Gauss Symbol and

Unification of Gravity and Electromagnetic Field

in Reissner-Nordstrom and Kerr-Newman Solution

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ABSTRACT

Solutions of unified theory equations of gravity and electromagnetism satisfy Einstein-Maxwell equation. Hence, solutions of the unified theory is Reissner-Nordstrom solution, Kerr-Newman solution in vacuum. In this careful point, Einstein gravity field equation's energy-momentum tensor has two sign. Therefore, according to the solution, the sign is treated. We found in revised Einstein gravity tensor equation, the condition is satisfied by 2-order contravariant metric tensor two times product and Gauss symbol..

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ACS Number:**04,04.90.+e,41.12**

Key words:General relativity theory,

Unified Theory;

2-order contravariant metric tensor two times product;

Gauss Symbol

Energy-momentum's sign

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1.Introduction

This theory's aim is that we discover the revised Einstein gravity equation had Reissner-Nordstrom solution and Kerr-Newman solution in vacuum. In this careful point, Einstein gravity field equation's energy-momentum tensor has two sign. Therefore, according to the solution, the sign is treated.

First, the revised Einstein gravity equation is in recent my paper,

$$R_{\mu\upsilon} - \frac{1}{2} g_{\mu\upsilon} R + \Lambda g_{\mu\upsilon} I (g^{\theta\theta})^2 = \pm \frac{8\pi G}{c^4} T_{\mu\upsilon} \tag{1}$$

In this time,

$$\Lambda = k\frac{GQ^2}{c^4}, \quad I = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \tag{2}$$

If we think new revised Einstein gravity field equation using Gauss symbol for replacing the constant matrix I in Eq(1)-(2).

$$\Lambda = k\frac{GQ^2}{c^4},$$

$$I = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \text{ is replaced by } (-1)^{[\frac{\mu+\upsilon}{4}]} \tag{3}$$

In this time, Gauss symbol $[x]$ is the greatest integer that is less than or equal to x. For example,

$[3.141592] = 3$, $[2.71828] = 2$, $[-0.802] = -1$,...

Hence, new term is in Kerr-Newman solution,

$$(\mu,\upsilon) = (0,0) \to (-1)^{[\frac{\mu+\upsilon}{4}]} = 1 \ ,(\mu,\upsilon) = (1,1) \to (-1)^{[\frac{\mu+\upsilon}{4}]} = 1$$

$$(\mu,\upsilon) = (0,3) \to (-1)^{[\frac{\mu+\upsilon}{4}]} = 1 \ (\mu,\upsilon) = (3,0) \to (-1)^{[\frac{\mu+\upsilon}{4}]} = 1$$

$$(\mu,\upsilon) = (2,2) \to (-1)^{[\frac{\mu+\upsilon}{4}]} = -1 \ ,(\mu,\upsilon) = (3,3) \to (-1)^{[\frac{\mu+\upsilon}{4}]} = -1 \tag{4}$$

Hence, new symmetric revised Einstein gravity field Equation is

$$R_{\mu\upsilon} - \frac{1}{2} g_{\mu\upsilon} R + \Lambda g_{\mu\upsilon} (-1)^{[\frac{\mu+\upsilon}{4}]} (g^{\theta\theta})^2 = \pm \frac{8\pi G}{c^4} T_{\mu\upsilon} \tag{5}$$

We take the covariant differential operator,

$$(R_{\mu\upsilon} - \frac{1}{2} g_{\mu\upsilon} R)_{;\mu} + \Lambda g_{\mu\upsilon} (-1)^{[\frac{\mu+\upsilon}{4}]} 2 g^{\theta\theta} g^{\theta\theta}{}_{;\mu} = \pm \frac{8\pi G}{c^4} T_{\mu\upsilon;\mu} = 0 \tag{6-i}$$

$$(R_{\mu\upsilon} - \frac{1}{2} g_{\mu\upsilon} R)_{;\upsilon} + \Lambda g_{\mu\upsilon} (-1)^{[\frac{\mu+\upsilon}{4}]} 2 g^{\theta\theta} g^{\theta\theta}{}_{;\upsilon} = \pm \frac{8\pi G}{c^4} T_{\mu\upsilon;\upsilon} = 0 \tag{6-ii}$$

