi\pi^2/16$$
$$\mathbf{1 = \Phi(\pi/4)^2}$$

∴ π is *not* "transcendental"

such an endeavor provides a *rational* means to *discern* what is *real* from what is *not* (the same needed to discern a *real* π from an *imaginary* "transcendental" one*).* Whereas the latter is a measure of *millenia* of *human ignorance,* the former *rationally clarifies* the *universal constancy(s)* of *both* Φ and π as: *not* two, but *one*.