# Gravino theory: a simple physical model to explain gravity 

András Miklós<br>andras.grav@gmail.com


#### Abstract

A simple physical model called gravino theory is presented in this paper to explain gravity. The theory is based on the following three fundamental assumptions: 1) the Universe is filled by extremely small particles that move with the speed of light and are weakly absorbed by the material; 2) this gravino gas emits very weak electromagnetic radiation; 3) the speed of light depends on the local drift velocity of the gravino gas. Consequences of the assumptions and interesting results obtained from the gravino theory are discussed in the paper.


## Introduction

This paper is not a scientific work in the usual form; it is rather an overview of the author's exceptional approach to the question of gravity. The new ideas appeared casually during the professional life of the author, but they were put aside for later consideration. Slowly an increasingly coherent theory evolved from these ideas. This theory can explain the well-known properties of gravity, and it provides simple explanations for other phenomena. Since the article refers only to knowledge widely used in physics, a list of references is not included.
There are two generally accepted theories to explain the phenomenon of gravity: Newton's classical theory and Einstein's general theory of relativity. Newton envisioned mutual attraction to the masses as a force of infinite range and infinitely fast propagation. According to the general theory of relativity, there is no gravitational force; the masses modify the geometry of the space and move along inertial orbits in the curved space. This theory can explain three phenomena for which Newton's theory cannot provide an explanation; the perihelion shift of the Mercury orbit, the deflection of the light beam near the Sun, and the redshift of the wavelength of light due to gravity.

Observations of the redshift of light from distant galaxies have shown that the farther away the galaxy is, the greater its redshift. Assuming a redshift due to the Doppler shift, Hubble concluded that the velocity of distancing of galaxies from the earth is proportional to their distance (Hubble's law). The expanding universe and the Big Bang theories are based on Hubble's law. Cosmic microwave background radiation (CMBR) at 2.73 K is considered as an experimental evidence for the Big Bang theory.
Recent observations that the expansion of the Universe accelerates have led to the assumption that there must be some other constituent ("dark energy") in the Universe, which creates a repulsive effect. It is calculated that the share of "dark energy" in the Universe should be around $70 \%$.

The author had an aversion to the fundamental concepts of both Newton's and Einstein's theory, so he considered how to find a simpler physical model capable of reproducing Newton's law and also explaining the three relativistic phenomena mentioned above. Instead of infinite-range forces and curved space, the author preferred a model based on local interactions. A brief summary of the developed theory and the conclusions to be drawn from it are presented in the following chapters.

## 1. Essentials of the new theory

The basic assumptions of the theory can be summarized as follows:
The space is filled by extremely small particles, which can fly very large distances through matter without any interaction. By analogy with the neutrino, these particles are called gravinos, while the presented new theory is called gravino theory.

The mean velocity of gravinos is $c_{0}$, the speed of light in vacuum.
The gravino gas has a very low average mass density $\rho$. However, the number density $n$ of the gravinos is large, because the mass $\mu$ of a single gravino is extremely small. The gravino gas can be treated similarly as a gas in the classical physics; although the gravinos move with a mean velocity $c_{0}$, their drift velocity $u$ (the average velocity in a small volume) is very small, or even zero. The gravino carries a mass $\mu$, a momentum $\mu c_{0}$ and an energy of $\mu c_{0}{ }^{2}$.

The gravinos can fly a long path through matter without any interaction. However, a very small part of them is absorbed. It means that the number of gravinos moving towards a body is slightly larger than that of moving away. Because of the gravino absorption the absorber body gains mass and energy. On the other hand, the gravino gas around the body loses mass and energy, and a flow of the gravino gas (drift) towards the absorber body develops. Therefore, the density of the gravinos increases towards the body, too.
The mass and energy of the matter in the Universe increase continuously due to the absorption of gravinos. However, the net linear momentum transferred to an isolated object in the gravino field is zero because of symmetry reasons.

The situation is different if a body of mass $m$ is close to another body of mass $M$. Then fewer gravino arrives at $m$ from the direction $M$ than from the opposite direction. Because of this asymmetry of the gravino flow a net linear momentum is transferred to $m$. As a result, $m$ will be pushed towards $M$ by the gravinos.

Let the gravino density be $\rho$, the mass flow density of the gravinos absorbed in mass $M$ be $\mathbf{q}$. The equation describing gravino absorption is then:

$$
\begin{equation*}
\oint \mathbf{q} d \mathbf{S}=-\frac{d M}{d t} \tag{1}
\end{equation*}
$$

where $\mathbf{S}$ is a closed spherical surface around $M$.
Since no such continuous mass increase has been observed so far, the growth can only be very slow. It is also assumed that the growth rate is constant, i.e.

$$
\begin{equation*}
\frac{d M}{d t}=\alpha M \tag{2}
\end{equation*}
$$

The dimension of the constant $\alpha$ is s ${ }^{-1}$ and its value must be very small. Such a constant exists already in cosmology, the Hubble constant. Therefore, it is plausible that, in a first attempt, the Hubble constant is chosen as the $\alpha$ mass growth constant of the gravino theory.
Therefore, in the following, $\boldsymbol{\alpha}=\mathbf{2 . 3 4 \cdot 1 0} \mathbf{- 1 8}^{\mathbf{- 1}} \mathbf{s}^{\mathbf{- 1}}$, which corresponds to $72 \mathrm{~km} / \mathrm{s} / \mathrm{Mpc}$. Let us determine the mass flow for the body of mass $m$. Denote the density of gravinos coming from $M$ by $\rho_{+}$, and the density of gravinos coming from the other side to $\rho$. Then the $r$-component of the mass flow

$$
\begin{equation*}
q_{r}=\rho_{+} c_{0}-\rho_{-} c_{0} \tag{3}
\end{equation*}
$$

and the drift velocity is

$$
\begin{equation*}
u_{r}=\frac{\rho_{+} c_{0}-\rho_{-} c_{0}}{\rho_{+}+\rho_{-}} \tag{4}
\end{equation*}
$$

It is assumed further that the number of gravinos moving in different spatial directions is equal. Therefore

$$
\begin{equation*}
\rho_{+}+\rho_{-}=\frac{\rho}{3} \quad \text { and } \quad \mathbf{q}=\frac{\rho}{3} \mathbf{u} \tag{5}
\end{equation*}
$$

The mass flow at position $r$ created by the gravino absorption of $M$ can be calculated from (1):

$$
\begin{equation*}
\mathbf{q}(\mathbf{r})=-\frac{\alpha M}{4 \pi r^{3}} \mathbf{r}=-\frac{\alpha\langle\sigma(r)\rangle \mathbf{r}}{3} \tag{6}
\end{equation*}
$$

Here $\langle\sigma(r)\rangle$ is the average material density in a sphere of radius $r$.

