Klein-Gordon Equation and Wave Function for Free Particle in Rindler Space-Time

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ABSTRACT

Klein-Gordon equation is a relativistic wave equation. It treats spinless particle. The wave function cannot use as a probability amplitude. We made Klein-Gordon equation in Rindler space-time. In this paper, we make free particle's wave function as the solution of Klein-Gordon equation in Rindler space-time.

PACS Number:**03.30.+p,03.65**

Key words: Wave Function;

Free Particle;

Klein-Gordon equation

Rindler Space-time;

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1. Introduction

At first, Klein-Gordon equation is for free particle field ϕ in inertial frame.

$$\frac{m^2c^2}{\hbar^2}\phi + \frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2} - \nabla^2\phi = 0$$

m is free particle's mass (1)

If we write wave function as solution of Klein-Gordon equation for free particle,[3]

$$\phi = A_0 \exp i(\omega t - \vec{k}\cdot\vec{x})$$

A_0 is amplitude, ω is angular frequency, $k = |\vec{k}|$ is wave number (2)

Energy and momentum is in inertial frame,[3]

$$E = \hbar\omega, \vec{p} = \hbar\vec{k} \quad (3)$$

Hence, energy-momentum relation is[3]

$$E^2 = \hbar^2\omega^2 = p^2c^2 + m^2c^4 = \hbar^2k^2c^2 + m^2c^4 \quad (4)$$

Or angular frequency- wave number relation is

$$\frac{\omega^2}{c^2} = k^2 + \frac{m^2c^2}{\hbar^2} \quad (5)$$

2. Klein-Gordon Equation and Wave Function for Free Particle Field in Rindler-Space-Time

Rindler coordinates are

$$ct = (\frac{c^2}{a_0} + \xi^1)\sinh(\frac{a_0}{c}\xi^0) \quad , \quad x = (\frac{c^2}{a_0} + \xi^1)\cosh(\frac{a_0}{c}\xi^0) - \frac{c^2}{a_0}$$

$$y = \xi^2, z = \xi^3 \quad (6)$$

We know Klein-Gordon equation in Rindler space-time.

Klein-Gordon equation is for free particle field ϕ_ξ in Rindler space-time,[1]

$$\frac{m^2c^2}{\hbar^2}\phi_\xi + \frac{1}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}\frac{\partial^2\phi_\xi}{(\partial\xi^0)^2} - \nabla_\xi{}^2\phi_\xi - \frac{\partial\phi_\xi}{\partial\xi^1}\frac{a_0}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)} = 0 \quad (7)$$

m is free particle's mass

For we write wave function as solution of Klein-Gordon equation for free particle in Rindler space-time, if we insert Eq(6) in Eq(2),[2,3]

$$\phi = A_0 \exp i(\omega t - \vec{k}\cdot\vec{x})$$

$$= \phi_{\xi} = A_0 \exp i[(\frac{c^2}{a_0} + \xi^1)\{\frac{\omega}{c}\sinh(\frac{a_0\xi^0}{c}) - k_1\cosh(\frac{a_0\xi^0}{c})\} + k_1\frac{c^2}{a_0} - k_2\xi^2 - k_3\xi^3] \quad (8)$$

Eq(8) is the solution's function of the wave equation, Eq(7) in Rindler space-time.

In this point, energy-momentum transformation is[2]

$$E = \hbar\omega = E_{\xi}\cosh(\frac{a_0\xi^0}{c}) + p_{\xi^1}c\sinh(\frac{a_0\xi^0}{c})$$
$$= \hbar\omega_{\xi}\cosh(\frac{a_0\xi^0}{c}) + \hbar k_{\xi^1}c\sinh(\frac{a_0\xi^0}{c}) \quad (9)$$

$$p_x = \hbar k_1 = \frac{E_{\xi}}{c}\sinh(\frac{a_0\xi^0}{c}) + p_{\xi^1}\cosh(\frac{a_0\xi^0}{c})$$
$$= \hbar\frac{\omega_{\xi}}{c}\sinh(\frac{a_0\xi^0}{c}) + \hbar k_{\xi^1}\cosh(\frac{a_0\xi^0}{c}) \quad (10)$$

$$p_y = \hbar k_2 = p_{\xi^2} = \hbar k_{\xi^2}, p_z = \hbar k_3 = p_{\xi^3} = \hbar k_{\xi^3} \quad (11)$$

In this time, we suppose $E_{\xi} = \hbar\omega_{\xi}, \vec{p}_{\xi} = \hbar\vec{k}_{\xi}$ .In this careful point is we know $\omega, \vec{k}$ are constant.

But, in Eq(9),E(10), ω_{ξ}, k_{ξ^1} are variable functions with ξ^0. Hence, $\omega_{\xi} = \omega_{\xi}(\xi^0), k_{\xi^1} = k_{\xi^1}(\xi^0)$

don't have to use in Eq(7),Eq(8). Energy-momentum relation is Rindler space-time,

$$E_{\xi}{}^2 - p_{\xi}{}^2c^2 = \hbar^2\omega_{\xi}{}^2 - \hbar^2k_{\xi}{}^2c^2 = m^2c^4 = E^2 - p^2c^2 = \hbar^2\omega^2 - \hbar^2k^2c^2 \quad (12)$$

3. Conclusion

We found the wave function of Klein-Gordon's free particle in Rindler space-time. The wave function cannot use as a probability amplitude. In this paper, the particle has to do a spinless particle.

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