In this time, in Kerr-Newman solution [4]

$$g_{\theta\theta} = 1 / g^{\theta\theta} = \rho^2 = r^2 + a^2 \cos^2\theta$$

$$\begin{aligned} g^{\theta\theta}{}_{;\rho} &= \frac{\partial g^{\theta\theta}}{\partial x^\rho} + 2\Gamma^\theta{}_{\sigma\rho} g^{\sigma\theta} = \frac{\partial g^{\theta\theta}}{\partial r} + 2\Gamma^\theta{}_{\theta r} g^{\theta\theta} \\ &= \frac{\partial}{\partial r}(\frac{1}{\rho^2}) + 2 \cdot \frac{r}{\rho^2} \cdot \frac{1}{\rho^2} = -2\frac{1}{\rho^3} \cdot \frac{r}{\rho} + \frac{2r}{\rho^4} = 0 \end{aligned} \tag{7}$$

$$\begin{aligned} g^{\theta\theta}{}_{;\rho} &= \frac{\partial g^{\theta\theta}}{\partial x^\rho} + 2\Gamma^\theta{}_{\sigma\rho} g^{\sigma\theta} = \frac{\partial g^{\theta\theta}}{\partial \theta} + 2\Gamma^\theta{}_{\theta\theta} g^{\theta\theta} \\ &= \frac{\partial}{\partial \theta}(\frac{1}{\rho^2}) - 2 \cdot \frac{1}{\rho^2} \cdot \frac{1}{2\rho} \frac{2a^2}{\rho} \cos\theta \sin\theta \frac{}{\rho^2} \\ &= -2 \cdot \frac{1}{\rho^3} \cdot -\frac{2a^2 \cos\theta \sin\theta}{\rho} - \frac{4a^2}{\rho^4} \cos\theta \sin\theta = 0 \end{aligned} \tag{8}$$

If $g^{\theta\theta}{}_{;\rho} = V_\rho$, the vector transformation is

$$0 = V_\rho = \frac{\partial x'^\alpha}{\partial x^\rho} V'_\alpha \quad , \quad V'_\alpha = 0 \tag{9}$$

Therefore, if the coordinate is not the spherical coordinate, the covariant differential of $g^{\theta\theta} = \frac{1}{\rho^2}$ is still zero in the changed coordinate

2. The new revised Einstein gravity equation and Reissner-Nordstrom solution

In this theory, Eq(5) can change the following equation. This time, energy-momentum tensor's sign is minus in Einstein equation in Reissner-Nordstrom space-time,

$$R_{\mu\upsilon} = -\frac{8\pi G}{c^4}(T_{\mu\upsilon} - \frac{1}{2} g_{\mu\upsilon} T^\lambda{}_\lambda) + \Lambda g_{\mu\upsilon} (-1)^{[\frac{\mu+\upsilon}{4}]} (g^{\theta\theta})^2 \tag{10}$$

In this time, in vacuum, Eq(5) is in Reissner-Nordstrom solution.

$$R_{\mu\upsilon} = \Lambda g_{\mu\upsilon} (-1)^{[\frac{\mu+\upsilon}{4}]} (g^{\theta\theta})^2 = \Lambda g_{\mu\upsilon} (-1)^{[\frac{\mu+\upsilon}{4}]} \frac{1}{r^4} \tag{11}$$

Reissner-Nordstrom solution of Einstein-Maxwell equation is

$$\begin{aligned} & g_{00} = -1 + \frac{2GM}{rc^2} - \frac{kGQ^2}{r^2c^4}, g_{11} = 1/(1 - \frac{2GM}{rc^2} + \frac{kGQ^2}{r^2c^4}) \\ & g_{22} = r^2, g_{33} = r^2 \sin^2\theta \end{aligned} \tag{12}$$