## 2. Gravitational force

The force acting on a body of mass $m$ is the product of the momentum current density $\mathbf{q} c_{0}$ and the cross section $A_{\rho}$ of the gravino absorption of the body:

$$
\begin{equation*}
\mathbf{F}(\mathbf{r})=\mathbf{q}(\mathbf{r}) c_{0} A_{\rho}=\mathbf{q}(\mathbf{r}) c_{0} \eta m \tag{7}
\end{equation*}
$$

Here, it is assumed that $\underline{A_{\rho}}$ is proportional to the mass. So finally

$$
\begin{equation*}
\mathbf{F}(\boldsymbol{r})=-\frac{\alpha c_{0} \eta}{4 \pi} \frac{M m}{r^{3}} \mathbf{r} \tag{8}
\end{equation*}
$$

which is equivalent to Newton's law of gravitation if

$$
\begin{equation*}
\frac{\alpha c_{0} \eta}{4 \pi}=G \tag{9}
\end{equation*}
$$

where $G=6,674 \cdot 10^{-11} \mathrm{Nm}^{2} / \mathrm{kg}^{2}$ is the gravitational constant.
From this formula, the value of the constant of gravino absorption cross section can be calculated as:

$$
\begin{equation*}
\eta=1.20 \frac{\mathrm{~m}^{2}}{\mathrm{~kg}} \tag{10}
\end{equation*}
$$

The properties of equation (8) are discussed in more details in Annex I. The body of mass $m$ moves around $M$ on an orbit given in equation (A20b).
Although equation (8) is the same as the Newton's equation of gravity, a more detailed investigation reveals small deviations due to the orbital movement of body $m$ in the gravitational field of $M$. In the gravino theory, the gravitational effect has the finite speed $c_{0}$; therefore, the direction of the force is not completely opposite to the direction of the position vector. Consequently, very small corrections appear in the distance and in the angular momentum. Because of the transversal move of $m$, the cross section for gravino absorption slightly increases, too. A more detailed discussion of these effects is presented in Annex I. The main result is a very small increase of the period of the orbital movement; a shift of the perihelion of the orbit. The result of the calculation (see A27) is the same as obtained from the general theory of relativity.

## 3. Drift velocity and gravino density

Suppose that a moving body can be released from the attraction of a body of mass $M$ if its velocity reaches or exceeds the drift velocity of gravinos. That is, the so-called escape velocity is equal to the drift velocity:

$$
\begin{equation*}
u=\sqrt{\frac{2 G M}{r}}=\sqrt{\frac{2 \alpha c_{0} \eta M}{4 \pi} \frac{M}{r}} \tag{11}
\end{equation*}
$$

Written in vector notation

$$
\begin{equation*}
\mathbf{u}(\mathbf{r})=-\sqrt{\frac{2 \alpha c_{0} \eta}{4 \pi} \frac{M}{r^{3}}} \mathbf{r}=-\sqrt{\frac{2 \alpha c_{0} \eta}{3}\langle\sigma(r)\rangle \mathbf{r}} \tag{12}
\end{equation*}
$$

From equations (6) and (12) we can determine the gravino density:

$$
\begin{equation*}
\rho(\mathbf{r})=\frac{\boldsymbol{\alpha}\langle\sigma(r)\rangle}{\sqrt{\frac{2 \alpha c_{0} \eta}{3}\langle\sigma(r)\rangle}}=\sqrt{\frac{3 \alpha}{2 c_{0} \eta}\langle\sigma(r)\rangle}=\sqrt{\sigma_{0}\langle\sigma(r)\rangle} \tag{13}
\end{equation*}
$$

The quantity $\sigma_{0}$ has the dimension of density. It can be written in another form using formula (9):

$$
\begin{equation*}
\sigma_{0}=\frac{3 \alpha^{2}}{8 \pi G}=\frac{3 H^{2}}{8 \pi G} \tag{14}
\end{equation*}
$$

That is, $\sigma_{0}$ is nothing but the critical mass density used in cosmology.
Calculated with $\alpha=H=72 \mathrm{~km} / \mathrm{s} / \mathrm{Mpc}$ :

$$
\begin{equation*}
\sigma_{0}=9.75 \cdot 10^{-27} \frac{\mathrm{~kg}}{\mathrm{~m}^{3}} \tag{15}
\end{equation*}
$$

The gravino density on the surface of the Earth is

$$
\begin{equation*}
\rho\left(R_{E}\right)=\sqrt{5510 * 9.75 \cdot 10^{-27}}=7.33 \cdot 10^{-12} \frac{\mathrm{~kg}}{\mathrm{~m}^{3}} \tag{16}
\end{equation*}
$$

Since the density of nucleons is about $2.3 \cdot 10^{17} \mathrm{~kg} / \mathrm{m}^{3}$, the gravino density at a proton would be $\rho=2.24 \cdot 10^{-9} \mathrm{~kg} / \mathrm{m}^{3}$. The maximum gravino density at neutron stars can be about three times this value.

## 4. Gravity far from the mass

Since it is assumed in the gravino theory that the gravinos fly apart in empty space, this movement at a great distance from the masses will lead to a drift outwards. This asymmetry in the distribution of gravinos results in a repulsive force. Modify formula (6) as follows:

$$
\begin{equation*}
\mathbf{q}=\frac{\alpha\left(M_{\rho}-M\right)}{4 \pi r^{3}} \mathbf{r}=\frac{\alpha(\langle\rho(r)\rangle-\langle\sigma(r)\rangle) \mathbf{r}}{3} \tag{17}
\end{equation*}
$$

Then the force on the body of mass $m$ will be

$$
\begin{equation*}
\mathbf{F}=\frac{\alpha c_{0} \eta m\left(M_{\rho}-M\right)}{4 \pi r^{3}} \mathbf{r}=\frac{\alpha c_{0} \eta m(\langle\rho(r)\rangle-\langle\sigma(r)\rangle) \mathbf{r}}{3} \tag{18}
\end{equation*}
$$

Far from the mass $M$ its gravitational force will be repulsive. The radius $R_{G}$ of the gravitational attraction of $M$ can be calculated from the condition $M_{\rho}\left(R_{G}\right)=M$.
According to (13), the gravino density outside $M$ can be written in the following form:

$$
\begin{equation*}
\rho(r)=\sqrt{\frac{3 \sigma_{0} M}{4 \pi}} r^{-\frac{3}{2}} \tag{19}
\end{equation*}
$$

The gravino mass in a sphere of radius $R_{\mathrm{G}}$ is given by the following formula:

$$
\begin{equation*}
M_{\rho}\left(R_{G}\right)=4 \pi \int \rho(r) r^{2} d r \cong 4 \pi \sqrt{\frac{3 \sigma_{0} M}{4 \pi}} \int_{R}^{R_{G}} \sqrt{r} d r \cong \sqrt{\frac{16 \pi \sigma_{0} M}{3}} \sqrt{R_{G}^{3}} \tag{20}
\end{equation*}
$$

The gravitational action radius $\boldsymbol{R}_{\boldsymbol{G}}$ can be calculated as:

$$
\begin{equation*}
R_{G}=\sqrt[3]{\frac{3 M}{16 \pi \sigma_{0}}} \tag{21}
\end{equation*}
$$

Using this formula, the Earth's gravitational action radius is $R_{\mathrm{GE}}=3.32 \cdot 10^{16} \mathrm{~m} \cong 3.5$ light years, and that of the Sun is $2.30 \cdot 10^{18} \mathrm{~m} \cong 244$ light years.
According to (21) the gravitational action radius of a proton is $R_{p r}=0.217 \mathrm{~m}$.
If the average distance of the protons (or hydrogen atoms) in the Universe is larger than 0.217 meter, no gravitational attraction occurs between them; the interaction with the gravinos results in mutual repulsion. This is the case for a very dilute gas containing less than 23.4 protons pro cubic meter (a density under $3.91 \times 10^{-26} \mathrm{~kg} / \mathrm{m}^{3}$ ).

