Novel explanation of the Active Galactic Nuclei.The Power Source of Quasars as a result of vacuum polarization by the gravitational singularities on the distributional BHs horizon.

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Abstract. In this paper we argue that the current paradigm for AGN and quasars essentially incomplete and rivision is needed. Remind that the current paradigm for AGN and quasars is that their radio emission is explained by synchrotron radiation from relativistic electrons that are Doppler boosted through bulk motion. In this model, the intrinsic brightness temperatures cannot exceed 10^{11} to 10^{12} K. Typical Doppler boosting is expected to be able to raise this temperature by a factor of 10.The observed brightness temperature of the most compact structures in BL Lac, constrained by baselines longer than $5.3G\lambda$, must indeed exceed 2×10^{13}K and can reach as high as $\sim 3 \times 10^{14}$K.This is difficult to reconcile with current incoherent synchrotron emission models from relativistic electrons, requiring alternative models such as emission from relativistic protons.However the proton, as we know, is 1836 times heavier than an electron and absolutely huge energy is required to accelerated it to sublight speed.These alternative models such as emission from relativistic protons can be suported by semiclassical gravity effect finds its roots in the singular behavior of quantum fields on curved distributional spacetimes presented by rotating gravitational singularities.

1. Introduction

The classical Cartan's structural equations show in a compact way the relation between a connection and its curvature, and reveals their geometric interpretation in terms of moving frames [1]-[2]. In order to study the mathematical properties of singularities, we need to study the geometry of manifolds endowed on the tangent bundle with a symmetric bilinear form which is allowed to become degenerate or singular (or both degenerate and singular) on semi Riemannian manifold (M,g) or on submanifolds of semi Riemannian manifold (M,g). But if the fundamental tensor

is allowed to be degenerate or singular, there are some obstructions in constructing the geometric objects normally associated to the fundamental tensor. Also, local orthonormal frames and coframes no longer exist, as well as the metric connection and its curvature operator.
Degenerate semi Riemannian manifolds arise naturally in the semi-Riemannian category: for example the restriction of a non-degenerate metric to a degenerate submanifold is a degenerate metric and the Killing-Cartan form on a non-semi-simple Lie Group is a degenerate metric.
Definition 1.1. (i) Semi Riemannian manifold (M,g) is nonclassical if the fundamental tensor g is allowed to be degenerate or singular, (ii) semi Riemannian manifold (M,g) is internally nonclassical if the fundamental tensor g is not allowed to be degenerate or singular but there exists semi Riemannian submanifold $(M',g'),M' \subsetneq M, g' = g|_{M'}$ such that the fundamental tensor g' is allowed to be degenerate or singular,(iii) otherwise we will be say that (M,g) is classical.
In nonclassical case the main problem arises from the degeneracy of the $\det(g_{ij}(\hat{x}))$ on some isolated points: $\det(g_{ij}(\hat{x}^0)) = 0, \hat{x}^0 \in M$ or some submanifold $\det(g_{ij}(\hat{x})) = 0$ for all $\hat{x} \in M' \subsetneq M$ and consequantly the corresponding Christoffel symbols bicome infinity.Let (M,g) be a nonclassical semi Riemannian manifold. Let $\Gamma_{\hat{x}^0}$be closed contour and let $\Sigma_{\Gamma_{\hat{x}^0}} \subset M$ be a surface spanning by $\Gamma_{\hat{x}^0}$, We assume now that christoffel symbols $\Gamma^i_{kl}(\hat{x})$ are smooth on $\Sigma_\Gamma \cup \Gamma \backslash \{\hat{x}^0\}$ and $\Gamma^i_{kl}(\hat{x}) \to \infty$ if $\hat{x} \to \hat{x}^0$. The classical formula for the change in a smooth vector $A_i(\hat{x})$ after parallel displacement around infinitesimal closed contour Γ

$$\Delta A_k(\Gamma) = \oint_\Gamma \delta A_k = \oint_\Gamma \Gamma^i_{kl}(\hat{x}) A_k dx^l. \tag{1.1}$$

