Bell's theorem refuted via elementary probability theory

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Abstract: Bell's theorem has been described as the most profound discovery of science. Let's see.

Introduction: Let β denote the EPR-Bohm experiment in Bell (1964). Let **B**(.) denote his equations (.). Let $A^{\pm}$ & $B^{\pm}$ denote the independent same-instance results in the line before **B**(1). Thus, from **B**(1), $A^{\pm}$ and $B^{\pm}$ are pairwise correlated via Bell's functions A & B and the twinned latent variables λ: so (2), the basis for our analysis, is relativistically causal. Then, reserving P for probabilities, let's replace Bell's expectation $P(\vec{a},\vec{b})$ in **B**(2) with its identity $E(a,b\,|\,\beta)$. So, from **B**(1), **B**(2), RHS **B**(3) and the line below **B**(3), this is Bell's theorem under β and relativistic causality:

$$E(a,b\,|\,\beta) = \smallint d\lambda\,\rho(\lambda)A(a,\lambda)B(b,\lambda) \neq -a\cdot b \text{ [sic];} \tag{1}$$

$$\text{with } A(a,\lambda) = \pm 1 = A^{\pm},\ B(b,\lambda) = \pm 1 = B^{\pm},\ A(a,\lambda)B(b,\lambda) = \pm 1. \tag{2}$$

Refutation: $E(a,b\,|\,\beta)$ is the average result under β with settings a & b. So, via LHS (1) & RHS (2):

$$E(a,b\,|\,\beta) = \smallint d\lambda\,\rho(\lambda)[(A(a,\lambda)B(b,\lambda) \equiv 1) - (A(a,\lambda)B(b,\lambda) \equiv -1)], \text{ separating the results.} \tag{3}$$

$$= P\left(A(a,\lambda)B(b,\lambda) = 1\right) - P\left(A(a,\lambda)B(b,\lambda) = -1\right), \text{ the weighted-sum of results.} \tag{4}$$

$$= \left[P\left(A^+B^+\right) + P\left(A^-B^-\right)\right] - \left[P\left(A^+B^-\right) + P\left(A^-B^+\right)\right], \text{ the weighted-sum of same-instance results } (\pm 1)\text{: for each } \lambda\text{-pair in (4) delivering its result in two ways.} \tag{5}$$

$$= P\left(A^+\right)P\left(B^+\,|A^+\right) + P\left(A^-\right)P\left(B^-\,|A^-\right) - P\left(A^+\right)P\left(B^-\,|A^+\right) - P\left(A^-\right)P\left(B^+\,|A^-\right)$$
$$\text{via the general product rule for the paired (same instance) results correlated as in (2).} \tag{6}$$

$$= \tfrac{1}{2}\left[P\left(B^+\,|\,A^+\right) + P\left(B^-\,|A^-\right) - P\left(B^-\,|A^+\right) - P\left(B^+\,|A^-\right)\right] \text{ for, with } \lambda \text{ a random latent variable, the marginal probabilities } \left[\text{like } P\left(A^+\right)\right] = \tfrac{1}{2}. \tag{7}$$

$$= \tfrac{1}{2}\left[\sin^2\tfrac{1}{2}(a,b) + \sin^2\tfrac{1}{2}(a,b) - \cos^2\tfrac{1}{2}(a,b) - \cos^2\tfrac{1}{2}(a,b)\right] \text{: replacing the probability functions in (7) with our } \beta\text{-based laws (akin to Malus' Law for light-beams).} \tag{8}$$

$$= \sin^2\tfrac{1}{2}(a,b) - \cos^2\tfrac{1}{2}(a,b) = -\cos(a,b) = -a\cdot b. \text{ So RHS (1) is refuted: QED.} \tag{9}$$

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Confirmation: We now use high-school mathematics to refute **B**(15), the inequality that Bell offered as proof of his theorem. Thus, from **B**(15), this is Bell's inequality (**BI**):

$$\textbf{BI:}\ |E(a,b)-E(a,c)|-1 \le E(b,c)\ \text{[sic]}: \tag{10}$$

$$\text{where } -1 \le E(a,b) \le 1,\ -1 \le E(a,c) \le 1,\ -1 \le E(b,c) \le 1. \tag{11}$$

$$\text{However: } E(a,b)[1+E(a,c)] \le 1+E(a,c);\ \text{for, if } V \le 1, \text{ and } 0 \le W, \text{ then } VW \le W. \tag{12}$$

$$\therefore E(a,b)-E(a,c)-1 \le -E(a,b)E(a,c). \tag{13}$$

$$\text{Similarly: } E(a,c)-E(a,b)-1 \le -E(a,b)E(a,c). \text{ Hence our own irrefutable inequality} \tag{14}$$

$$\textbf{WI:}\ |E(a,b)-E(a,c)|-1 \le -E(a,b)E(a,c). \tag{15}$$

$$\text{So, with test-settings } -\pi < (a,c) < \pi; (a,b) = (b,c) = \frac{(a,c)}{2} = \frac{x}{2}, \tag{16}$$

$$\text{and, via (9), with test-functions } E(a,b) = E(b,c) = -\cos\left(\frac{x}{2}\right), E(a,c) = -\cos(x): \tag{17}$$

$$\text{copy and paste this next expression into WolframAlpha}^{\circledR}\text{; free-online, see References.} \tag{18}$$

$$\text{plot}|cos(x)-cos(x/2)|-1\&\&-cos(x/2)\&\&-cos(x)cos(x/2), 0 \le x \le \pi \tag{19}$$

Then click **[=]**. Note that (16) is quite general: for it allows $(a,b),(b,c)$ and (a,c) to be co-planar at any workable orientation to the line-of-flight axis. Thus, under (16), we see that **WI** is never false and **BI** is false almost everywhere. So **BI**, Bell's supposed proof of his theorem, is false too. QED.

Conclusions: (i) Bell's theorem, (1), is refuted via elementary probability theory. (ii) Bell's related inequality—**B**(15), our (10), the basis for (1)—is refuted via high-school mathematics. (iii) In (8)—via our heuristic debt to Étienne-Louis Malus (1775-1812)—we provide the first of a family of laws that refute Bell's theorem elsewhere. (iv) Via (2), all our laws and results are consistent with relativistic causality. (v) Note: we identify Bell's error—his false move from **B**(14)—and provide the said-to-be-impossible functions A and B elsewhere.

References:

1. Bell, J. S. (1964). "On the Einstein Podolsky Rosen paradox." Physics 1, 195-200. http://cds.cern.ch/record/111654/files/vol1p195-200_001.pdf

2. WolframAlpha®. "WolframAlpha: computational intelligence." https://www.wolframalpha.com