If the density of the material is the same as the above value, there is neither an attractive nor a repulsive interaction between the protons (hydrogen atoms); the particles move freely in space. Call this material density $\sigma_{\text {free }}$ :

$$
\begin{equation*}
\sigma_{\text {free }}=3.91 \cdot 10^{-26} \frac{\mathrm{~kg}}{\mathrm{~m}^{3}} \tag{22}
\end{equation*}
$$

To determine more accurately the gravino density and drift velocity, convert (17) as follows:

$$
\begin{equation*}
\mathbf{q}=\frac{\alpha(\sqrt{\langle\rho(r)\rangle}+\sqrt{\langle\sigma(r)\rangle})(\sqrt{\langle\rho(r)\rangle}-\sqrt{\langle\sigma(r)\rangle}) \mathbf{r}}{3} \tag{23}
\end{equation*}
$$

Multiply the first term in parentheses and divide the second one by $\sqrt{\sigma_{0}}$ :

$$
\begin{equation*}
\mathbf{q}=\frac{\left(\sqrt{\sigma_{0}\langle\rho(r)\rangle}+\sqrt{\sigma_{0}\langle\sigma(r)\rangle}\right)}{3}\left(\sqrt{\frac{\langle\rho(r)\rangle}{\sigma_{0}}}-\frac{\sqrt{\langle\sigma(r)\rangle}}{\sigma_{0}}\right) \alpha \mathbf{r} \tag{24}
\end{equation*}
$$

Comparing (24) with (5), one can conclude that the modified values of drift velocity and gravino density are:

$$
\begin{equation*}
\mathbf{u}(\mathbf{r})=\left(\sqrt{\frac{\langle\rho(r)\rangle}{\sigma_{0}}}-\frac{\sqrt{\langle\sigma(r)\rangle}}{\sigma_{0}}\right) \alpha \mathbf{r} \tag{25}
\end{equation*}
$$

$$
\begin{equation*}
\rho(r)=\left(\sqrt{\sigma_{0}\langle\rho(r)\rangle}+\sqrt{\sigma_{0}\langle\sigma(r)\rangle}\right)=\left(\sqrt{\frac{\langle\rho(r)\rangle}{\sigma_{0}}}+\sqrt{\frac{\langle\sigma(r)\rangle}{\sigma_{0}}}\right) \sigma_{0} \tag{26}
\end{equation*}
$$

Equation (25) corresponds to the Hubble Law of cosmology in the gravino theory. It does not describe the expansion of the universe, but the outward drift of the gravino gas. The Hubble constant also depends on the average gravino density and the average material density as follows:

$$
\begin{equation*}
H=\left(\sqrt{\frac{\langle\rho(r)\rangle}{\sigma_{0}}}-\frac{\sqrt{\langle\sigma(r)\rangle}}{\sigma_{0}}\right) \alpha \tag{27}
\end{equation*}
$$

Since, as one of the assumptions of the gravino theory, the present value of the Hubble constant is chosen for the gravino absorption coefficient $\alpha$, the value of the difference described in the parentheses is currently 1 . A simple solution could be:

$$
\begin{equation*}
H=(3-2) \alpha \tag{28}
\end{equation*}
$$

Then

$$
\begin{equation*}
\frac{\langle\rho(r)\rangle}{\sigma_{0}}=9 \quad \frac{\langle\sigma(r)\rangle}{\sigma_{0}}=4 \tag{29}
\end{equation*}
$$

The ratio of the average gravino density to the total density is

$$
\begin{equation*}
\Omega_{\rho}=\frac{\langle\rho(r)\rangle}{\langle\rho(r)\rangle+\langle\sigma(r)\rangle}=\frac{9}{13}=0.6923 \tag{30}
\end{equation*}
$$

and of the average material density is

$$
\begin{equation*}
\Omega_{\sigma}=\frac{\langle\sigma(r)\rangle}{\langle\rho(r)\rangle+\langle\sigma(r)\rangle}=\frac{4}{13}=0.3077 \tag{31}
\end{equation*}
$$

The density ratios $\Omega_{\rho}$ and $\Omega_{\sigma}$ are very close to the values $\Omega_{\Lambda}$ and $\Omega_{\mathrm{m}}$ of the dark energy and material density parameters ( $\Omega_{\Lambda}=0.6911 \pm 0.0062$ and $\Omega_{\mathrm{m}}=0.3089 \pm 0.0062$ ) of modern cosmology.

## This cannot be an accidental coincidence!

It is also interesting that in the gravino theory the average density of material in the Universe given in (29)

$$
\begin{equation*}
4 \sigma_{0}=3.90 \cdot 10^{-26} \frac{\mathrm{~kg}}{\mathrm{~m}^{3}} \tag{32}
\end{equation*}
$$

is almost exactly equal to the value of $\sigma_{\text {free }}$ calculated from the gravitational action radius of the proton (see Eq. (22)).

## 5. The 2.73 K background radiation

Another new idea of the gravino theory is the assumption that the gravino gas around the Earth emits weak electromagnetic radiation. It is assumed that a power $d W=w d V$ is emitted from a $d V$ volume element.

Because of the spherical symmetry of the gravino gas around the Earth, the power density $w$ of the radiation is only a function of the distance $r$.
Suppose further that the volume element $d V$ is a thin shell with a surface $d A$ and a thickness $d r$. Then the intensity radiated perpendicular to the surface $d A$ in the direction of the Earth is

$$
\begin{equation*}
d I(r)=\frac{(w(r) / 2) d r}{\pi} \tag{33}
\end{equation*}
$$

(half of the power is radiated to Earth, half to the opposite direction.)
The intensity reaching the Earth at a small solid angle $d \Omega$ does not depend directly on distance, because although the intensity decreases proportionally with the square of the distance, the radiating surface $d A$ increases with the square of the distance $\left(d A=r^{2} d \Omega\right)$. Therefore, the total intensity that hits the Earth's surface is

$$
\begin{equation*}
I\left(R_{E}\right)=\frac{1}{2 \pi} \int_{R_{E}}^{\infty} w(r) d r \tag{34}
\end{equation*}
$$

regardless of the direction from which the radiation comes.
It is assumed that the power density of electromagnetic radiation is proportional to the energy density of the gravino gas, and the proportionality factor is the same $\alpha$ as was used so far.

$$
\begin{equation*}
w(r)=\alpha \rho c_{0}^{2} \tag{35}
\end{equation*}
$$

Calculated with equations (13) and (35) the total intensity on Earth is

$$
\begin{equation*}
I\left(R_{E}\right)=\frac{\alpha c_{0}^{2}}{2 \pi} \int_{R_{E}}^{\infty} \rho(r) d r=\frac{\alpha c_{0}^{2}}{2 \pi} 2 R_{E} \rho\left(R_{E}\right) \tag{36}
\end{equation*}
$$