no longer hold since $\Delta A_k(\Gamma) = \infty$.
In mathematical literature more than 50 yers accepted that a nonclassical semi Riemannian manifold mentioned above impossible treated classically, i.e. by using canonical apparatus of the Riemannian geometry. However in the contemporary mathematical literature, manifolds with degenerate metric tensors have been studied only fore some special case called a Reinhart manifold [3]-[4].
In order to avoid these difficultness with divergence $\Delta A_k(\Gamma_{\hat{x}^0}) = \infty$, etc. we consider the canonical imbedding $(M, g_{i,j,0}(\hat{x})) \hookrightarrow (M, (g_{i,j,\varepsilon}(\hat{x}))_\varepsilon)$, and we extend the classical formula (1.1) from a nonclassical semirimannian manifold $(M, g_{i,j,0}(\hat{x}))$ up to Colombeau manifold $(M, (\det(g_{ij,\varepsilon}))_\varepsilon)$, where $(g_{ij,\varepsilon}) \in \mathcal{G}_\delta(\mathbb{R}^n), i,j = 1,\ldots,n$ [5]-[8].
In contemporary mathematics, a Colombeau algebra of Colombeau generalized functions is an algebra of a certain kind containing the space of Schwartz distributions. While in classical distribution theory a general multiplication of distributions is not possible, classical Colombeau algebras provide a rigorous framework for this [9]-[11].
Through whole this paper we shall apply the following definitions and notations [1].
Definition 1.2. The algebra moderate functions $C^\infty_M(\mathbb{R}^n)$ on $\mathbb{R}^n$ is the algebra of families of smooth functions $(f_\varepsilon(x))_\varepsilon \triangleq (f_\varepsilon(x))_\varepsilon, x \in \mathbb{R}^n, \varepsilon \in (0,\delta], \delta \leq 1$ (smooth ε-regularisations, where ε is the regularization parameter), such that: (i) for all compact subsets K of $\mathbb{R}^n$ and all multiindices α, there is an $N > 0$ such that

$$\sup_{x\in K}\left|\frac{\partial^{|\alpha|}f_\varepsilon(x)}{(\partial x_1)^{\alpha_1}\cdots(\partial x_n)^{\alpha_n}}\right| = O(\varepsilon^{-N}), \varepsilon\to 0, \quad (1.2)$$

with addition and multiplication defined by natural way:

$$(f_\varepsilon(x))_\varepsilon + (g_\varepsilon(x))_\varepsilon = (f_\varepsilon(x) + g_\varepsilon(x))_\varepsilon \quad (1.3)$$

and

$$(f_\varepsilon(x))_\varepsilon \times (g_\varepsilon(x))_\varepsilon = (f_\varepsilon(x)\times g_\varepsilon(x))_\varepsilon. \quad (1.4)$$

Definition 1.3.The ideal $\mathcal{N}_\delta(\mathbb{R}^n)$ of negligible functions is defined in the same way but with the partial derivatives instead bounded by $O(\varepsilon^N)$ for all $N>0$, i.e.

$$\sup_{x\in K}\left|\frac{\partial^{|\alpha|}f_\varepsilon(x)}{(\partial x_1)^{\alpha_1}\cdots(\partial x_n)^{\alpha_n}}\right| = O(\varepsilon^{N}), \varepsilon\to 0. \quad (1.5)$$

Definition 1.4.The Colombeau Algebra $\mathcal{G}_\delta(\mathbb{R}^n)$ [1] is defined as the quotient algebra

$$\mathcal{G}_\delta(\mathbb{R}^n) = C^\infty_M(\mathbb{R}^n)/\mathcal{N}_\delta(\mathbb{R}^n). \quad (1.6)$$

Elements of $\mathcal{G}_\delta(\mathbb{R}^n)$ are denoted by:

$$u = \mathbf{cl}[(u_\varepsilon)_\varepsilon] \triangleq (u_\varepsilon)_\varepsilon + \mathcal{N}_\delta(\mathbb{R}^n). \quad (1.7)$$

Embedding of distributions

The space of Schwartz distributions $\mathcal{D}'(\mathbb{R}^n)$ can be embedded into the Colombeau algebra $\mathcal{G}_\delta(\mathbb{R}^n)$ by (component-wise) convolution with any element $(\varphi_\varepsilon)_\varepsilon$ of the algebra $\mathcal{G}_\delta(\mathbb{R}^n)$ having as representative a ε-net, i.e. a family of smooth functions φ_ε (δ-net) such that $\varphi_\varepsilon\to\delta$ in $\mathcal{D}'(\mathbb{R}^n)$ as $\varepsilon\to 0$. Note that the embedding $\iota : \mathcal{D}'(\mathbb{R}^n)\hookrightarrow\mathcal{G}_\delta(\mathbb{R}^n)$ is non-canonical, because it depends on the choice of the δ-net. However note that embedding $\mathcal{D}'(\mathbb{R}^n)\hookrightarrow\mathcal{G}_\delta(\mathbb{R}^n)$ does not meant the full equivalence of the Schwartz distributions and corresponding by embedding Colombeau generalized functions. In contrast with the Schwartz distributions Colombeau generalized functions has well defined walue at any point $x\in\mathbb{R}^n$ these point values of the Colombeau generalized functions is the Colombeau generalized numbers $\widetilde{\mathbb{R}}_\delta$ [1].

