Bell’s theorem refuted via elementary probability theory

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Abstract: Bell’s theorem has been described as the most profound discovery of science. Let’s see.

Introduction: Let $\beta$ denote the thought-experiment in Bell (1964). Let $B(.)$ denote his equations (.). Let the causally-independent same-instance results in $B(1)$ be $A^\pm$ and $B^\pm$, pairwise correlated via the functions $A$ & $B$ and the variable $\lambda$. Then, reserving $P$ for probabilities, let’s replace Bell’s expectation $P(\bar{a}, \bar{b})$ in $B(2)$ with its identity $E(a, b|\beta)$. So, from $B(1)$, $B(2)$, RHS $B(3)$ and the line below $B(3)$—with $\Lambda$ denoting the space of $\lambda$—here’s Bell’s theorem ($\text{BT}$) in our notation:

$$\text{BT: } E(a, b|\beta) = \int_{\Lambda^+} d\lambda \, p(\lambda) A(a, \lambda) B(b, \lambda) - \int_{\Lambda^-} d\lambda \, p(\lambda) A(a, \lambda) B(b, \lambda)$$

(1)

with $A(a, \lambda) = \pm 1 \equiv A^\pm$, $B(b, \lambda) = \pm 1 \equiv B^\pm$, $A(a, \lambda) B(b, \lambda) = \pm 1$.

(2)

Refutation: Via RHS (2), and independent of the functions $A$ and $B$, we divide $\Lambda$ into two subsets: $\Lambda^+$ is the space that delivers $A(a, \lambda) B(b, \lambda) = 1$, $\Lambda^-$ delivers $A(a, \lambda) B(b, \lambda) = -1$. Thus, from (1):

$$E(a, b|\beta) = \int_{\Lambda^+} d\lambda \, p(\lambda) A(a, \lambda) B(b, \lambda) - \int_{\Lambda^-} d\lambda \, p(\lambda) A(a, \lambda) B(b, \lambda)$$

(3)

$$= P(AB = 1 | a, b, \Lambda^+) - P(AB = -1 | a, b, \Lambda^-), \text{ the weighted-sum of } AB \text{ results.}$$

(4)

$$= [P(A^+ B^+) + P(A^- B^-)] - [P(A^+ B^-) + P(A^- B^+)], \text{ with conditions suppressed,}$$

the weighted-sum of the same-instance results (±1) that deliver each $AB$ result. (5)

$$= P(A^+) P(B^+ | A^+) + P(A^-) P(B^- | A^-) - P(A^+) P(B^- | A^+) - P(A^-) P(B^+ | A^-)$$

via the product rule for the paired (same-instance) results correlated as in (2). (6)

$$= \frac{1}{2} \left[ P(B^+ | A^+) + P(B^- | A^-) - P(B^- | A^+) - P(B^+ | A^-) \right] \text{ for, with}$$

$\lambda$ a random latent variable, the marginal probabilities $[\text{like } P(A^+)] = \frac{1}{2}$. (7)

$$= \frac{1}{2} \left[ \sin^2 \frac{1}{2}(a, b) + \sin^2 \frac{1}{2}(a, b) - \cos^2 \frac{1}{2}(a, b) - \cos^2 \frac{1}{2}(a, b) \right]: \text{ replacing the probability}$$

functions in (7) with our $\beta$-based laws (akin to Malus’ Law for light-beams). (8)

$$= \sin^2 \frac{1}{2}(a, b) - \cos^2 \frac{1}{2}(a, b) = -\cos(a, b) = -a \cdot b. \text{ So RHS (1) is refuted: QED.}$$

(9)

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**Confirmation:** We now refute B(15), Bell’s inequality (BI), offered by Bell as proof of his theorem.

\[ \text{BI: } |E(a, b) - E(a, c)| - 1 \leq E(b, c) \text{ [sic]} \text{; ie, B(15) in our notation,} \]  
(10)

where \(-1 \leq E(a, b) \leq 1, -1 \leq E(a, c) \leq 1, -1 \leq E(b, c) \leq 1. \)  
(11)

However: \(E(a, b)[1 + E(a, c)] \leq 1 + E(a, c); \text{ for, if } V \leq 1, \text{ and } 0 \leq W, \text{ then } VW \leq W. \)  
(12)

\[ \therefore E(a, b) - E(a, c) - 1 \leq -E(a, b)E(a, c). \]  
(13)

Similarly: \(E(a, c) - E(a, b) - 1 \leq -E(a, b)E(a, c). \) Hence our irrefutable inequality  
(14)

\[ \text{WI: } |E(a, b) - E(a, c)| - 1 \leq -E(a, b)E(a, c). \]  
(15)

So, with test-settings \(0 < (a, c) < \pi; (a, b) = (b, c) = \frac{(a, c)}{2} = \frac{x}{2}, \) and, via (9),  
(16)

with test-functions \(E(a, b) = E(b, c) = -\cos \left( \frac{x}{2} \right), E(a, c) = -\cos(x) ; \) please \(\text{copy, paste and test this next expression in WolframAlpha®; free-online, see References.} \)  
(17)

\[ \text{plot} |\cos(x) - \cos(x/2)| - 1 \& -\cos(x/2)\& -\cos(x)\cos(x/2), 0 \leq x \leq \pi \]  
(19)

Thus, under the generality of (16)-(17): \(^2\) (i) For \(0 < x < \pi, (10) \) is everywhere false, (15) is everywhere true. (ii) For \(x = 0\) and \(x = \pi, (10) \) and (15) are true. (iii) Let the unnumbered relations between B(14) and B(15) be B(14a)-B(14c). (iv) Then Bell’s error is his move from true B(14a) to false B(14b): for B(14b) leads to false B(15). (v) In other words, given the common LHS in (10) and (15): Bell’s error equates irrefutable \(-E(a, b)E(a, c) \) from (15) to false \(E(b, c) \) from (10); hence, as above, Bell’s equality only holds at \(x = 0 \) and \(x = \pi. \) That is: when Bell’s \(-\cos \left( \frac{x}{2} \right) = -\cos \left( \frac{x}{2} \right) \cos(x). \)

**Conclusions:** (i) Bell’s theorem (1) and Bell’s inequality (10) are refuted. (ii) In (8), via our heuristic debt to Malus, we provide the first of a family of laws that refute Bell’s theorem in other settings.

**References:**


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\(^2\) Under \(\beta, \) the plane of the coplanar angles need not be orthogonal to the line-of-light axis: just not parallel to it.