Infinite Twin Primes from Twin Surfaces

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September 12, 2020

Abstract

A proof that there are infinitely many Twin Primes is presented in 5 sections. Section 1 is a brief statement of the problem and general approach to the proof. Part 2 describes a concise condition that can be used to prove or disprove that there are infinite Twins. Sections 3 and 4 show that the condition affirms infinite twins, and the last segment is a short summary conclusion.

1 A Statement of the Problem and the Approach to the Proof

The Twin Prime Conjecture is the well known topic within the field of mathematics regarding whether or not there are an infinite number of prime numbers separated by a difference of 2. This proof uses a strategy that is most generally split into 2 halves. First, 2 surfaces are determined such that choosing any natural number which is not found on either surface generates a Twin Prime Pair. Then, the proof shows that there are infinite such natural number generators.

As stated in the paper's abstract, the proof is more specifically presented in 5 sections, this being the brief statement of the problem and general approach to the solution. Part 2 describes a concise condition that can be used to prove or disprove that there are infinite Twins. Sections 3 and 4 show that the condition is true, and the last segment is a short summary conclusion.

2 Infinite Twins from 2 Surfaces

The proof begins with the given that all primes except the number 2 are odd. This means that all primes, other than the number 2, can be expressed in the form of 2n+1 for some natural number n. Next, use the fact that all odd numbers except 1 are either prime or an odd composite. This means that all odd numbers greater than 1, which are not odd composites, are prime. Thirdly, use the fact that all odd composites are the product of 2 odd numbers greater than 1. These 3 givens are expressed as equation 1, which is interpreted as stating that primes are all odds greater than 1 that are not odd composites, for positive natural number inputs n,a, and b.

$$\text{primes} = 2n + 1 \neq (2a + 1)(2b + 1) \tag{1}$$

Since twin primes have a difference of 2, if $2n + 1$ is the smaller prime of a pair, then $2n + 3$ is the larger. Using the same logic as equation 1 means that $2n + 3$ must also not be an odd composite. This is stated in equation 2 for positive natural inputs n, c, and d.

$$\text{upper twin} = 2n + 3 \neq (2c + 1)(2d + 1) \tag{2}$$

Simplifying equations 1 and 2 gives equations 3 and 4 respectively.

$$n \neq 2ab + a + b \tag{3}$$

$$n \neq 2cd + c + d - 1 \tag{4}$$

Equations 3 and 4 represent basic surfaces in 3 coordinates, or rather "anti-surfaces" due to the does not equal sign, stating what the values can not be. Moreover, the second surface, eq.4, represents the same surface as the first, eq.3, only slid down by a value of 1. Figures 1 and 2 show truncated tables of the positive values of the surfaces, beginning outside the origin for natural inputs ≥ 1.

Figure 1: 2ab+a+b

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	4	7	10	13	16	19	22	25	28	31	34	37	40	43	46
2	7	12	17	22	27	32	37	42	47	52	57	62	67	72	77
3	10	17	24	31	38	45	52	59	66	73	80	87	94	101	108
4	13	22	31	40	49	58	67	76	85	94	103	112	121	130	139
5	16	27	38	49	60	71	82	93	104	115	126	137	148	159	170
6	19	32	45	58	71	84	97	110	123	136	149	162	175	188	201
7	22	37	52	67	82	97	112	127	142	157	172	187	202	217	232
8	25	42	59	76	93	110	127	144	161	178	195	212	229	246	263
9	28	47	66	85	104	123	142	161	180	199	218	237	256	275	294
10	31	52	73	94	115	136	157	178	199	220	241	262	283	304	325
11	34	57	80	103	126	149	172	195	218	241	264	287	310	333	356
12	37	62	87	112	137	162	187	212	237	262	287	312	337	362	387
13	40	67	94	121	148	175	202	229	256	283	310	337	364	391	418
14	43	72	101	130	159	188	217	246	275	304	333	362	391	420	449
15	46	77	108	139	170	201	232	263	294	325	356	387	418	449	480