The proper time of spherical coordinates is

$$d\tau^2 = A(t,r)dt^2 - \frac{1}{c^2}[B(t,r)dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\varphi^2]$$

$$(\mu,\upsilon) = (0,0) \to (-1)^{[\frac{\mu+\upsilon}{4}]} = 1 \ ,(\mu,\upsilon) = (1,1) \to (-1)^{[\frac{\mu+\upsilon}{4}]} = 1$$

$$(\mu,\upsilon) = (2,2) \to (-1)^{[\frac{\mu+\upsilon}{4}]} = -1, (\mu,\upsilon) = (3,3) \to (-1)^{[\frac{\mu+\upsilon}{4}]} = -1 \qquad (13)$$

If we use Eq(11),Eq(13), we obtain the Ricci-tensor equations.[3]

$$R_{tt} = -\frac{A''}{2B} + \frac{A'B'}{4B^2} - \frac{A'}{Br} + \frac{A'^2}{4AB} + \frac{\ddot{B}}{2B} - \frac{\dot{B}^2}{4B^2} - \frac{\dot{A}\dot{B}}{4AB} = -\Lambda A \frac{1}{r^4} \qquad (14)$$

$$R_{rr} = \frac{A''}{2A} - \frac{A'^2}{4A^2} - \frac{A'B'}{4AB} - \frac{B'}{Br} - \frac{\ddot{B}}{2A} + \frac{\dot{A}\dot{B}}{4A^2} + \frac{\dot{B}^2}{4AB} = \Lambda B \frac{1}{r^4} \qquad (15)$$

$$R_{\theta\theta} = -1 + \frac{1}{B} - \frac{rB'}{2B^2} + \frac{rA'}{2AB} = -\Lambda r^2 \frac{1}{r^4} = -\Lambda \frac{1}{r^2} \qquad (16)$$

$$R_{\varphi\varphi} = R_{\theta\theta} \sin^2\theta \qquad (17)$$

$$R_{tr} = -\frac{\dot{B}}{Br} = 0 \qquad (18)$$

$$R_{t\theta} = R_{t\varphi} = R_{r\theta} = R_{r\varphi} = R_{\theta\varphi} = 0 \qquad (19)$$

In this time, $' = \frac{\partial}{\partial r}$, $\cdot = \frac{1}{c}\frac{\partial}{\partial t}$

If we calculate,

$$\frac{R_{tt}}{A} + \frac{R_{rr}}{B} = -\frac{1}{Br}(\frac{A'}{A} + \frac{B'}{B}) = -\frac{(AB)'}{rAB^2} = 0 \qquad (20)$$

Hence, we obtain this result.

$$A = \frac{1}{B} \qquad (21)$$

If Eq(21) inserts Eq(16),

$$R_{\theta\theta} = -1 + \frac{1}{B} - \frac{rB'}{2B^2} + \frac{rA'}{2AB} = -1 + (\frac{r}{B})' = -\Lambda \frac{1}{r^2} \qquad (22)$$

If we solve Eq(22),

$$\frac{r}{B} = r + C + \frac{\Lambda}{r}$$

$$\to \ A = \frac{1}{B} = 1 - \frac{2GM}{rc^2} + \frac{kGQ^2}{r^2c^4}, \ \Lambda = k\frac{GQ^2}{c^4}, C = -\frac{2GM}{c^2} \qquad (23)$$

Therefore, in vacuum, the spherical solution of the revised Einstein gravity equation is Reissner-

Nordstrom solution.

$$d\tau^2 = (1-\frac{2GM}{rc^2}+\frac{kGQ^2}{r^2c^4})dt^2 - \frac{1}{c^2}[\frac{dr^2}{(1-\frac{2GM}{rc^2}+\frac{kGQ^2}{r^2c^4})} + r^2d\theta^2 + r^2\sin^2\theta d\varphi^2] \tag{24}$$