Designation 1.1. (**I**) We denote by $\widetilde{\mathbb{R}}_\delta, \delta\leq 1$ the ring of real Colombeau generalized numbers. Recall that by definition $\widetilde{\mathbb{R}}_\delta = \mathcal{E}_\delta(\mathbb{R})/\mathbf{N}_\delta(\mathbb{R})$ where [34],[36],[37]:

$$\begin{aligned}\mathcal{E}_\delta(\mathbb{R}) &= \{(x_\varepsilon)_\varepsilon\in\mathbb{R}^{(0,\delta)}|(\exists a\in\mathbb{R}_+)(\exists\varepsilon_0\in(0,1))(\forall\varepsilon\leq\varepsilon_0)[|x_\varepsilon|\leq\varepsilon^{-a}]\},\\ \mathcal{N}_\delta(\mathbb{R}) &= \{(x_\varepsilon)_\varepsilon\in\mathbb{R}^{(0,\delta)}|(\forall a\in\mathbb{R}_+)(\exists\varepsilon_0\in(0,1))(\forall\varepsilon\leq\varepsilon_0)[|x_\varepsilon|\leq\varepsilon^{a}]\}.\end{aligned} \quad (1.8)$$

(**II**) In this subsection we will be write for short $\widetilde{\mathbb{R}}$ instead $\widetilde{\mathbb{R}}_\delta$.

Notice that the ring $\widetilde{\mathbb{R}}$ arises naturally as the ring of constants of the Colombeau algebras $\mathcal{G}_\delta(\Omega)$. Recall that there exists natural embedding $\widetilde{r} : \mathbb{R}\hookrightarrow\widetilde{\mathbb{R}}$ such that for all $r\in\mathbb{R}, \widetilde{r} = (r_\varepsilon)_\varepsilon$ where $r_\varepsilon\equiv r$ for all $\varepsilon\in(0,1]$. We say that r is standard number and abbreviate $r\in\mathbb{R}$ for short. The ring $\widetilde{\mathbb{R}}$ can be endowed with the structure of a partially ordered ring: for $r,s\in\widetilde{\mathbb{R}}$ $\eta\in\mathbb{R}_+, \eta\leq\delta$ we abbreviate $r\leq_{\widetilde{\mathbb{R}},\eta} s$ or simply $r\leq_{\widetilde{\mathbb{R}}} s$ if and only if there are representatives $(r_\varepsilon)_\varepsilon$ and $(s_\varepsilon)_\varepsilon$ with $r_\varepsilon\leq s_\varepsilon$ for all $\varepsilon\in(0,\eta]$.

Colombeau generalized number $r\in\widetilde{\mathbb{R}}$ with representative $(r_\varepsilon)_\varepsilon$ we abbreviate

$\mathbf{cl}[(r_\varepsilon)_\varepsilon]$.

Definition 1.5. (i) Let $\breve{\delta} = \mathbf{cl}[(\delta_\varepsilon)_\varepsilon] \in \widetilde{\mathbb{R}}$. We say that $\breve{\delta}$ is infinite small Colombeau generalized number and abbreviate $\breve{\delta} \approx_{\widetilde{\mathbb{R}}} \widetilde{0}$ if there exists representative $(\delta_\varepsilon)_\varepsilon$ and some $q \in \mathbb{N}$ such that $|\delta_\varepsilon| = O(\varepsilon^q)$ as $\varepsilon \to 0$. (ii) Let $\Delta \in \widetilde{\mathbb{R}}$. We say that Δ is infinite large Colombeau generalized number and abbreviate $\Delta =_{\widetilde{\mathbb{R}}} \widetilde{\infty}$ if $\Delta_{\widetilde{\mathbb{R}}}^{-1} \approx_{\widetilde{\mathbb{R}}} \widetilde{0}$. (iii) Let $\mathbb{R}_{\pm\infty}$ be $\mathbb{R} \cup \{\pm\infty\}$ We say that $\Theta \in \widetilde{\mathbb{R}}_{\pm\infty}$ is infinite Colombeau generalized number and abbreviate $\Theta =_{\widetilde{\mathbb{R}}} \pm\infty_{\widetilde{\mathbb{R}}}$ if there exists representative $(\Theta_\varepsilon)_\varepsilon$ where $|\Theta_\varepsilon| = \infty$ for all $\varepsilon \in (0,1]$. Here we abbreviate $\mathcal{E}_\delta(\mathbb{R}_{\pm\infty}) = \mathcal{E}_\delta(\mathbb{R} \cup \{\pm\infty\})$, $\mathcal{N}_\delta(\mathbb{R}_{\pm\infty}) = \mathcal{N}_\delta(\mathbb{R} \cup \{\pm\infty\})$ and $\widetilde{\mathbb{R}}_{\pm\infty} = \mathcal{E}_\delta(\mathbb{R}_{\pm\infty})/\mathcal{N}_\delta(\mathbb{R}_{\pm\infty})$