Figure 2: 2cd+c+d-1

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	3	6	9	12	15	18	21	24	27	30	33	36	39	42	45
2	6	11	16	21	26	31	36	41	46	51	56	61	66	71	76
3	9	16	23	30	37	44	51	58	65	72	79	86	93	100	107
4	12	21	30	39	48	57	66	75	84	93	102	111	120	129	138
5	15	26	37	48	59	70	81	92	103	114	125	136	147	158	169
6	18	31	44	57	70	83	96	109	122	135	148	161	174	187	200
7	21	36	51	66	81	96	111	126	141	156	171	186	201	216	231
8	24	41	58	75	92	109	126	143	160	177	194	211	228	245	262
9	27	46	65	84	103	122	141	160	179	198	217	236	255	274	293
10	30	51	72	93	114	135	156	177	198	219	240	261	282	303	324
11	33	56	79	102	125	148	171	194	217	240	263	286	309	332	355
12	36	61	86	111	136	161	186	211	236	261	286	311	336	361	386
13	39	66	93	120	147	174	201	228	255	282	309	336	363	390	417
14	42	71	100	129	158	187	216	245	274	303	332	361	390	419	448
15	45	76	107	138	169	200	231	262	293	324	355	386	417	448	479

What these 2 surfaces dictate is the following. Surface 1, eq.3, restricted to the natural domain, is all n such that $2n+1$ will be an odd composite. Therefore, choosing any natural value for n that is not on that surface, i.e. values not from the table represented by figure 1, will make $2n+1$ a prime number. In fact, the natural numbers n that are not from the natural range of surface 1, generate all the primes except the number 2, without exception, and when ordered, generate them in order.

Similarly, surface 2 is all n such that $2n+3$ will be an odd composite. So, choosing a natural value n that is not found on either surface, ensures that both $2n+1$ and $2n+3$ are prime, and it generates a Twin Pair. In fact, like surface 1 does alone for the primes, choosing a number not on either, generates all Twin Pairs without exception, and when ordered, generates them in order. Since choosing a natural value n that is on neither surface will always generate a unique Twin Pair, showing that there are infinite such generating values proves the Twin Prime Conjecture.

At this point, it may intuitively seem that there are an infinite number of Twin Primes, since one can always find another value not on either surface $2ab+a+b$ or $2ab+a+b-1$, and this makes sense. As a quick initial verification, try the first few values. The first value not on either chart is the number 1. Applying $2n+1$ gives 3, corresponding to 3, 5, the first Twin Pair. The next missing value is 2. This time $2n+1$ gives 5, corresponding to 5 and 7. Let's do 2 more. Next is $n=5$, since 3 and 4 do appear, and $2(5)+1$ is 11 for 11 and 13. The last example, is the next missing number $n=8$, giving $2n+1=17$ for 17 and 19. Remember, these charts are only portions of the full surfaces which extend infinitely, so you'll have to consult expanded versions when using them to confirm values beyond 45 in the first row.

To reiterate, in order to prove the Twin Prime conjecture, it must be shown that there is an infinite number of natural numbers not in the range of either surface when their domains are natural numbers. In concept it's straightforward, but for me, this is easier said than done. Over time, I have found a few similar strategies to do so, some better in ways, or easier to explain than others, and it is here that I describe what is currently the easiest of those for me to explain. I suspect that others know, or can devise, more direct methods to show it compared to the technique that I offer below.

3 The Values Within or Outside the Range of the Surfaces

The general method that is used to show that there are infinite numbers outside both ranges takes the following path. Hold one variable constant, in order to decompose the surfaces into an infinite number of lines, and thus assign each line to a row as shown in the tables. Show that there are infinite numbers outside the range of each specific row/line. Next, show that there are infinite numbers outside the range of any 2 adjacent rows. Finally, show that there are infinite numbers outside the range of any number of consecutive rows.

