An Injection Locked Oscillator as Adaptive Filter

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Summary: An Injection Locked Synchronous Oscillator is a narrowband digital filter that tracks its frequency to a variable frequency signal. This allows the detection of weak signals of variable frequency despite a strong noise background. The signal is processed in the time domain with negligible phase shift. The calculation effort is much lower than with comparable filter types.

Introduction: The following program is a building block for the analysis of NMR data in the earth's magnetic field. The precession frequency of the protons, which varies around 2140 Hz in Central Europe, is digitized with a 24-bit ADC. Without further measures, the Nyquist theorem would require a sampling rate of at least 5000 Hz, and long-term measurements would generate a daily storage volume of more than 1 GB. Too much data for a manageable amount of information.

Preliminary studies have shown that the magnetic field usually changes so slowly that a sampling frequency below ten Hertz is sufficient. Therefore, the signal frequency is reduced to 40 Hz before digitization and storage. Before analysis, the program mixes the frequency of the stored data down to five Hertz. The frequency changes do not alter the signal content, but have several advantages: The necessary small bandwidth of the filters can be realized more easily and the computing time and memory requirements are reduced by a factor of 400.

The spectrogram in Fig. 1 shows that the frequency of proton NMR in the Earth's magnetic field can change by about one Hertz within a few seconds, which is a multiple of the desired bandwidth. The extremely weak NMR signals in magnetic fields below 1 mT are difficult to detect in noise and because their frequency changes constantly and unpredictably, common narrow-band filters are unsuitable. Automatic tracking is needed to ensure that the filter corridor follows the frequency of the input signal. If the bandwidth is too small and the center frequency is constant, the signal will leave the filter corridor and is no longer detectable. If the bandwidth is increased, too much noise will pass through the filter and the signal will disappear in the noise.

An Injection Locked Oscillator (ILO) can solve this problem because it can be easily synchronized by an external signal. A common ILO can regenerate noisy signals of constant frequency very well, but fails if the signal frequency leaves the narrow corridor of the hold-in range. Therefore, a usable circuit must meet the following requirements:

• The bandwidth should not exceed 0.1 Hz to sufficiently improve the signal-to-noise ratio. Since high filter qualities often produce unwanted side effects like ringing artifacts, the required bandwidth forces filter frequencies below 10 Hz.
• The frequency of the ILO must automatically follow that of the signal even if the bandwidth is exceeded several times. Fig. 1 shows that the signal frequency moves in a range that exceeds the bandwidth up to 20 times.
• If the signal amplitude is too low during some oscillations, the frequency and phase of the oscillator may change only slightly during this time, so that the oscillator locks in immediately as soon as the signal amplitude is sufficient again.

The function of an ILO without frequency tracking is described in detail here [1]. In the following, we will only deal with the extension by a frequency tracking.

Fig 1: NMR signal after selective filtering with an ILO (BW=0.1 Hz).

1 H. Weidner, Detection of Weak Signals with an Injection Locked Oscillator, 2020
In the spectrogram in Fig. 2 you can see that an ILO without frequency tracking only locks-in if the variable signal frequency randomly falls within the narrow catch range. Otherwise the ILO oscillates at its preset, almost constant frequency and ignores the lower frequency signal. If the signal frequency deviates too much from the preset frequency of the ILO after lock-in, the phase difference between signal and ILO exceeds the critical value $\pi/2$ and the synchronization ends.

There are two solutions to permanently synchronize an ILO despite large frequency changes of the signal:

1. The frequency of the ILO is increased to widen the lock-in range, which is about 5% of the average frequency. But this measure also increases the bandwidth and therefore the signal-to-noise ratio deteriorates.
2. The current frequency of the ILO is measured continuously and its programmed target frequency is corrected to reduce the phase difference between signal and ILO. With this method it is possible to follow very wide frequency shifts of the signal in spite of a small bandwidth.

