Measuring Cosmological Parameters by Using Uncertainty Principles in the $S^3 \times S^1$ Space-Time

Moninder Singh Modgil\(^1\)

Abstract

Finite size of the $S^3$ universe suggests that the maximum spatial uncertainty $\langle \Delta x \rangle_{\text{Max}} = 2\pi R_{\text{Univ}}$, where $R_{\text{Univ}}$ is the radius of the universe. It follows from the Position-Momentum uncertainty principle $\Delta x \Delta p \geq \hbar / 2\pi$, that there exists a minimum uncertainty in the momentum - i.e., $\langle \Delta p \rangle_{\text{Min}} = \hbar / (2\pi)^2 R_{\text{Univ}}$. Similarly, the finite duration $T$ of the $S^1$ Time Cycle, suggests that the maximum temporal uncertainty is $\langle \Delta t \rangle_{\text{Max}} = T$. It follows from the Time-Energy uncertainty principle $\Delta E \Delta t \geq \hbar / 2\pi$, that there exists a minimum uncertainty in the energy - i.e., $\langle \Delta E \rangle_{\text{Min}} = \hbar / 2\pi T$. These considerations suggest the following conclusions - (1) Quantum states with $\Delta E \leq \langle \Delta E \rangle_{\text{Min}}$ and $\Delta p \leq \langle \Delta p \rangle_{\text{Min}}$ will be indistinguishable, (2) It should be possible to determine radius of the finite $S^3$ universe, i.e., $R_{\text{Univ}} = \hbar / [(2\pi)^2 \langle \Delta p \rangle_{\text{Min}}]$, by locally measuring $\langle \Delta p \rangle_{\text{Min}}$, and (3) Determine duration of the universe's time cycle, $T = \hbar / 2\pi \langle \Delta E \rangle_{\text{Min}}$, by locally measuring $\langle \Delta E \rangle_{\text{Min}}$.

\(^1\) PhD in Physics from Indian Institute of Technology, Kanpur, India. 
B.Tech. (Hons) in Aeronautical Engineering from Indian Institute of Technology, Kharagpur, India. 
Presently working as Senior Data Scientist in the company PROVANA, located in Noida, India. 
Email: msmodgil@gmail.com
Measuring Cosmological Parameters by Using Uncertainty Principles in the $S^3 \times S^1$ Space-Time

In context of quantum mechanics and uncertainty principles - one can ask the question, what could be the maximum spatial and temporal uncertainty. Arguably, the maximum spatial uncertainty cannot be larger than the size of the universe - for a static $S^3$ universe. For an expanding universe, the uncertainty should have the particle horizon as the maximum spatial uncertainty. For a cyclic universe, the temporal uncertainty cannot be larger than the duration of the time cycle. Segal [1] has extensively studied cosmology on $S^3 \times S^1$ space-time ($S^3$ space and $S^1$ time) - in which both the spatial factor as well as the temporal factor are finite. His motivation is to develop cosmology on the spatial part of the Einstein Cylinder ($S^i \times R^j$). He introduces $S^j$ times and uses the Uni-energy operator to arrive at cosmological red-shift of distant galaxies. Guillemin [2], develops periodicity for particles propagating along the null geodesics - with the null geodesics being closed curves.

Finite size of the $S^3$ universe suggests that the maximum spatial uncertainty should be -

\[(\Delta x)_{\text{Max}}=2\pi R_{\text{Univ}},\]

where $R_{\text{Univ}}$ is the radius of the universe. It follows from the Position-Momentum uncertainty principle

\[\Delta x \Delta p \geq \hbar/2\pi,\]

that there exists a minimum uncertainty in the momentum - i.e.,

\[(\Delta p)_{\text{Min}}=\hbar/(2\pi)^2 R_{\text{Univ}}.\]

Similarly, the finite duration $T$ of the $S^1$ Time Cycle, suggests that the maximum temporal uncertainty is -

\[(\Delta t)_{\text{Max}}=T\]

It follows from the Time-Energy uncertainty principle,

\[\Delta E \Delta t \geq \hbar/2\pi\]

that there exists a minimum uncertainty in the energy - i.e.,

\[(\Delta E)_{\text{Min}}=\hbar/2\pi T\]

These consideration suggest the following conclusions -
(1) Quantum states with $\Delta E \leq (\Delta E)_{\text{Min}}$ and $\Delta p \leq (\Delta p)_{\text{Min}}$ will be indistinguishable,

(2) It should be possible to determine radius of the finite $S^3$ universe, i.e.,

$$R_{\text{Univ}} = \frac{h}{(2\pi)^2 (\Delta p)_{\text{Min}}}$$

by locally measuring $(\Delta p)_{\text{Min}}$, and

(3) Determine duration of the universe's time cycle,

$$T = \frac{h}{2\pi (\Delta E)_{\text{Min}}}$$

by locally measuring $(\Delta E)_{\text{Min}}$.

References