Examine the form of the values generated in each row on both tables. These are the result of choosing a row number and setting one variable for the input of the surfaces to that value using equations 3 and 4. Notice that the surfaces are symmetric from the variable's standpoints, that is diagonalized, and so it doesn't matter which variable you use for this purpose. In this case, a and c were chosen as rows, and b and d for columns. The values can be written as 1 line per row per table, such that an infinite family of lines represent all the values on either surface.

Since the values on the 2nd surface are simply 1 less than those on the first, the y intercepts are one less for the corresponding lines. Look at the values in rows one, where a and c are held constant and set equal to 1. That is, $2(1)b+1+b$ and $2(1)d+1+d-1$. The surfaces simplify into the following lines.

All values appearing in row 1 of either surface.

$$3x + 1 \quad \text{and} \quad 3x + 0 \tag{5}$$

This can be repeated for any row, and shows the following pattern. The next 3 rows are shown as a reference.

All values appearing in row 2 of either surface.

$$5x + 2 \quad \text{and} \quad 5x + 1 \tag{6}$$

All values appearing in row 3 of either surface.

$$7x + 3 \quad \text{and} \quad 7x + 2 \tag{7}$$

All values appearing in row 4 of either surface.

$$9x + 4 \quad \text{and} \quad 9x + 3 \tag{8}$$

Notice that the slopes are the set of odd numbers, that the y intercepts differ by 1 between surfaces per row, and that they also increase by 1 down the rows.

Now take a look at the form of the values NOT generated in the rows by the surfaces; that is, outside the range of the surfaces. There is an increasing odd amount of lines and values per row, determined by the row number, which represent all the values not appearing in that row on either surface.

All values not appearing in row 1 of either surface.

$$3x + 2 \tag{9}$$

All values not appearing in row 2 of either surface.

$$5x + 3, 5x + 4, 5x + 5 \tag{10}$$

All values not appearing in row 3 of either surface.

$$7x+4, 7x+5, 7x+6, 7x+7, 7x+8 \quad (11)$$

Notice that the slopes are again the set of odd numbers, that the y intercepts span a consecutive range between surfaces per row, and that they increase by 1 for the elements of that range down the rows.

In order for an integer n to not be in the range of either surface, it must not appear in any row on either table for the surfaces. Equations 9, 10, and 11 show the general pattern for all values not in any specific row. The next step is to use that pattern to find values that do not occur in any and all rows. Also note, that due to the modular nature of the sets of lines, increasing the y intercept further causes a congruence with an earlier line and set of values in that row. For example, $5x+6$, which would be the next value in eq.10, generates the same range of values as $5x+1$, eq.6, except the first value of course.

3.1 Values not Occurring on any Row or Rows

To find at least one set of values not in any row on either surface, and thus not on either surface, begin by noticing for rows 1, that all the values greater than the first elements of those rows, which do not appear in either row, are all the natural values of the line $3x+2$, eq.9. That is, that the infinite set {5,8,11,14,17,20,...}, and so on, are not on either row 1. Put that aside for the moment and look now at the 2nd row.

Using eq.6, $5x+1$ and $5x+2$ are excluded, because they produce values on the 2nd rows. However, eq.10 shows that there are 3 sets of values that are never on rows 2. The first set is $5x+3$. Repeating this for rows 3, for the first set of values never on the rows, yields $7x+4$, for rows 4, it yields $9x+5$, for rows 5, $11x+6$, and so on.

This means that numbers in the intersection of the sets $3x+2$ and $5x+3$ are not on the first 2 rows of either surface. Numbers from that intersection that are also in the set $7x+4$ are then not on the first 3 rows. Continuing the process means that finding an infinite number of values in the intersection of the sets of all those lines shows that there is an infinite number of values not on any row, and therefore, not on either surface.

4 Infinite Values not Within the Range

Going forward, it is very handy when explaining, to have a table of the values generated by these lines, in order to help visualize the relations between each row or check some values. The general formula of the family of lines that were chosen for each row r, showing values on neither surface for that row, is equation 12.

$$(2r+1)x+r+1 \quad (12)$$

In Figure 3, values that would continue to the right have been broken into 2 more groups and moved below so that more values could be shown. The lines for the rows are labeled on the left column.