3. Kerr-Newman Solution and New Revised Gravity Field Equation

In this theory, Eq(5) can change the following equation. In this time, energy-momentum tensor's sign is plus in Einstein equation in Kerr-Newman space-time,

$$R_{\mu\upsilon} = \frac{8\pi G}{c^4}(T_{\mu\upsilon} - \frac{1}{2}g_{\mu\upsilon}T^{\lambda}{}_{\lambda}) + \Lambda(-1)^{[\frac{\mu+\upsilon}{4}]}g_{\mu\upsilon}(g^{\theta\theta})^2 \tag{25}$$

In this time, in vacuum, Eq(25) is in Kerr-Newman solution,

$$R_{\mu\upsilon} = \Lambda g_{\mu\upsilon}(-1)^{[\frac{\mu+\upsilon}{4}]}(g^{\theta\theta})^2 = \Lambda g_{\mu\upsilon}(-1)^{[\frac{\mu+\upsilon}{4}]}\frac{1}{\rho^4} \tag{26}$$

We think Kerr-Newman equation and solution.

Kerr-Newman solution is

$$ds^2 = g_{00}c^2dt^2 + 2g_{03}cdtd\phi + g_{11}dr^2 + g_{22}d\theta^2 + g_{33}d\phi^2$$

$$(\mu,\upsilon) = (0,0) \rightarrow (-1)^{[\frac{\mu+\upsilon}{4}]} = 1 \ ,(\mu,\upsilon) = (1,1) \rightarrow (-1)^{[\frac{\mu+\upsilon}{4}]} = 1$$

$$(\mu,\upsilon) = (0,3) \rightarrow (-1)^{[\frac{\mu+\upsilon}{4}]} = 1 \ (\mu,\upsilon) = (3,0) \rightarrow (-1)^{[\frac{\mu+\upsilon}{4}]} = 1$$

$$(\mu,\upsilon) = (2,2) \rightarrow (-1)^{[\frac{\mu+\upsilon}{4}]} = -1 \ ,(\mu,\upsilon) = (3,3) \rightarrow (-1)^{[\frac{\mu+\upsilon}{4}]} = -1$$

$$,\rho^2 = r^2 + a^2\cos^2\theta \tag{27}$$

Hence, Kerr-Newman equation is [5]

$$R_{tt} = \Lambda g_{00}\frac{1}{\rho^4} \tag{28}$$

$$R_{rr} = \Lambda g_{11}\frac{1}{\rho^4} \tag{29}$$

$$R_{\theta\theta} = -\Lambda g_{22}\frac{1}{\rho^4} = -\Lambda\frac{1}{\rho^2} \tag{30}$$

$$R_{\phi\phi} = -\Lambda g_{33}\frac{1}{\rho^4} \tag{31}$$

$$R_{t\phi}=R_{\phi t}=\Lambda g_{03}\frac{1}{\rho^4}=\Lambda g_{30}\frac{1}{\rho^4} \quad (32)$$

$$R_{\phi t}=R_{t\phi}=\Lambda g_{03}\frac{1}{\rho^4}=\Lambda g_{30}\frac{1}{\rho^4} \quad (33)$$

Otherwise $R_{\mu\upsilon}=0$

In this time, for proving Eq(28)-Eq(32), Maxwell tensor equations of Einstein-Maxwell equation are

$$-\Lambda g_{\mu\upsilon}(-1)^{[\frac{\mu+\upsilon}{4}]}(g^{\theta\theta})^2=R_{\mu\upsilon}-\frac{1}{2}g_{\mu\upsilon}R=\frac{8\pi G}{c^4}T_{\mu\upsilon}$$
$$=\frac{2G}{c^5}(F_{\mu\rho}F_{\upsilon}^{\ \rho}-\frac{1}{4}g_{\mu\upsilon}F_{\rho\sigma}F^{\rho\sigma})$$
$$=\frac{2G}{c^5}(g_{\mu\upsilon}F_{\upsilon\rho}F^{\upsilon\rho}-\frac{1}{4}g_{\mu\upsilon}F_{\rho\sigma}F^{\rho\sigma}) \quad (34)$$