Considering this total intensity on Earth as thermal radiation, the Stefan-Boltzmann equation can be written as follows

$$
\begin{equation*}
I\left(R_{E}\right)=a T^{4} \tag{37}
\end{equation*}
$$

and

$$
\begin{equation*}
T^{4}=\frac{\alpha c_{0}^{2} R_{E} \rho\left(R_{E}\right)}{\pi a} \tag{38}
\end{equation*}
$$

where $a=5.6687 \cdot 10^{-8} \mathrm{~kg} / \mathrm{s}^{3} \mathrm{~K}^{4}$ is the Stefan-Boltzmann constant. Substituting the gravino density from Eq. (16) and the value of the Earth's radius $\left(6.371 \cdot 10^{6} \mathrm{~m}\right)$ for $T$ the following value is obtained:

$$
\begin{equation*}
T=\left(\frac{2.34 \cdot 10^{-18} \cdot\left(3 \cdot 10^{8}\right)^{2} \cdot 6.371 \cdot 10^{6} \cdot 7.33 \cdot 10^{-12}}{\pi \cdot 5.6687 \cdot 10^{-8}}\right)^{\frac{1}{4}}=2.726 \mathrm{~K} \tag{39}
\end{equation*}
$$

This value is equal to the $T_{C M B}$ value of the cosmic microwave background radiation in modern cosmology. Interesting, right?

It may be assumed that the energy of a gravino is probably in the order of thermal energy $k T$. (The value of $k T$ from (38) is $3.81 \cdot 10^{-23} \mathrm{~J}=0.24 \mathrm{meV}$ ). Starting from this, the gravino mass should be in the order of $10^{-40} \mathrm{~kg}$. The average particle density of gravinos in the Universe is then in the order of $10^{14}$ gravinos $/ \mathrm{m}^{3}, 12-13$ orders of magnitude higher than the average
density of material particles. The gravino particle density would be about $10^{28}$ gravino $/ \mathrm{m}^{3}$ on the Earth's surface and $10^{31}$ gravino $/ \mathrm{m}^{3}$ near a nucleus. That is, the particle density of the gravino gas is many orders of magnitude higher everywhere than the particle density of the matter.

## 6. Light propagation and redshift

It is supposed in the gravino theory that the gravino gas also affects the propagation of light. Suppose that the speed of light depends on the drift speed of the gravinos as follows:

$$
\begin{equation*}
c(r)=\sqrt{c_{0}^{2}-u(r)^{2}}=c_{0} \sqrt{1-\frac{u(r)^{2}}{c_{0}^{2}}} \tag{40}
\end{equation*}
$$

As a result, passing a body of mass $M$, the light beam deviates from its original direction. The detailed calculation can be found in Annex II. Here only the result is cited: the light beam passing at distance $a$ next to the mass $M$ deviates from its original direction by an angle of

$$
\begin{equation*}
2 \Delta \varphi=2 \frac{u^{2}(a)}{c_{0}^{2}}=\frac{4 G M}{c_{0}^{2} a} \tag{41}
\end{equation*}
$$

This value is exactly the same as that calculated on the basis of the general theory of relativity. No wonder, because according to equations (11) and (A10)

$$
\begin{equation*}
\frac{u^{2}(a)}{c_{0}^{2}}=\frac{2 G M}{a c_{0}^{2}}=\frac{r_{g}}{a} \tag{42}
\end{equation*}
$$

where $r_{g}$ is the so-called Schwarzschild radius. For this reason, the calculation presented in Annex II leads to the same result as the calculation according to the general theory of relativity.
However, the interpretation is very different. In the gravino theory, the dependence of the speed of light on gravino drift velocity causes light deflection, not the curvature of space.

In the gravino theory, the deflection of the light beam in gravitational field is a similar phenomenon to the phenomenon of the mirage on Earth; both phenomena are caused by the location-dependent speed of light.

The gravitational redshift of light can also be explained by the change of the speed of light. Assuming that the energy flow represented by the photons is constant during propagation and that the product of wavelength and frequency is also constant, that is

$$
\begin{equation*}
E_{S} c_{S}=E_{M} c_{M} \quad \text { and } \quad \lambda_{S} v_{S}=\lambda_{M} v_{M}=c_{0} \tag{43}
\end{equation*}
$$

for the position of the source (index S ) and measurement (index M ).
Using the relation $\mathrm{E}=h \nu$ we get the following result:

$$
\begin{equation*}
\frac{v_{M}}{v_{S}}=\frac{c_{S}}{c_{M}} \quad \text { and } \quad \frac{\lambda_{M}}{\lambda_{S}}=\frac{c_{M}}{c_{S}} \tag{44}
\end{equation*}
$$

According to the general theory of relativity, the redshift due to gravity between the location $r_{S}$ of the light source and the location $r_{M}$ of the observation in the gravitational field of the mass $M$ is given by the following formula:

$$
\begin{equation*}
\frac{\lambda_{M}}{\lambda_{S}}=\frac{v_{S}}{v_{M}}=\sqrt{\frac{1-\frac{2 G M}{c_{0}^{2} r_{M}}}{1-\frac{2 G M}{c_{0}^{2} r_{S}}}} \tag{45}
\end{equation*}
$$

Equation (45) is exactly the same as (44) because, according to eq. (11) $2 G M / r=u(r)^{2}$. Thus

$$
\begin{equation*}
\frac{\lambda_{M}}{\lambda_{S}}=\frac{v_{S}}{v_{M}}=\sqrt{\frac{c_{0}^{2}-u\left(r_{M}\right)^{2}}{c_{0}^{2}-u\left(r_{S}\right)^{2}}}=\frac{c_{M}}{c_{S}} \tag{46}
\end{equation*}
$$

The wavelength change (Doppler effect) of light from a moving body is taken into account as follows:

$$
\begin{equation*}
\frac{\lambda_{M}}{\lambda_{S}}=\frac{1+\frac{\mathrm{v}}{c_{0}}}{\sqrt{1-\frac{u\left(r_{S}\right)^{2}}{c_{0}^{2}}}} \tag{47}
\end{equation*}
$$

where $v$ is the speed of the moving light source ( v positive if the source is moving away, negative if it is approaching). If the source velocity is equal to the drift velocity of the gravinos, then

$$
\begin{equation*}
\frac{\lambda_{M}}{\lambda_{S}}=\sqrt{\frac{1+\frac{\mathrm{v}}{c_{0}}}{1-\frac{\mathrm{v}}{c_{0}}}} \tag{4}
\end{equation*}
$$

This is nothing more than the well-known formula for the Doppler effect in the theory of special relativity.