Figure 3: (2r+1)x+r+1

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
1	3x+2	5	8	11	14	17	20	23	26	29	32	35	38	41	44	47
2	5x+3	8	13	18	23	28	33	38	43	48	53	58	63	68	73	78
3	7x+4	11	18	25	32	39	46	53	60	67	74	81	88	95	102	109
4	9x+5	14	23	32	41	50	59	68	77	86	95	104	113	122	131	140
5	11x+6	17	28	39	50	61	72	83	94	105	116	127	138	149	160	171
6	13x+7	20	33	46	59	72	85	98	111	124	137	150	163	176	189	202
7	15x+8	23	38	53	68	83	98	113	128	143	158	173	188	203	218	233
8																
9																
10	3x+2	50	53	56	59	62	65	68	71	74	77	80	83	86	89	92
11	5x+3	83	88	93	98	103	108	113	118	123	128	133	138	143	148	153
12	7x+4	116	123	130	137	144	151	158	165	172	179	186	193	200	207	214
13	9x+5	149	158	167	176	185	194	203	212	221	230	239	248	257	266	275
14	11x+6	182	193	204	215	226	237	248	259	270	281	292	303	314	325	336
15	13x+7	215	228	241	254	267	280	293	306	319	332	345	358	371	384	397
16	15x+8	248	263	278	293	308	323	338	353	368	383	398	413	428	443	458
17																
18																
19	3x+2	95	98	101	104	107	110	113	116	119	122	125	128	131	134	137
20	5x+3	158	163	168	173	178	183	188	193	198	203	208	213	218	223	228
21	7x+4	221	228	235	242	249	256	263	270	277	284	291	298	305	312	319
22	9x+5	284	293	302	311	320	329	338	347	356	365	374	383	392	401	410
23	11x+6	347	358	369	380	391	402	413	424	435	446	457	468	479	490	501
24	13x+7	410	423	436	449	462	475	488	501	514	527	540	553	566	579	592
25	15x+8	473	488	503	518	533	548	563	578	593	608	623	638	653	668	683

4.1 Infinite Values in the Intersection of Adjacent Rows

When comparing 2 rows, the full intersection of the sets is of interest, requiring each line to get its own variable. For the first 2 rows, the relationship is equation 13.

$$3x + 2 = 5y + 3 \tag{13}$$

This has the integer solutions in eq.14 for an integer s.

$$x = 5s + 2 \quad y = 3s + 1 \tag{14}$$

The integer solutions and adjacent row relation are used to calculate the positions and values of the intersection between those rows. Starting with $s = 0$ in eq.14, and using the resulting values in eq.13, shows that the first element of row 2, 8 in this case, will map to the 2nd element of row one, again 8, and that every 3rd element thereafter on row 2, will map to every 5th element thereafter on row 1. Checking the $s = 1$ case gives $y = 4$ and $x = 7$, and we indeed see the 4th element on row 2, value 23, mapping to the 7th element of row 1. The next shared value in this instance would be $s = 2$ with row 2 element 7 value 38, and row 1 element 12 value 38.

This establishes an infinite number of values in the intersection of these 2 rows, 1 for each s, and therefore shows an infinite number of values not on the first 2 rows of the surfaces. Using the general formula for rows from eq.12 allows for the solution between any 2 adjacent rows. Using rows r and $r + 1$, in the same way as was done in eq.13 for rows 1 and 2, gives eq.15.

$$(2r+1)x+r+1=(2(r+1)+1)y+(r+1)+1 \tag{15}$$

This has the integer solutions in eq.16 for a row r and an integer s.

$$x=(2r+3)s+r+1 \quad y=(2r+1)s+r \tag{16}$$

This verifies that there is an infinite number of solutions in the intersection of any two adjacent rows, and therefore an infinite number of values not on those 2 rows of the surfaces.

4.2 Specific Shared Values For the First 3 Rows

The goal is to show an intersection common to all rows. To do this, it helps to show the common values through the first 3 rows. When the term position is used below, it generally refers to the ordinal value or location of a member within a set, not the actual value of that element. This can also be thought of as the column value in Figure 3, though not to confuse things, rather what would be those column's proper values were they not all offset by 1 to the right due to the labels in column A as pictured.