In Kerr-Newman coordinates, if we calculate Eq(34) of g_{03}, g_{30} by g_{30} [4,7]

$$\frac{8\pi G}{c^4}T_{\mu\upsilon}=\frac{2G}{c^5}(g_{\mu\upsilon}F_{\upsilon\rho}F^{\upsilon\rho}-\frac{1}{4}g_{\mu\upsilon}F_{\rho\sigma}F^{\rho\sigma})$$
$$=\frac{8\pi G}{c^4}T_{30}=\frac{8\pi G}{c^4}T_{03}=\frac{2G}{c^5}(g_{30}F_{01}F^{01}-\frac{1}{4}g_{30}(2B^2-2E^2))$$
$$=\frac{2G}{c^5}(-g_{30}E^2-\frac{1}{2}g_{30}(B^2-E^2))=-\frac{G}{c^5}(B^2+E^2)g_{30}=-k\frac{G}{c^4}\frac{Q^2}{\rho^4}g_{30}$$
$$=-\Lambda g_{30}(-1)^{[\frac{0+3}{4}]}\frac{1}{\rho^4}=-\Lambda g_{30}\frac{1}{\rho^4}=-\Lambda g_{03}\frac{1}{\rho^4}=-g_{03}(-1)^{[\frac{0+3}{4}]}\frac{1}{\rho^4}$$

$$E=F_{01}=-F_{10}=\frac{Q(r^2-a^2\cos^2\theta)}{\rho^4},\quad B=F_{23}=-F_{32}=\frac{2Qar\cos\theta}{\rho^4}$$
$$B^2+E^2=\frac{Q^2}{\rho^4} \quad (35)$$

In this careful point, if we calculate Eq(34) by g_{03}, the calculation is wrong.

In Kerr-Newman coordinates, if we calculate Eq(34) of g_{00} [4,7]

$$\frac{2G}{c^5}(g_{\mu\upsilon}F_{\upsilon\rho}F^{\upsilon\rho}-\frac{1}{4}g_{\mu\upsilon}F_{\rho\sigma}F^{\rho\sigma})$$
$$=\frac{2G}{c^5}(g_{00}F_{01}F^{01}-\frac{1}{4}g_{00}(2B^2-2E^2))$$

$$= \frac{2G}{c^5}(-g_{00}E^2 - \frac{1}{2}g_{00}(B^2 - E^2)) = -\frac{G}{c^5}(B^2 + E^2)g_{00} = -k\frac{G}{c^4}\frac{Q^2}{\rho^4}g_{00}$$

$$= -\Lambda g_{00}(-1)^{[\frac{0}{4}]}\frac{1}{\rho^4} = -\Lambda g_{00}\frac{1}{\rho^4}$$

$$E = F_{01} = -F_{10} = \frac{Q(r^2 - a^2\cos^2\theta)}{\rho^4}, \quad B = F_{23} = -F_{32} = \frac{2Qar\cos\theta}{\rho^4}$$

$$B^2 + E^2 = \frac{Q^2}{\rho^4} \tag{36}$$

In Kerr-Newman coordinates, if we calculate Eq(34) of g_{11} [4,7]

$$\frac{2G}{c^5}(g_{\mu\upsilon}F_{\ \ \upsilon\rho}F^{\upsilon\rho} - \frac{1}{4}g\ \ F_{\mu\upsilon}F_{\rho\sigma}^{\ \ \rho\sigma})$$

$$= \frac{2G}{c^5}(g_{11}F_{10}F^{10} - \frac{1}{4}g_{11}(2B^2 - 2E^2))$$

$$= \frac{2G}{c^5}(-g_{11}E^2 - \frac{1}{2}g_{11}(B^2 - E^2)) = -\frac{G}{c^5}(B^2 + E^2)g_{11} = -k\frac{G}{c^4}\frac{Q^2}{\rho^4}g_{11}$$