## 7. Other consequences

### 7.1 Gravino density and drift velocity inside of the Earth

In the gravino theory, equations (25) and (26) for drift velocity and gravino density are generally valid, also for the interior of bodies. Since the gravino density on the Earth's surface is in the order of $10^{-12} \mathrm{~kg} / \mathrm{m}^{3}$, the terms on the right side of equations (25) and (26), which depend on gravino density $\rho$, are negligible. Therefore

$$
\begin{equation*}
\mathbf{u}(\mathbf{r})=-\sqrt{\frac{\langle\sigma(r)\rangle}{\sigma_{0}}} \alpha \mathbf{r} \tag{49}
\end{equation*}
$$

and

$$
\begin{equation*}
\rho(r)=\sqrt{\sigma_{0}\langle\sigma(r)\rangle} \tag{50}
\end{equation*}
$$

In almost all cases, it is sufficient to use the approximate formulas (48) and (49) because the average densities of condensed materials are in the order of $10^{3} \mathrm{~kg} / \mathrm{m}^{3}$, while the maximum
density of gravinos, as seen above, is in the order of $10^{-9} \mathrm{~kg} / \mathrm{m}^{3}$. Therefore, the above approximations can be safely used for planets, stars, comets and dust particles, and the complete equations (25) and (26) are only needed when examining very dilute gas clouds. Inside the Earth, the gravino density increases slowly toward the center of the Earth, as the average density increases from $5510 \mathrm{~kg} / \mathrm{m}^{3}$ to about $13000 \mathrm{~kg} / \mathrm{m}^{3}$. Because of the square root dependence, the gravino density at the centre of the Earth is only about one and a half times as high as on the Earth's surface.
The drift speed, on the other hand, decreases towards the centre of the Earth and becomes zero there; the same number of gravinos arrives here from all directions, the symmetry is restored, therefore the drift disappears.

### 7.2 Gain of mass

According to the gravino theory, the mass of every solid body, including the mass of the Earth, grows in time due to gravino absorption. The solution of differential equation (2) is an exponential function

$$
\begin{equation*}
M(t)=M\left(t_{0}\right) e^{\alpha\left(t-t_{0}\right)} \tag{51}
\end{equation*}
$$

The growth is very slow at first; $1 \%$ weight gain takes about 135 million years. However, it is conceivable that the slow increase in the mass and volume of the Earth has led to the Earth's solid crust breaking into tectonic plates.

### 7.3 Black holes

Assume that a body with very large mass $M$ and radius $R$ may absorb all the gravinos flying towards it. In this case, the average (drift) velocity of the gravinos, according to equation (4), will be $c_{0}$, the speed of light in vacuum. Consequently, the speed of light on the surface of the body becomes zero according to equation (40). The light cannot leave the body, so it becomes an invisible black body.
By substituting $u$ by $c_{0}$ into (42) the following relations can be derived for such a body:

$$
\begin{equation*}
\frac{r_{g}}{R}=\frac{u(R)^{2}}{c_{0}^{2}}=1 \tag{52}
\end{equation*}
$$

that is, a body becomes a black body if its radius is equal to the Schwarzschild radius. This result is also the same as the one obtained from the general theory of relativity.

## 8. Summary

A simple physical model, called gravino theory, to explain gravity is presented in this paper.
The gravino theory is based on three fundamental and a few additional assumptions. The three fundamental assumptions are as follows:

1) The space is filled by extremely small particles moving at the speed of light. These particles are absorbed in a very small proportion in the material, increasing its mass and energy.
2) The gravino gas emits weak electromagnetic radiation, the power density of which is proportional to the energy density of the gravino gas.
3) The speed of light is not constant, but depends on the drift velocity of the gravino gas.

Additional assumptions of gravino theory:

- The gravino absorption constant of the matter is $\alpha=2.34 \cdot 10^{-18} \mathrm{~s}^{-1}$, the current value of the Hubble constant.
- The gravino absorption cross section of matter is proportional to the mass.
- In the immediate vicinity of a material body the drift velocity is equal to the escape velocity of the Newtonian gravity.
- The gravino gas expands in outer space, so at great distances the drift is directed outward and its velocity increases with distance.

From the 3 basic assumptions and additional assumptions listed above, the following consequences can be drawn:
$\checkmark$ Due to gravino absorption, the mass and energy of material bodies increase slowly but steadily.
$\checkmark$ The gravino gas pushes the material bodies towards each other. The derived law of force is formally identical to Newton's law of gravity.
$\checkmark$ Since it is mediated by gravinos, the gravitational effect propagates at a finite velocity $c_{0}$, the speed of light in vacuum.
$\checkmark$ The direction of the force points always in the direction of the drift speed of the gravino gas, so it can be not only attractive, but also repulsive.
$\checkmark$ The gravitational attraction has a finite action radius because the outward drift of the gravino gas overcompensates the inward drift over a long distance.
$\checkmark$ Near a material body, the gravino's density is proportional to the square root of the average material density.
$\checkmark$ The numerical value of the density constant ( $\sigma_{0}$ ) in Equation (13) is equal to the critical density of modern cosmology.
$\checkmark$ By comparing the drift velocity of gravino gas (see equation 25) with Hubble's law, it is possible to calculate the ratio of the average gravino density and the ratio of the average material density to the total density of the Universe.
$\checkmark$ Due to the spherically symmetrical density distribution of the gravino gas near the Earth, the gravino gas surrounding the Earth (or any other celestial body) radiates an electromagnetic radiation to the Earth's surface, the intensity of which is constant and does not depend on the direction of observation.
$\checkmark$ Because the speed of light depends on the local drift velocity of the gravino gas, two effects can be observed: the light beam deflects from its original direction and a redshift occurs.

The main results obtained from gravino theory and presented in this article are listed below:

- Equation (8) derived for the attraction between bodies is formally identical to the Newton equation. By making the constants of the two equations equal, the constant $\eta$ of the gravino absorption cross section is determined as $\eta=1.20 \mathrm{~m}^{2} / \mathrm{kg}$.
- In gravino theory, small corrections occur in Equation (8) due to the orbital motion of the body. These corrections lead to a slight shift in the perihelion of the orbit. The result of the calculation presented in Annex I is the same as the result obtained in the general theory of relativity.
- Density ratios $\Omega_{\rho}=9 / 13$ and $\Omega_{\sigma}=4 / 13$, calculated from a comparison of equation (25) for gravino drift velocity and Hubble's law, are the same as the density ratios $\Omega_{\Lambda}$ and $\Omega_{\mathrm{m}}$ in modern cosmology for "dark energy" and material, respectively.
- The material density calculated from the gravitational action radius of the proton (hydrogen atom) is equal to the material density $4 \sigma_{0}$ determined from the agreement of Hubble's law and the gravino drift velocity.
- The formula for the deflection of a light beam near an attractive mass, which was derived in Annex II by applying the Fermat principle (see (41) and (A46)), is the same as the result obtained in the general theory of relativity.
- For gravitational redshift, the same result is obtained in gravino theory as in general relativity.
- The combination of the classical Doppler effect and of the gravino-drift-dependent light speed gives the same formula for redshift as the special theory of relativity (if the speed of the light source is the same as the drift speed), but it also allows significantly different redshifts if the speed of drift and the speed of the light source are different. This may explain why many galaxies can be found whose redshift of light does not match the value predicted by Hubble's law.
- Regarding the electromagnetic radiation arriving from the gravino gas that surrounds the Earth as a blackbody radiation, a temperature of 2.726 K was derived. This value is in good agreement with the measurement results obtained for cosmic microwave background radiation.

According to gravino theory, every material body is exposed to the black body radiation from the surrounding gravino gas. However, equation (38) reveals that the temperature of the radiation depends on the average mass density and radius of the body, i.e it must be different for each body. Measurements of the temperature of radiation on other planets of the solar system could validate this prediction of the gravino theory.