It has already been shown that all values from the first row, $3x+2$, are not on the first rows of the surfaces. It was also shown that an infinite number of elements will map between any row r and row $r+1$. The question now, is which specific elements map between rows? Start by examining the second row, $5x+3$. Eq.14 showed that the elements that map from $5x+3$ in the second row, into the first row, are in the $3s+1$ positions of the second row. That is, the positions {1,4,7,10,...} of row 2 that correspond to the values {8,23,38,53,...}.

Now repeat the question, this time asking not only which values will map from row 3 to row 2, but which values will map from row 3 to the specific values in row 2 that were mentioned, and thus allow them to intersect with row 1 also? Using eq.16 with $r=2$ gives the location of the overall elements that map from row 3 to row 2 as the elements in the $y=5s+2$ positions of the 3rd row. It also shows that they map to $x=7s+3$ positions in the 2nd row. Since it was shown that all $3s+1$ locations in the 2nd row map to the first, this means that whenever $3x+1=7y+3$, eq.17, an element in a $5s+2$ position of row 3 maps to a position in row 2 that will then go on to map to row 1.

$$3x+1=7y+3 \tag{17}$$

This has the integer solutions in eq.18 for an integer s.

$$x=7s+3 \quad y=3s+1 \tag{18}$$

Since there are integer solutions of $x=7s+3$ and $y=3s+1$, it also means that there is an infinite number of such elements. However, because of the $3x+1=7y+3$ requirement, it must now be noted that though an infinite number do map, not all of those elements in $5s+2$ positions on the 3rd row will map to a location on the 2nd row that continues on to row 1.

As an example, the first integer solution for y is $y = 3s + 1$ with $s = 0$, giving $y = 1$. Since the $5s + 2$ positions on the 3rd row map to the $7s + 3$ spots on the 2nd, it means that the $5(1) + 2 = 7$th element on row 3 maps to the $7(1) + 3 = 10$th element on row 2. When checked, the value 53 maps between those locations, and also continues on and is found in the first row. The next solution, with $s = 1$, gives $y = 4$. This translates to the $5(4)+2 = 22$nd element on row 3 mapping to the $7(4) + 3 = 31$st element on the 2nd row. Again, when checked, the value 158 maps between those locations, and also continues on and is found in the first row.

4.3 Intersections Across Subsequent Rows

In order to proceed to the 4th and subsequent rows, the question must first be answered of which specific subset of elements from the $5s + 2$ positions in the 3rd row map to the proper positions in the 2nd row. Also note that the $s = 0$ position actually represents the first member of a set, $s = 1$ the second, and so on, and as such, that the ordinal location value of an element within a set is 1 greater than that integer s when spoken of in terms of being the first, second, or "xth" element of that set. As shown in the above example, the first value on row 3 that meets all the requirements is 53. Inserting the $5s + 2$ positions mapping from row 3 to row 2 into its row value of $7x + 4$ for x, gives $35s + 18$. Set it equal to the first intersection of all 3 rows, 53, eq.19.

$$7(5s + 2) + 4 = 35s + 18 = 53 \tag{19}$$

Solving for s gives $s = 1$, which corresponds to the 2nd member of the $5s+2$ subset, which remember, are the positions of values that map from row 3 to row 2. Repeating the process for the next value of 158, gives $s = 4$, corresponding to the 5th member of that subset. Solving for all values that map back to row 1 gives $s = 3t + 1$ for a generic integer t, which again could also be spoken of as being the $3x+2$ member of the $5s + 2$ subset for an integer x. That is, in terms of the members, that the $3x + 2$ positions of the $5s + 2$ locations in row 3, are those that continue to row 2 in positions that will then continue on to row 1. To help avoid confusion, and to make it more explicit, the $5s + 2$ locations are columns {2,7,12,17,22,27,32,37,42,...}, and the $3x + 2$ elements {2,5,8,11,...}, of those locations, are then the corresponding columns {7,22,37,...}.