$$= -\Lambda g_{11}(-1)^{[\frac{1+1}{4}]}\frac{1}{\rho^4} = -\Lambda g_{11}\frac{1}{\rho^4}$$

$$E = F_{01} = -F_{10} = \frac{Q(r^2 - a^2\cos^2\theta)}{\rho^4}, \quad B = F_{23} = -F_{32} = \frac{2Qar\cos\theta}{\rho^4}$$

$$B^2 + E^2 = \frac{Q^2}{\rho^4} \tag{37}$$

In Kerr-Newman coordinates, if we calculate Eq(34) of g_{22} [4,7]

$$\frac{2G}{c^5}(g_{\mu\upsilon}F_{\ \ \upsilon\rho}F^{\upsilon\rho} - \frac{1}{4}g\ \ F_{\mu\upsilon}F_{\rho\sigma}^{\ \ \rho\sigma})$$

$$= \frac{2G}{c^5}(g_{22}F_{23}F^{23} - \frac{1}{4}g_{22}(2B^2 - 2E^2))$$

$$= \frac{2G}{c^5}(g_{22}B^2 - \frac{1}{2}g_{22}(B^2 - E^2)) = \frac{G}{c^5}(B^2 + E^2)g_{22}$$

$$= k\frac{G}{c^4}\frac{Q^2}{\rho^4}g_{22} = -\Lambda g_{22}(-1)^{[\frac{2+2}{4}]}\frac{1}{\rho^4} = \Lambda g_{22}\frac{1}{\rho^4}$$

$$E = F_{01} = -F_{10} = \frac{Q(r^2 - a^2\cos^2\theta)}{\rho^4}, \quad B = F_{23} = -F_{32} = \frac{2Qar\cos\theta}{\rho^4}$$

$$B^2 + E^2 = \frac{Q^2}{\rho^4} \tag{38}$$

In Kerr-Newman coordinates, if we calculate Eq(34) of g_{33} [4,7]

$$\frac{2G}{c^5}(g_{\mu\upsilon} F_{\upsilon\rho} F^{\upsilon\rho} - \frac{1}{4} g_{\mu\upsilon} F_{\rho\sigma} F^{\rho\sigma})$$

$$= \frac{2G}{c^5}(g_{33}F_{32}F^{32} - \frac{1}{4} g_{33}(2B^2 - 2E^2))$$

$$= \frac{2G}{c^5}(g_{33}B^2 - \frac{1}{2} g_{33}(B^2 - E^2)) = \frac{G}{c^5}(B^2 + E^2)g_{33} = k\frac{G}{c^4}\frac{Q^2}{\rho^4} g_{33}$$

$$= -\Lambda g_{33}(-1)^{[\frac{3+3}{4}]} \frac{1}{\rho^4} = \Lambda g_{33} \frac{1}{\rho^4}$$

$$E = F_{01} = -F_{10} = \frac{Q(r^2 - a^2\cos^2\theta)}{\rho^4}, \quad B = F_{23} = -F_{32} = \frac{2Qar\cos\theta}{\rho^4}$$

$$B^2 + E^2 = \frac{Q^2}{\rho^4} \tag{39}$$

According to Eq(35), in Kerr-Newman solution, the electromagnetic energy-momentum tensor's formula is equal with the new revised Einstein equation's added term of our unified theory. Hence, we know if we solve the revised Einstein's equation, we can obtain Kerr-Newman solution.

4. Conclusion

We found the revised Einstein equation of unified theory (the gravity and electromagnetic field).This theory's strong point is 4-dimensional theory. This theory is different from 5-dimensional Kaluza-Klein theory. But as the method of describing universe, Einstein normal gravity equation is equal with the revised Einstein equation of the unified theory because the electric charge has to be zero. .

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