EUCLID’S FORMULA IMPLIES FERMAT’S LAST THEOREM FOR INTEGRAL EXPONENT > 2

PHILIP AARON BLOOM, VERSION 1, BRAINEMAIL1@GMAIL.COM

Abstract. Euclid’s formula holds for the set of all coprime Pythagorean triples per a well-established proof by rational points on the unit circle. We generalize Euclid’s formula to directly imply Fermat’s Last Theorem for any given integral exponent greater than two.

A well-known algebraic identity, Euclid’s formula, holds for the set of all coprime Pythagorean triples, as determined by the well-established demonstration of rational points on a unit circle. Euclid’s formula, with \(t, s\) positive integral, \(t > s\), can be equivalently rewritten as:

(1) \((t^2 - s^2)^2 + (2ts)^2 = (t^2 + s^2)^2\).

In identity (1) we can substitute \(t^n\) for \(t\) and \(s^n\) for \(s\), such that \(n\) has any given value of positive integers, to yield:

(2) \((t^n)^2 - (s^n)^2)^2 + (2(t^n)(s^n)^2) = ((t^n)^2 + (s^n)^2)^2\).

Equation (2) reduces to:

(3) \((t^n - s^n)^2 + (2t^n s^n)^2 = (t^n + s^n)^2\).

We transform the Pythagorean triple for which (3) holds, \((t^n - s^n), (2t^n s^n), (t^n + s^n)\) into the Fermat triple for which (4), below, holds, \((t^n - s^n)^{\frac{n}{2}}, (2t^n s^n)^{\frac{n}{2}}, (t^n + s^n)^{\frac{n}{2}}\):

(4) \( (t^n - s^n)^{\frac{n}{2}} + (2t^n s^n)^{\frac{n}{2}} = (t^n + s^n)^{\frac{n}{2}} \).

Equation (4) can be reduced, as follows:

(5) \( (t^n - s^n)^{\frac{n}{2}} + (2t^n s^n)^{\frac{n}{2}} = (t^n + s^n)^{\frac{n}{2}} \).

Note in (5) that middle term \(2t^n s^n\) can not be integral for \(n > 2\) with \(t, s\) integral, and \(2t^n\) being rational solely for the natural numbers \(n = 1, 2\). Hence, for \(n > 2\), the set of all coprime Fermat triples is null. Thus, for \(n > 2\), the set of all integral Fermat triples is null. Q.E.D.