This can now be put in terms of the row 3, $7x + 4$ set directly, and can answer the question at the beginning of this section. Inserting the $3s+1$ integer relation that selects continuing $5s + 2$ positions into $5s + 2$, which remember represents all positions that map from row 3 to 2 in general, now puts directly in terms of the $7x + 4$ set, only those locations which also map to row 1. This is equation 20.

$$5(3s + 1) + 2 = 15s + 7 \tag{20}$$

This states that the $15s + 7$ elements of the $7x + 4$ set are those that map through row 2 to row 1; that is, row 3, columns {7,22,37,52,...}. Now that it is known which values on row 3 intersect with both rows 1 and 2, the process can be repeated asking which values on row 4 will map to those specific locations on row 3.

4.3.1 Row 4

Using $r = 3$ in eq.16 gives the location of the overall elements that map from row 4 to row 3 as the elements in the $y = 7s + 3$ positions of the 4th row. It also shows that they map to the $x = 9s + 4$ positions in the 3rd row. This means that wherever $9x + 4 = 15y + 7$, an element in a $7s + 3$ position of row 4 maps to a position in row 3 that will then go on to map through to row 1 and intersect all 4 rows.

$$9x + 4 = 15y + 7 \tag{21}$$

This has the integer solutions in eq.22 for an integer s.

$$x = 5s + 2 \quad y = 3s + 1 \tag{22}$$

Like before, the question is which specific subset of elements from the $7s + 3$ positions of the 4th row map to the proper positions in the 3rd row. For $s = 0$ in eq.22 gives $4y = 1$, and then $15y + 7 = 22$. The 22nd value of row 3, and the first value to intersect all 4 rows, is 158. Insert the $7s + 3$ position value that maps from row 4 to row 3 into its row 4 element value of $9x + 5$ for x, and set it equal to 158, eq.23.

$$9(7s + 3) + 5 = 63s + 32 = 158 \tag{23}$$

Solving for s in eq.23 gives $s = 2$, which corresponds to the 3rd member of the $7s + 3$ subset. Repeating the process for $s = 1$ in eq.22 gives the next value of 473, and gives $s = 7$ when it's used in eq.23, corresponding to the 8th member of that subset. Solving for all values gives the relation $s = 5t + 2$, for a generic integer t, which again could also be spoken of as being the $5x + 3$ member of the set for an integer x.

This can now be put in terms of the row 4, $9x + 5$ set directly. Inserting the $5s + 2$ integer relation that selects continuing $7s + 3$ positions into $7s + 3$, now puts directly in terms of the $9x + 5$ set, only those locations which also map to row 1. This is equation 24.

$$7(5s + 2) + 3 = 35s + 17 \tag{24}$$

This states that the $35s+17$ elements of the $9x+5$ set are those that intersect the first 4 rows. By now, you may begin to see the pattern, and/or, it begins to emerge. Continue the technique for the transition from row 5 to row 4.

4.3.2 Row 5

Using $r = 4$ in eq.16 gives the location of the overall elements that map from row 5 to row 4 as the elements in the $y = 9s + 4$ positions of the 5th row. It also shows that they map to the $x = 11s + 5$ positions in the 4th row. This means that whenever $11x + 5 = 35y + 17$, an element in a $9s + 4$ position of row 5 maps to a position in row 4 that will then go on to map through to row 1 and intersect all 5 rows.

$$11x + 5 = 35y + 17 \tag{25}$$

This has the integer solutions in eq.26 for an integer s.

$$x = 35s + 17 \quad y = 11s + 5 \tag{26}$$

As previously, the question is which specific subset of elements from the $9s + 4$ positions of the 5th row map to the proper locations in the 4th row. For $s = 0$ in eq.26 gives $y = 5$, and then $35y + 7 = 192$. The 192nd value of row 4, and the first value to intersect 5 rows, is 1733. Insert the $9s + 4$ position value that maps from row 5 to row 4 into its row 5 element value of $11x + 6$ for x, and set it equal to 1733, eq.27.

$$11(9s + 4) + 6 = 99s + 50 = 1733 \tag{27}$$

Solving for s in eq.27 gives $s = 17$, which corresponds to the 18th member of the $9s + 4$ subset. Repeating the process for $s = 1$ in eq.26 gives the next value of 5198, and gives $s = 52$ when it's used in eq.27, corresponding to the 53rd member of that subset. Solving for all values gives the relation $35t + 17$, for a generic integer t, which again could also be spoken of as being the $35x + 18$ member of the set for an integer x.