Gravino theory explains gravity and the other related effects (change of the speed of light, light deflection, redshift, blackbody radiation) by the local interaction of matter and the gravino gas.
The gravino theory can explain the most important results of the Newtonian gravity and of the general relativity theory of Einstein without the need for a distant force (Newton) or a massdependent spatial geometry (Einstein).
By providing a simple explanation for the phenomenon of temperature background radiation and cosmic redshift, the gravino theory challenges the two most important experimental proofs of the theories of the expanding Universe and the Big Bang.
In gravino theory, the share of gravino gas density in the total density of the Universe is the same as the share of dark energy in total density according to modern cosmology. Therefore, gravino gas may be the dark energy of modern cosmology.

Finally, the author wishes to emphasize that the presented gravino theory is only a physical model based on 'ad hoc' assumptions. The author does not know what gravinos are or do they exist at all, and if they do, why they fly apart in space, why they are absorbed in matter, why they affect the speed of light, and why they emit electromagnetic radiation. However, the conclusions to be drawn from the theory based on these 'ad hoc' assumptions agree very well with experimentally validated results of existing theories. This is the usual way to gain scientific knowledge when direct observation is no longer possible. We build a model, draw specific conclusions from the model, and compare them with the results of the observations.
In this respect, the gravino theory is promising, so the author believes it is worthwhile to study this theory more closely and develop it further. If other researchers are encouraged to contribute, the present publication will have achieved its goal.

## Annex I.

## Perihelion shifts of planetary orbits

Equation (8) describing the gravitational attraction can only be considered as a first approximation because it was not taken into account that the body of mass $m$ is in motion. Let us therefore examine the properties of the equation more closely.

Dividing both sides of the equation by $m$ gives the equation for acceleration:

$$
\begin{equation*}
\mathbf{a}(r)=\ddot{\mathbf{r}}(r)=-\frac{\alpha c_{0} \eta}{4 \pi} \frac{M}{r^{3}} \mathbf{r} \tag{A1}
\end{equation*}
$$

Here the time derivation is denoted by a dot. Multiply both sides of the equation by the vector r:

$$
\begin{equation*}
\mathbf{r} \times \mathbf{a}(r)=\mathbf{r} \times \ddot{\mathbf{r}}(r)=-\frac{\alpha c_{0} \eta}{4 \pi} \frac{M}{r^{3}} \mathbf{r} \times \mathbf{r}=0 \tag{A2}
\end{equation*}
$$

Because

$$
\begin{equation*}
\mathbf{r} \times \ddot{\mathbf{r}}(r)=\frac{d}{d t}(\mathbf{r} \times \dot{\mathbf{r}})-(\dot{\mathbf{r}} \times \dot{\mathbf{r}})=\frac{d}{d t}(\mathbf{r} \times \dot{\mathbf{r}}) \tag{A3}
\end{equation*}
$$

therefore

$$
\begin{equation*}
(\mathbf{r} \times \dot{\mathbf{r}})=\mathbf{h} \tag{A4}
\end{equation*}
$$

where $\mathbf{h}$ is a time-independent vector. This equation means that the angular momentum is constant during motion.

The motion takes place in a plane perpendicular to the vector $\mathbf{h}$, so in the following a plane polar coordinate system will be used. The mass $M$ is located at the origin. The coordinates of the position, velocity, and acceleration vectors of the moving mass $m$ are given by the following formulas:

$$
\begin{gather*}
\mathbf{r}=(r, 0)  \tag{A5a}\\
\dot{\mathbf{r}}=(\dot{r}, r \dot{\varphi})  \tag{A5b}\\
\ddot{\mathbf{r}}=\left(\ddot{r}-r^{2} \dot{\varphi}, 2 \dot{r} \dot{\varphi}+r \ddot{\varphi}\right) \tag{A5c}
\end{gather*}
$$

Substituting the coordinates of the position vector and the velocity vector into (A4) gives the following result:

$$
\begin{equation*}
r^{2} \dot{\varphi}=h \tag{A6}
\end{equation*}
$$

Suppose that during the movement of the mass $m$ the angular rotation depends only on time, and the distance $r$ depends on the angle of rotation. That is

$$
\begin{equation*}
\frac{d}{d t} r(\varphi(t))=\frac{d r}{d \varphi} \frac{d \varphi}{d t}=\frac{h}{r^{2}} r^{\prime} \tag{A7}
\end{equation*}
$$

Here ' indicates the derivation according to $\varphi$.
Using equations (A6) and (A7), calculate the components of the velocity and acceleration vectors:

$$
\begin{gather*}
\dot{\mathbf{r}}=\left(-h\left(\frac{1}{r}\right)^{\prime}, \frac{h}{r}\right)  \tag{A8a}\\
\ddot{\mathbf{r}}=\left(-\frac{h^{2}}{r^{2}}\left(\left(\frac{1}{r}\right)^{\prime \prime}+\left(\frac{1}{r}\right)\right), 0\right) \tag{A8b}
\end{gather*}
$$

Substitute the acceleration (A8b) into Equation (A7):

$$
\begin{equation*}
-\frac{h^{2}}{r^{2}}\left(\left(\frac{1}{r}\right)^{\prime \prime}+\left(\frac{1}{r}\right)\right)=-\frac{\alpha c_{0} \eta}{4 \pi} \frac{M}{r^{2}} \tag{A9}
\end{equation*}
$$

This form of Newton's gravitational equation can be used very well to calculate the orbits of bodies moving in gravitational field.

After this brief introduction, let us examine how this equation changes in the gravino theory if the orbital motion of body $m$ is also taken into account.

Consider the case where the small mass $m$ revolves around the large mass $M$. Since the motion is relative, it may be supposed that $m$ does not move, while $M$ moves relative to $m$. It must be taken into account that gravinos move at a finite speed. The following Fig. 1 helps explain the situation. As long as the gravinos move from $M$ to $m, M$ will continue to move. The force is in the opposite direction to the vector $\mathbf{s}$, while the direction of the position vector $\mathbf{r}$ will be slightly different from the direction of $\mathbf{s}$. The travel time of gravinos is $d t=s / c_{0}$, during this time $M$ travels a distance $v s / c_{0}$.


Fig. 1.: Explanatory figure for the text.
The length of the position vector $\mathbf{r}$ is given by the following formula:

$$
\begin{equation*}
r=s \sqrt{1+\frac{\mathrm{v}_{\varphi}^{2}}{c_{0}^{2}}} \tag{A10}
\end{equation*}
$$

The equation of motion is modified as follows:

$$
\begin{equation*}
-\frac{h^{2}}{r^{2}}\left(\left(\frac{1}{r}\right)^{\prime \prime}+\left(\frac{1}{r}\right)\right)=-\frac{\alpha c_{0} \eta}{4 \pi} \frac{M}{s^{2}} \tag{A11}
\end{equation*}
$$

Because the direction of the vector $\mathbf{s}$ and the vector $\mathbf{r}$ is different, Equation (A3) is also modified:

$$
\begin{equation*}
\mathbf{r} \times \ddot{\mathbf{r}}=\frac{d}{d t}(\mathbf{r} \times \dot{\mathbf{r}})=-\frac{\alpha c_{0} \eta}{4 \pi} \frac{M}{s^{3}}(\mathbf{r} \times \mathbf{s}) \tag{A12}
\end{equation*}
$$

