Bell's theorem refuted via elementary probability theory

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Abstract: Bell's theorem has been described as the most profound discovery of science. Let's see.

Introduction: Let β denote the thought-experiment in Bell (1964). Let **B**(.) denote his equations (.). Let the causally-independent same-instance results in the line before **B**(1) be $A^{\pm}$, $B^{\pm}$ (since Bell uses $A\,\&\,B$ for functions and results). So $A^{\pm}$ and $B^{\pm}$ are pairwise correlated via Bell's functions A & B and the latent variable λ. Our analysis is thus based on relativistic causality (aka *locality* and *local causality* in Bellian terminology). Then, reserving P for probabilities, let's replace Bell's expectation $P(\vec{a},\vec{b})$ in **B**(2) with its identity $E(a,b|\beta)$. So, from **B**(1), **B**(2), RHS **B**(3) and the line below **B**(3):

This is Bell's theorem, **BT**: $E(a,b|\beta) = \int d\lambda\, \rho(\lambda)A(a,\lambda)B(b,\lambda) \neq -a{\cdot}b$ [sic]; (1)

with $A(a,\lambda) = \pm 1 \equiv A^{\pm}$, $B(b,\lambda) = \pm 1 \equiv B^{\pm}$, $A(a,\lambda)B(b,\lambda) = \pm 1 \equiv AB$ in short form. (2)

Refutation: Via RHS (2), and independent of the functions A and B, we distribute (1)'s integrand (identified in short form as $AB = \pm 1$) over two subsidiary integrands: $AB = 1$ and $AB = -1$. Thus:

$$E(a,b|\beta) = \int d\lambda\, \rho(\lambda)[(A(a,\lambda)B(b,\lambda)\,|AB=1) - (A(a,\lambda)B(b,\lambda)\,|AB=-1)] \quad (3)$$

$= P(AB=1) - P(AB=-1)$, the weighted-sum of the binary AB results ± 1. (4)

$= \left[P\left(A^+B^+\right)+P\left(A^-B^-\right)\right]-\left[P\left(A^+B^-\right)+P\left(A^-B^+\right)\right]$, the weighted-sum of same-instance results ± 1: for each AB-pair in (4) delivers its result in two ways. (5)

$= P\left(A^+\right)P\left(B^+\,|A^+\right)+P\left(A^-\right)P\left(B^-\,|A^-\right)-P\left(A^+\right)P\left(B^-\,|A^+\right)-P\left(A^-\right)P\left(B^+\,|A^-\right)$ via the product rule for the paired (same-instance) results correlated as in (2). (6)

$= \frac{1}{2}\left[P\left(B^+\,|\,A^+\right)+P\left(B^-\,|A^-\right)-P\left(B^-\,|A^+\right)-P\left(B^+\,|A^-\right)\right]$ for, with λ a random latent variable, the marginal probabilities $\left[\text{like } P\left(A^+\right)\right] = \frac{1}{2}$. (7)

$= \frac{1}{2}\left[\sin^2\frac{1}{2}(a,b)+\sin^2\frac{1}{2}(a,b)-\cos^2\frac{1}{2}(a,b)-\cos^2\frac{1}{2}(a,b)\right]$: replacing the probability functions in (7) with our β-based laws (akin to Malus' Law for light-beams). (8)

$= \sin^2\frac{1}{2}(a,b)-\cos^2\frac{1}{2}(a,b) = -\cos(a,b) = -a{\cdot}b$. So RHS (1) is refuted: QED. (9)

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Confirmation: We now refute **B**(15), Bell's inequality (**BI**), offered by Bell as proof of his theorem.

$$\textbf{BI:}\ |E(a,b)-E(a,c)|-1 \leq E(b,c)\ \text{[sic]: ie, from } \textbf{B}(15) \text{ in our terms} \tag{10}$$

$$\text{where } -1 \leq E(a,b) \leq 1,\ -1 \leq E(a,c) \leq 1,\ -1 \leq E(b,c) \leq 1. \tag{11}$$

$$\text{However: } E(a,b)[1+E(a,c)] \leq 1+E(a,c);\ \text{for, if } V \leq 1, \text{ and } 0 \leq W, \text{ then } VW \leq W. \tag{12}$$

$$\therefore E(a,b)-E(a,c)-1 \leq -E(a,b)E(a,c). \tag{13}$$

$$\text{Similarly: } E(a,c)-E(a,b)-1 \leq -E(a,b)E(a,c). \text{ Hence our irrefutable inequality} \tag{14}$$

$$\textbf{WI:}\ |E(a,b)-E(a,c)|-1 \leq -E(a,b)E(a,c). \tag{15}$$

$$\text{So, with test-settings } 0 < (a,c) < \pi; (a,b) = (b,c) = \frac{(a,c)}{2} = \frac{x}{2}, \text{ and, via (9),} \tag{16}$$

$$\text{with test-functions } E(a,b) = E(b,c) = -\cos\left(\frac{x}{2}\right), E(a,c) = -\cos(x):\ \text{please} \tag{17}$$

$$\text{copy, paste and test this next expression in WolframAlpha}^{\circledR}\text{; free-online, see References.} \tag{18}$$

$$\text{plot}|cos(x)-cos(x/2)|-1\&\&-cos(x/2)\&\&-cos(x)cos(x/2), 0 \leq x \leq \pi \tag{19}$$

Thus, for $0 < x < \pi$ under (16): (15) is everywhere true, (10) is everywhere false. So Bell's claimed proof of his theorem is false and refuted. Further: (i) (16)'s generality allows $(a,b), (b,c), (a,c)$ to be co-planar, so long as the line-of-flight axis intersects that plane at a point. (ii) (19) is readily formatted for other tests. (iii) Identify the unnumbered relations between **B**(14) and **B**(15) as **B**(14a)-**B**(14c): then Bell's error is his move from true **B**(14a) to false **B**(14b): for the latter leads to false **B**(15).

Conclusions: (i) Bell's theorem (1) and Bell's inequality (10) are refuted. (ii) In (8), via our heuristic debt to Malus, we provide the first of a family of laws that refute Bell's theorem in other settings. (iii) Via (2), our laws and results are consistent with relativistic causality: the union of locality and causality that Bell incorrectly rejected. (iv) For us, the physical significance of λ is that it relates to a particle's total angular momentum: so, akin to **B**(13), $A(a,\lambda) = -B(a,\lambda) = -A(a,-\lambda)$, etc, where the first and last terms reflect the pairwise conservation of total angular momentum: $\lambda + (-\lambda) = 0$.

References:

1. Bell, J. S. (1964). "On the Einstein Podolsky Rosen paradox." Physics 1, 195-200. http://cds.cern.ch/record/111654/files/vol1p195-200_001.pdf
2. WolframAlpha®. "WolframAlpha: computational intelligence." https://www.wolframalpha.com