This can now be put in terms of the row 5, $11x + 6$ set directly. Inserting the $35s + 17$ integer relation that selects continuing $9s + 4$ positions into $9s + 4$, now puts directly in terms of the $11x + 6$ set, only those locations which also map to row 1. This is equation 28.

$$9(35s + 17) + 4 = 315s + 157 \tag{28}$$

This states that the $315s + 157$ elements of the $11x + 6$ set are those that intersect the first 5 rows.

4.3.3 Row 6

At this point, the technique for finding the next set of values is established, and a general formula can be described. When doing so, it is also helpful to have the information from row 6, and rather than walk through the procedure again, the associated equations for row 6 are simply provided as follows.

$$13x + 6 = 315y + 157 \tag{29}$$

$$x = 315s + 157 \quad y = 13s + 6 \tag{30}$$

$$13(11s + 5) + 7 = 22523 \tag{31}$$

$$11(315s + 157) + 5 = 3465s + 1732 \tag{32}$$

4.4 The General Formula for All Rows

To generate the formula for all rows, examine equations 17, 21, 25, and 29. For ease, these are relisted as eq.33, which also includes the next corresponding relation from row 7.

$$\begin{aligned} \text{Row 3} \quad 3x + 1 &= 7y + 3 \\ \text{Row 4} \quad 9x + 4 &= 15y + 7 \\ \text{Row 5} \quad 11x + 5 &= 35y + 17 \\ \text{Row 6} \quad 13x + 6 &= 315y + 157 \\ \text{Row 7} \quad 15x + 7 &= 3465y + 1732 \end{aligned} \tag{33}$$

Now ask, from where do the values in these equations emerge, and what is being compared between the left and right side of the equations? The process began with all values of $3x+2$ on the first row not being on the first rows of the surfaces. From there, it was determined that the $3x + 1$ values on row 2 were the ones that intersected the first row. Using $r = 2$ with x in eq.16 showed the members mapping in from row 3 to row 2 into the $7x+3$ positions. In eq.17, the set was arbitrarily assigned by me into the right side of the relation as $7y + 3$ as to set the precedent going forward of the lesser valued parameters on the left when comparing sets. These are the sets represented and compared in Row 3 of eq.33, and they went on to generate the $15y + 7$ set, using equations 17-20, as seen in the right side of Row 4 of eq.33.

From that point forward, the left equations of the comparisons are from eq.16 for x with $r = R - 1$. That is, the left side of the Row 4 comparison in eq.33 uses the $r = 3$ value with x in eq.16, the Row 5 uses $r = 4$, and so on.

As for the right sides of the comparisons, the Row 4 sets went on to generate the $35y + 17$ set, using equations 21-24, as seen in Row 5 of eq.33. From that point forward, the right equations are generated using the right set from the previous row as the input for the sets from eq.16, but this time for y and with $r = R-2$. That is, the right side of the Row 6 comparison in eq.33 is generated by inserting the right side of Row 5 into the y value in eq.16 with $r = 4$, the right side of the Row 7 comparison in eq.33 is generated by inserting the right side of Row 6 into the y value in eq.16 with $r = 5$, and so on. Because of the r to $r+1$ relation in eq.16, this turns out to be the same as inserting a given Right side set into the previous Left side for x. For example, $9(35y+17)+4 = 315y+157$, and $11(315y + 157) + 5 = 3465y + 1732$.

