A Sieve for Goldbach Conjecture

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Abstract

In this article, we find a new sieve for pair of primes whose summation equals to a given Even Number.

When we sieve numbers 2 and 3 to the whole natural numbers by removing all the multiplies of 2 or 3, the remaining numbers are 1 and all pairs of \{6k-1, 6k+1; k=1,2,3,....\}. The remaining numbers set is a group of multiply.

We call the integer \(k\) as the id of the pair (6k-1, 6k+1).

By multiplication, two pairs of \((6k_1 - 1, 6k_1 + 1)\), and \((6k_2 - 1, 6k_2 + 1)\), can generate four new numbers,6k-1, or 6k+1, with id k takes four new integers of \((6k_1 \pm 1)k_2 \pm k_1\) when \(k_1 \neq k_2\), while four new k will reduce to three when \(k_1 = k_2\).

We have a new sieve for Goldbach Conjecture based on the above observation,

For any large Even Number of forms as, 6N, or 6N+2, or 6N-2. we sieve the whole integer set I of id integers by the numbers of 6k_1-1, and 6k_1+1 for \(k_1 = 1, 2, ..., m\), by removing all the numbers of \(i \in I; i = k_1, \text{or} N-k_1, \text{mod} (6k_1 + 1), \text{or} i = -k_1, \text{or} N+k_1, \text{mod} (6k_1 - 1), k_1 \leq m\);
the remaining numbers are set of \{i \in I; i \neq k_1,\text{or}\ N-k_1, \text{mod} (6k_1 + 1), \text{and} i \neq -k_1,\text{or}\ N+k_1, \text{mod} (6k_1 - 1), k_1 \leq m\};

If we limit our sieve upto this large number N, we have \(m = \lceil \sqrt{N/6} \rceil\), here [a] means the largest integer less than a.

Theorem;

By using the above sieve when sieve all \((6k_1 \pm 1), k_1 \leq m\) for the first \(N\) integers, \((0, N)\), the total number of the remaining numbers inside \((0, N)\) is larger than \(N \times \prod_{5 \leq p \leq (6m+1)}(1 - 2/p)\).

The remaining numbers less than \(N\) is the set, \(\{i < N; i \neq k_1,\text{or}\ N-k_1, \text{mod} (6k_1 + 1), \text{and} i \neq -k_1,\text{or}\ N+k_1, \text{mod} (6k_1 - 1), k_1 \leq m\}\);

Each remaining number \(i\) and \(N-i\) are id’s for possible primes of \((6i \pm 1)\), and \((6(N - i) \pm 1)\)

When the even Number is 6\(N+2\), take the possible pairs of, \((6i + 1)\), and \((6(N - i) + 1)\);

When the even Number is 6\(N\), take the possible pairs of, \((6i + 1)\), and \((6(N - i) - 1)\); or \((6i - 1)\), and \((6(N - i) + 1)\);

When the even Number is 6\(N-2\), take the possible pairs of, \((6i - 1)\), and \((6(N - i) - 1)\);

For example, if \(N= 100\), we get \(m=4\). Here we only need to sieve by primes of, 5,7,11,13,17,19,23,which are less than \((6m+1)\);
The total remaining number is larger than $100 \cdot (1-\frac{2}{5})(1-\frac{2}{7})(1-\frac{2}{11})(1-\frac{2}{13})(1-\frac{2}{17})(1-\frac{2}{19})(1-\frac{2}{23})$, which is about 21;

with $i$ equals to 5, 10, 12, 13, 17, 23, 27, 30, 32, 37, 38, 45, 55, 62, 63, 68, 70, 73, 77, 87, 90, 95;

here the actual total remaining id’s is 22. For example, when $i=5$, $N-i=95$, the pair primes is (31, 571) which add up to 602; or pair primes of (29, 569) which add up to 698; or pair primes of (29, 571), or (31, 569) which add up to 600.

This proves the Goldbach conjecture.