**Frequency tracking**: The signal frequency can usually not be determined directly and before filtering because the amplitude is too small and too noisy. Therefore the program (assuming synchronization) calculates the actual frequency $f_{\text{new}}$ of the ILO at regular intervals (e.g. after ten oscillations) and compares it with the previous value $f_{\text{old}}$. To keep the difference minimal, the program calculates corrected parameters "z" and "v" of the ILO again and again (the tasks of these two values are described in [1]). In this way, the setpoint frequency of the ILO follows the actual signal frequency.

For the calculation there are proven methods of control engineering which must be adapted to the signal quality:

- If the frequency changes slowly and a good signal-to-noise ratio is present, a **PD controller** will give good results. You can also detect fast changes because the differential component "anticipates" them and reacts immediately. The many small corrections increase the phase noise of the output signal.
- If the SNR is poor, a P or PI controller can give better results because they are blind to fast changes. Therefore the output signal is characterized by low phase noise.
- The "**linear prediction**" method gives comparatively moderate results and has not been studied in detail.

The executable program of a tracked ILO is a minimal framework, contains only the necessary steps and no precautions against data errors:
%3-Ring oscillator with frequency tracking
%the file ys contains the data from the ADC; The sampling rate is Fs=48;

ys1=0.6e-7*ys; %Normalize range of values: -0.1<ys1<0.1;
ILO=zeros(length(ys1),3); %result channels
ILO(1,3)=1e-3; %Starter for the iterations
b=3.5; %Hz Startfreq of the lock-in range, experimentally determine or guess
z = min(roots([-2.07,9.6953,0.2468-b]));
zk=1-z; %speed up the program
v=polyval([-2.03572,5.344,-7.355],z); %amplification of each stage

for k=2:20 %short leader because of frequency tracking
    b=v*({1/(1+exp(-ILO(k-1,3))))-0.5); ILO(k,1)=z*b+zk*ILO(k-1,1)+ys1(k);
    b=v*({1/(1+exp(-ILO(k-1,1))))-0.5); ILO(k,2)=z*b+zk*ILO(k-1,2);
    b=v*({1/(1+exp(-ILO(k-1,2))))-0.5); ILO(k,3)=z*b+zk*ILO(k-1,3);
end
while k>2 %where is the last zero crossing?
    if ILO(k-1,3)<0 && ILO(k,3)>0, break, end
k=k-1;
end, k0=k+ILO(k,3)/(ILO(k-1,3)-ILO(k,3)); zaehl=1;
for k=21:size(ILO,1) %main program
    b=v*({1/(1+exp(-ILO(k-1,3))))-0.5); ILO(k,1)=z*b+zk*ILO(k-1,1)+ys1(k);
    b=v*({1/(1+exp(-ILO(k-1,1))))-0.5); ILO(k,2)=z*b+zk*ILO(k-1,2);
    b=v*({1/(1+exp(-ILO(k-1,2))))-0.5); ILO(k,3)=z*b+zk*ILO(k-1,3);
end
if ILO(k-1,3)<0 && ILO(k,3)>0, zaehl=zaehl+1;
    if zaehl>5 %Length of an oscillation packet 3..20
        k1=k+ILO(k,3)/(ILO(k-1,3)-ILO(k,3)); %compute zero crossing
        b=(zaehl-1)*Fs/(k1-k0); %actual frequency
        b = min(roots([-2.07,9.6953,0.2468-b]));
        z=(z+b)/2; %PI controller
        %z=(3*z+b)/4; %slower PI
        %z=b+(b-z)/5; %PD controller, very nervous
        v=polyval([-2.03572,5.344,-7.355],z); %update gain
        zk=1-z; k0=k1; zaehl=1;
    end
end

spectrogram(ILO(:,3),300,240,2^11,Fs); beep

Fig 3: Error-free synchronization of the ILO by the signal thanks to frequency tracking. The raw data for Fig 2 and Fig 3 are identical.