The Left side sets for comparison in row R, from Row 4 onward, are simply $(2R+1)x+R$. For the Right side sets and Row 5 onward, the slopes m of the sets are the products of the first $R-3$ consecutive odd numbers beginning with the number 5, and the y intercepts are $(m-1)/2$. That is, the slope for Row 5 is $5x7$, for Row 6 it's $5x7x9$, for Row 7 it's $5x7x9x11$, and so on, and the intercepts are half of 1 less than those slopes.

The left set for row R≥ 4.

$$(2R+1)x+R \tag{34}$$

The right set for row R≥ 5.

$$\left(\prod_{k=2}^{R-2}(2k+1)\right)y+\frac{\left(\prod_{k=2}^{R-2}(2k+1)\right)-1}{2} \tag{35}$$

The product can also be expressed as:

$$\frac{(2R-3)!}{3*2^{R-2}*(R-2)!} \tag{36}$$

Setting eq.34 equal to eq.35 has the following integer solutions in eq.37 for Row R and an integer s.

$$x=\left(\prod_{k=2}^{R-2}(2k+1)\right)s+\frac{\left(\prod_{k=2}^{R-2}(2k+1)\right)-1}{2} \qquad y=(2R+1)s+R \tag{37}$$

Equation 38 is eq.37 in terms of the factorial expression.

$$x=\left(\frac{(2R-3)!}{3*2^{R-2}*(R-2)!}\right)s+\frac{\left(\frac{(2R-3)!}{3*2^{R-2}*(R-2)!}\right)-1}{2} \qquad y=(2R+1)s+R \tag{38}$$

This shows that there is an infinite number of natural number solutions in the intersection of any and all rows R. Note that it does not give the specific elements in that set, but simply proves the existence of the set. It shows that there is no row that exists such that an infinite subset can not be mapped from the first row, through the intersection of all previous rows, to that row. Because these are numbers that are not in the range of either surface, there is an infinite amount of natural numbers not in the range of those surfaces. Therefore, since all numbers not in the range of those surfaces generate unique Twin Pairs, there is an infinite number of Twin Prime Pairs.

4.4.1 Some Notes About the Integer Solutions

Some notes should be included about the nature of the integer solutions. The first note involves the behaviors of eq.35 and eq.37/38. Eq.37/38 gives an infinite set of integer solutions based on the integer s, which is enough for generating infinite sets in a given Row, however, in cases where the slopes in comparison between x and y have a common divisor, it does not give all integer solutions. This happens every 3rd row. The full integer solutions for those Rows simplify to have smaller slopes, and thus generate even more sets, however, adjusting to include all of those complicates eq.35, and it is not necessary, since it's still using every 3rd solution from those infinite sets for those rows as stated; which is of course still an infinite subset. It actually means that there are even more elements in the infinite intersection than the ones shown by eq.37/38. This is similar to the second note.

Remember that this entire process was done using only the first set of values that were not in each row. Recall from equations 10 and 11 how each row has an increasing number of sets not in that row. While the first row must use $3x + 2$, there are an infinite number of other combinations with different sets from other rows that generate their own infinite intersections.

Lastly, using a given row R for eq.37/38, and then generating integer solutions with integers s for all rows $\leq R$, does significantly increase the chance that the corresponding n will be a Twin Pair generator, but does not guarantee it. However, because the set is diagonalized, for any n that is generated for a given R, it is never needed to check values past row x, where x is the ceiling to the solution for $2x^2 + 2x + 1 = n$, the equation for the diagonal.

5 Conclusion

In summary, the proof used the fact that all Prime numbers greater than 2 are odd numbers that are not odd composites. It generated 2 surfaces from that requirement, one for each member of a Twin Pair, and showed that numbers not in the range of both surfaces always generate unique Twin Pairs. It then showed that there is an infinite number of elements in the set outside of that range, and therefore that there are infinite Twin Primes.

I hope you enjoyed the proof. If you know or find a more concise method to show an infinite number of natural elements not on the surfaces, or would like to discuss the proof in some other manner, such as improvements, corrections, or errors, I would be interested to know.

Q.E.D.