Bell’s theorem refuted via elementary probability theory

Gordon Stewart Watson

Abstract: Bell’s theorem has been described as the most profound discovery of science. Let’s see.

Introduction: Let $\beta$ denote the EPR-Bohm experiment in Bell (1964). Let $B(.)$ denote his equations (.). Let $A^\pm$ & $B^\pm$ denote the independent same-instance results in the line before $B(1)$. Thus, from $B(1)$, $A^\pm$ and $B^\pm$ are pairwise correlated via Bell’s functions $A$ & $B$ and the twinned latent variables $\lambda$: so $\beta$, the basis for our analysis, is relativistically causal. Then, reserving $P$ for probabilities, let’s replace Bell’s expectation $P(\bar{a}, \bar{b})$ in $B(2)$ with its identity $E(a, b | \beta)$. So, from $B(1)$, $B(2)$, RHS $B(3)$ and the line below $B(3)$, this is Bell’s theorem under $\beta$ and relativistic causality:

\[
E(a, b | \beta) = \int d\lambda \rho(\lambda)A(a, \lambda)B(b, \lambda) = -a \cdot b \text{ [sic];} \tag{1}
\]

with $A(a, \lambda) = \pm 1 = A^\pm, B(b, \lambda) = \pm 1 = B^\pm, A(a, \lambda)B(b, \lambda) = \pm 1. \tag{2}$

Refutation: $E(a, b | \beta)$ is the average result under $\beta$ with settings $a$ & $b$. So, via LHS (1) & RHS (2):

\[
E(a, b | \beta) = \int d\lambda \rho(\lambda)[A(a, \lambda)B(b, \lambda) \equiv 1] - (A(a, \lambda)B(b, \lambda) \equiv -1)], \text{ separating the results.} \tag{3}
\]

\[
P(A(a, \lambda)B(b, \lambda) = 1) - P(A(a, \lambda)B(b, \lambda) = -1), \text{ the weighted-sum of results.} \tag{4}
\]

\[
[P(A^+B^+) + P(A^-B^-)] - [P(A^+B^-) + P(A^-B^+)], \text{ the weighted-sum of same-instance results (±1): for each } \lambda \text{-pair in (4) delivering its result in two ways.} \tag{5}
\]

\[
P(A^+)P(B^+ | A^+) + P(A^-)P(B^- | A^-) - P(A^+)P(B^- | A^+) - P(A^-)P(B^+ | A^-) \tag{6}
\]

\[
= \frac{1}{2} [P(B^+ | A^+) + P(B^- | A^-) - P(B^- | A^+) - P(B^+ | A^-)] \text{ for, with} \lambda \text{ a random latent variable, the marginal probabilities} \text{ [like } P(A^+)] = \frac{1}{2}. \tag{7}
\]

\[
\frac{1}{2} \left[\sin^2 \frac{1}{2}(a, b) + \sin^2 \frac{1}{2}(a, b) - \cos^2 \frac{1}{2}(a, b) - \cos^2 \frac{1}{2}(a, b)\right] : \text{ replacing the probability functions in (7) with our } \beta \text{-based laws (akin to Malus’ Law for light-beams).} \tag{8}
\]

\[
= \sin^2 \frac{1}{2}(a, b) - \cos^2 \frac{1}{2}(a, b) = -\cos(a, b) = -a \cdot b. \text{ So RHS (1) is refuted: QED.} \tag{9}
\]
**Confirmation:** We now use high-school mathematics to refute \( B(15) \), the inequality that Bell offered as proof of his theorem. Thus, from \( B(15) \), this is Bell’s inequality (\( BI \)):

\[
BI: |E(a, b) - E(a, c)| - 1 \leq E(b, c) \text{ [sic]} : \quad (10)
\]

where \(-1 \leq E(a, b) \leq 1, -1 \leq E(a, c) \leq 1, -1 \leq E(b, c) \leq 1.\) \( (11)\)

However: \( E(a, b)[1 + E(a, c)] \leq 1 + E(a, c); \) for, if \( V \leq 1, \) and \( 0 \leq W, \) then \( VW \leq W.\) \( (12)\)

\[
: E(a, b) - E(a, c) - 1 \leq -E(a, b)E(a, c). \quad (13)
\]

Similarly: \( E(a, c) - E(a, b) - 1 \leq -E(a, b)E(a, c). \) Hence our own irrefutable inequality \( (14)\)

\[
WI: |E(a, b) - E(a, c)| - 1 \leq -E(a, b)E(a, c). \quad (15)
\]

So, with test-settings \(-\pi < (a, c) < \pi; (a, b) = (b, c) = \left(\frac{a, c}{2}\right) = \frac{x}{2}\); \( (16)\)

and, via \( (9)\), with test-functions \( E(a, b) = E(b, c) = -\cos\left(\frac{x}{2}\right), E(a, c) = -\cos(x) : \) \( (17)\)

copy and paste this next expression into WolframAlpha®; free-online, see References. \( (18)\)

\[
\text{plot}\{\cos(x) - \cos(x/2)\} - 1 \& \& -\cos(x/2)\&\& -\cos(x)\cos(x/2), 0 \leq x \leq \pi \quad (19)
\]

Then click \([=]\). Note that \( (16)\) is quite general: for it allows \((a, b), (b, c)\) and \((a, c)\) to be co-planar at any workable orientation to the line-of-flight axis. Thus, under \( (16)\), we see that \( WI \) is never false and \( BI \) is false almost everywhere. So \( BI \), Bell’s supposed proof of his theorem, is false too. QED.

**Conclusions:** (i) Bell’s theorem, \( (1)\), is refuted via elementary probability theory. (ii) Bell’s related inequality—\( B(15) \), our \( (10)\), the basis for \( (1)\)—is refuted via high-school mathematics. (iii) In \( (8)\)—via our heuristic debt to Étienne-Louis Malus (1775-1812)—we provide the first of a family of laws that refute Bell’s theorem elsewhere. (iv) Via \( (2)\), all our laws and results are consistent with relativistic causality. (v) Note: we identify Bell’s error—his false move from \( B(14)\)—and provide the said-to-be-impossible functions \( A \) and \( B \) elsewhere.

**References:**