Calculating the vector product $(\mathbf{r} \times \mathbf{s})$, the following result is obtained:

$$
\begin{equation*}
\frac{d}{d t}\left(r^{2} \dot{\varphi}\right)=-\frac{\alpha c_{0} \eta}{4 \pi} \frac{M}{s^{3}}\left(s^{2} \frac{\mathrm{v}_{\varphi}}{c_{0}}\right)=-\frac{\alpha c_{0} \eta}{4 \pi} \frac{M}{c_{0}^{2}} \frac{\mathrm{v}_{\varphi}}{d t} \tag{A13}
\end{equation*}
$$

In the right side of the equation $c_{0} d t$ is written instead of $s$. Integrating equation (A31) over time the following equation can be derived:

$$
\begin{equation*}
r^{2} \dot{\varphi}=-\frac{\alpha c_{0} \eta}{4 \pi} \frac{M}{c_{0}^{2}} \mathrm{v}_{\varphi}+h=-\frac{\alpha c_{0} \eta}{4 \pi} \frac{M}{c_{0}^{2} r} r^{2} \dot{\varphi}+h \tag{A14}
\end{equation*}
$$

Express the quantity $r^{2} d \varphi / d t$ from (A32):

$$
\begin{equation*}
r^{2} \dot{\varphi}=\frac{h}{1+\frac{\alpha c_{0} \eta}{4 \pi} \frac{M}{c_{0}^{2} r}}=h^{*} \tag{A15}
\end{equation*}
$$

This result means that the angular momentum is not constant but changes slightly during the orbital motion.

Because of the orbital motion one more correction must be taken into account; the absorption cross section of gravinos also increases. During the travel time of the gravinos through $m$ the surface moves slightly away and the path of the gravinos inside $m$ also becomes longer. Both effects increase the absorption cross section by a factor of $\left(1+\mathrm{v}_{\varphi}{ }^{2} / \mathrm{c}_{0}{ }^{2}\right)^{1 / 2}$. Therefore the modified equation of motion can be written as follows:

$$
\begin{equation*}
\left(\frac{1}{r}\right)^{\prime \prime}+\left(\frac{1}{r}\right)=\frac{\alpha c_{0} \eta}{4 \pi}\left(1+\frac{\mathrm{v}_{\varphi}^{2}}{c_{0}^{2}}\right) \frac{M}{h^{* 2}} \frac{r^{2}}{s^{2}}=\frac{G M}{h^{2}}\left(1+\frac{\mathrm{v}_{\varphi}^{2}}{c_{0}^{2}}\right)^{2}\left(1+\frac{G M}{c_{0}^{2} r}\right)^{2} \tag{A16}
\end{equation*}
$$

The correction terms are very small, so their squares are neglected. Thus the equation will be:

$$
\begin{equation*}
\left(\frac{1}{r}\right)^{\prime \prime}+\left(\frac{1}{r}\right)=\frac{G M}{h^{2}}\left(1+\frac{2 \mathrm{v}_{\varphi}^{2}}{c_{0}^{2}}+\frac{2 G M}{c_{0}^{2} r}\right)=\frac{1}{r_{0}}+\frac{r_{g}}{r^{2}}+\frac{2 \varepsilon}{r} \tag{A17}
\end{equation*}
$$

where the following notations

$$
\begin{equation*}
\frac{1}{r_{0}}=\frac{G M}{h^{2}} ; \quad \varepsilon=\frac{(G M)^{2}}{h^{2} c_{0}^{2}} ; \quad r_{g}=\frac{2 G M}{c_{0}^{2}} \tag{A18}
\end{equation*}
$$

and the approximation $\mathrm{v} \varphi=h / r$ are applied.
Rearranging the $r$-dependent terms to the left side gives the following equation:

$$
\begin{equation*}
\left(\frac{1}{r}\right)^{\prime \prime}+(1-2 \varepsilon)\left(\frac{1}{r}\right)-r_{g}\left(\frac{1}{r}\right)^{2}=\frac{1}{r_{0}} \tag{A19}
\end{equation*}
$$

The solution of the original equation without correction terms is given by:

$$
\begin{equation*}
\frac{1}{r}=\frac{1}{r_{0}}+A e^{ \pm i \varphi} \tag{A20a}
\end{equation*}
$$

The orbit of $m$ around $M$ can be given as

$$
\begin{equation*}
r(\varphi)=\frac{r_{0}}{1-A r_{0} \cos \varphi} \tag{A20b}
\end{equation*}
$$

The orbit is an ellipse, parabola or hyperbola for $A r_{0}<1, A r_{0}=1$ or $A r_{0}>1$, respectively.
Try to find the solution of the full equation (A19) in the following form:

$$
\begin{equation*}
\frac{1}{r}=\frac{1}{b}+y(\varphi)=\frac{1}{b}+A e^{ \pm i \lambda \varphi} \tag{A21}
\end{equation*}
$$

The substitution of (A22) into (A19) lead to the following equation:

$$
\begin{equation*}
y^{\prime \prime}+\left(1-2 \varepsilon-\frac{2 r_{g}}{b}\right) y-r_{g} y^{2}+\frac{1-2 \varepsilon}{b}-\frac{r_{g}}{b^{2}}=\frac{1}{r_{0}} \tag{A22}
\end{equation*}
$$

Separating the constants and the $\varphi$-dependent terms, the following two equations are resulted:

$$
\begin{gather*}
r_{g}\left(\frac{1}{b}\right)^{2}-(1-2 \varepsilon)\left(\frac{1}{b}\right)+\frac{1}{r_{0}}=0  \tag{A23a}\\
y^{\prime \prime}+\left(1-2 \varepsilon-\frac{2 r_{g}}{b}\right) y-r_{g} y^{2}=0 \tag{A23b}
\end{gather*}
$$

The solution of the quadratic equation for $1 / b$ is approximately:

$$
\begin{equation*}
\frac{1}{b} \cong \frac{1}{r_{0}}-\frac{\varepsilon^{2}}{r_{g}} \cong \frac{1}{r_{0}} \tag{A24}
\end{equation*}
$$

In Equation (A23b), the square term is neglected. Then the following solution is obtained for the parameter $\lambda$ in the exponent of the exponential function:

$$
\begin{equation*}
\lambda=\sqrt{1-2 \varepsilon-\frac{2 r_{g}}{r_{0}}} \tag{A25}
\end{equation*}
$$

For a whole period of movement

$$
\begin{equation*}
\lambda \varphi=\sqrt{1-2 \varepsilon-\frac{2 r_{g}}{r_{0}}} \cdot \varphi=2 \pi \tag{A26}
\end{equation*}
$$

The shift of the perihelion during one period is

$$
\begin{equation*}
\Delta \varphi \cong 2 \pi\left(\varepsilon+\frac{r_{g}}{r_{0}}\right)=2 \pi\left(\frac{(G M)^{2}}{h^{2} c_{0}^{2}}+\frac{2 G M}{c_{0}^{2}} \frac{G M}{h^{2}}\right)=6 \pi \frac{(G M)^{2}}{h^{2} c_{0}^{2}} \tag{A27}
\end{equation*}
$$

This result is the same as the perihelion shift obtained from the general theory of relativity.

## Annex II.

## Deflection of a light ray near to a heavy body

Since the gravino field around a heavy body is inhomogeneous, the path of a light ray must be determined by the Fermat principle. That is, the light propagates between two points along a path where the propagation time is minimal. Let's assume that a light ray approaches the body from a great distance. Far away from the body, the light path can be regarded as a straight line. This line and the center of the body define a plane in the space. The light path remains in this plane during the propagation. Thus the path can be described as a two dimensional curve in the polar coordinate system $(r, \varphi)$, with the heavy body in the origin. The travel time $T$ along this path can be given as

$$
\begin{equation*}
T(r, \varphi)=\int_{A}^{B} \frac{d s}{c(r)}=\min . \tag{A28}
\end{equation*}
$$

The path can be determined by variational calculus. The variation of $T$ when considering different possible paths between A and B should be zero:

$$
\begin{equation*}
\delta T(r, \varphi)=\delta \int_{A}^{B} \frac{d s}{c(r)}=\frac{1}{c_{0}} \delta \int_{A}^{B} \frac{d s}{\frac{c(r)}{c_{0}}}=\frac{1}{c_{0}} \delta \int_{A}^{B} \frac{\sqrt{r(\varphi)^{2}+r^{\prime}(\varphi)^{2}}}{\sqrt{1-\frac{u(r)^{2}}{c_{0}^{2}}}} d \varphi=0 \tag{A29}
\end{equation*}
$$

where $r$ ' denotes the derivative $d r / d \varphi$.
According to the variational calculus the variation of a path integral can be calculated as

$$
\begin{equation*}
\delta \int_{A}^{B} f\left(r, \frac{d r}{d \varphi}, \varphi\right)=\int_{A}^{B} \delta f d \varphi=0 \tag{A30}
\end{equation*}
$$

This integral will be zero for each possible path only if $\delta f=0$. The variation of $f$ can be calculated as

$$
\begin{equation*}
\delta f=\frac{\partial f}{\partial r}-\frac{d}{d \varphi} \frac{\partial f}{\partial r^{\prime}}=0 \tag{A31}
\end{equation*}
$$

In our case

$$
\begin{equation*}
f=\frac{\sqrt{r^{2}+r^{\prime 2}}}{g(r)} \tag{A32}
\end{equation*}
$$

where

$$
\begin{equation*}
g(r)=\sqrt{1-\frac{u(r)^{2}}{c_{0}^{2}}} \tag{A33}
\end{equation*}
$$

The variation of $f$ can be calculated as

$$
\begin{equation*}
\delta f=\frac{r^{2} r^{\prime \prime}-2 r r^{\prime}-r^{3}}{\sqrt{\left(r^{2}+r^{\prime 2}\right)^{3}}} g+\frac{r^{2}}{\sqrt{\left(r^{2}+r^{\prime 2}\right)}} \frac{\partial g}{\partial r}=0 \tag{A34}
\end{equation*}
$$

Introducing the new variable $y$ with the definition $y=r^{\prime} / r$ the above differential equation can be written in the following form:

$$
\begin{equation*}
y^{\prime}=\left(1+y^{2}\right)\left(g-r \frac{\partial g}{\partial r}\right) \tag{A35}
\end{equation*}
$$

In a homogeneous space the velocity of light is constant, i.e. $g=1$ and $\partial g / \partial r=0$. Therefore

$$
\begin{equation*}
\frac{d y}{1+y^{2}}=d \varphi \tag{A36}
\end{equation*}
$$

and

$$
\begin{equation*}
r=\frac{a}{\cos \varphi} \tag{A37}
\end{equation*}
$$

which is the equation of a straight line perpendicular to the $\varphi=0$ line and has a distance $a$ from the origin of the coordinate system. If $\varphi \rightarrow \pm \pi / 2, r \rightarrow \infty$.
This form of $r$ can be used for calculating the deviation of the light ray in the gravitational field. In this case the correction term $g$ of the light velocity is given as

$$
\begin{equation*}
g=\sqrt{1-\frac{u(r)^{2}}{c_{0}^{2}}}=\sqrt{1-\frac{2 G M}{c_{0}^{2} r}}=\sqrt{1-\frac{r_{g}}{r}} \tag{A38}
\end{equation*}
$$

where $r_{g}$ is the Schwarzschild radius of mass $M$.
The second term on the right side of the differential equation (A36) can be approximated as

$$
\begin{equation*}
g-r \frac{\partial g}{\partial r}=\sqrt{1-\frac{r_{g}}{r}}-\frac{\frac{r_{g}}{r}}{2 \sqrt{1-\frac{r_{g}}{r}}}=\frac{1-\frac{r_{g}}{r}-\frac{r_{g}}{2 r}}{\sqrt{1-\frac{r_{g}}{r}}} \cong 1-\frac{r_{g}}{r} \cong 1-\frac{r_{g}}{a} \cos \varphi \tag{A39}
\end{equation*}
$$

By substituting (A39) into (A35) the differential equation can be written as

$$
\begin{equation*}
\frac{d y}{1+y^{2}}=\left(1-\frac{r_{g}}{a} \cos \varphi\right) d \varphi \tag{A40}
\end{equation*}
$$

By integrating the above equation the following expression can be derived:

$$
\begin{equation*}
\tan ^{-1} y=\varphi-\frac{r_{g}}{a} \sin \varphi \tag{A41}
\end{equation*}
$$

Then $y$ can be written as

$$
\begin{equation*}
y=\frac{r^{\prime}}{r}=\tan \left(\varphi-\frac{r_{g}}{a} \sin \varphi\right) \tag{A42}
\end{equation*}
$$

The path of the light ray can be determined by integrating the above equation. Since $r_{g} / a$ is very small the integral can be written as

$$
\begin{equation*}
\ln r \cong \ln \cos \left(\varphi-\frac{r_{g}}{a} \sin \varphi\right)+\text { const } . \tag{A43}
\end{equation*}
$$

Thus, the light path will be

$$
\begin{equation*}
r(\varphi) \cong \frac{a}{\cos \left(\varphi-\frac{r_{g}}{a} \sin \varphi\right)} \tag{A44}
\end{equation*}
$$

The shape of the path calculated by Eq. (A16) is shown in Fig. 2.


Fig. 2: Deflection of a light ray passing near to a mass located in the center.

Now $r \rightarrow \infty$ if $\left(\varphi-\left(r_{\mathrm{g}} / a\right) \sin \varphi\right) \rightarrow \pm \pi / 2$. Let's assume $\varphi= \pm(\pi / 2+\Delta \varphi)$. Then $\sin \varphi= \pm \cos \Delta \varphi$. Thus

$$
\begin{equation*}
|\Delta \varphi|=\frac{r_{g}}{a} \cos \Delta \varphi \cong \frac{r_{g}}{a}=\frac{u(a)^{2}}{c_{0}^{2}} \tag{A45}
\end{equation*}
$$

The angle deflection of the light ray is

$$
\begin{equation*}
2 \Delta \varphi=2 \frac{u(a)^{2}}{c_{0}^{2}}=\frac{4 G M}{c_{0}^{2} a} \tag{A46}
\end{equation*}
$$

the same value as the deflection according to the general theory of relativity.