Definition 1.6. (Standard Part Mapping). (i) The standard part mapping $\mathbf{st} : \widetilde{\mathbb{R}} \to \mathbb{R}$ is defined by the formula:

$$\mathbf{st}(x) = \sup\left\{r \in \mathbb{R} | r \leq_{\widetilde{\mathbb{R}}} x\right\}. \tag{1.9}$$

If $x \in \widetilde{\mathbb{R}}$, then $\mathbf{st}(x)$ is called the standard part of x.

(ii) The mapping $\mathbf{st} : \widetilde{\mathbb{R}} \to \mathbb{R} \cup \{\pm\infty\}$ is defined by (i) and by $\mathbf{st}(x) = \pm\infty$ for $x \in \widetilde{\mathbb{R}}$ and for $x \in \widetilde{\mathbb{R}}_{\pm\infty}$, respectively.

Definition 1.7.Let $(f_\varepsilon(x))_\varepsilon \in \mathcal{G}_\delta(\mathbb{R}^n)$ and $\breve{x} \in \mathbb{R}$, then $\mathbf{cl}[(f_\varepsilon(\breve{x}))_\varepsilon] \in \widetilde{\mathbb{R}}$. We will say that Colombeau generalized number $\mathbf{cl}[(f_\varepsilon(\breve{x}))_\varepsilon]$ is a point values of Colombeau generalized function $(f_\varepsilon(x))_\varepsilon$ at point $\breve{x} \in \mathbb{R}^n$.

Definition 1.8. (i) Let $u = \mathbf{cl}[(u_\varepsilon(x))_\varepsilon]$ be the Colombeau generalized function such that $(u_\varepsilon(x))_\varepsilon \in \mathcal{G}_\delta(\mathbb{R}^n)$ and let μ be a vector $(\mu_\varepsilon)_\varepsilon = (\mu_{1,\varepsilon},\ldots,\mu_{i,\varepsilon},\ldots,\mu_{n,\varepsilon})_\varepsilon =$
$= ((\mu_{1,\varepsilon})_\varepsilon,\ldots,(\mu_{i,\varepsilon})_\varepsilon,\ldots,(\mu_{n,\varepsilon})_\varepsilon)$ where $\mathbf{cl}[(\mu_{i,\varepsilon})_\varepsilon] \in \widetilde{\mathbb{R}}_\delta, i = 1,\ldots,n$ are Colombeau generalized numbers with the representatives $(\mu_{i,\varepsilon})_\varepsilon \in \mathcal{E}_\delta(\mathbb{R})$.
Thus, we have a mapping $\tilde{u} : \tilde{\mathbb{R}}^n \to \widetilde{\mathbb{R}}_\delta$ that is defined in a natural way by the following formula:

$$\tilde{u}[(\mu_\varepsilon)_\varepsilon] = (u_\varepsilon(\mu_\varepsilon))_\varepsilon \in \mathcal{E}_\delta(\mathbb{R}). \tag{1.10}$$

(ii) Let $u_1 = (u_{1,\varepsilon}(x))_\varepsilon$ and $u_2 = (u_{2,\varepsilon}(x))_\varepsilon$ Colombeau generalized functions such that $u_1,u_2 \in \mathcal{E}_\delta^\infty(\mathbb{R}^n)$. The algebra moderate function $\mathcal{E}_\delta^\infty\left(\tilde{\mathbb{R}}_\delta^n\right)$ on $\tilde{\mathbb{R}}_\delta^n$ is the algebra of functions $\tilde{u} : \mathcal{E}_\delta^\infty\left(\tilde{\mathbb{R}}_\delta^n\right) \to \mathcal{E}_\delta\left(\widetilde{\mathbb{R}}_\delta\right)$ defined by Eq.(1.10) such that, for all compact subsets K of $\tilde{\mathbb{R}}_\delta^n$ and all multi indices $\alpha = (\alpha_1,\ldots,\alpha_i,\ldots,\alpha_n)$, there are $N > 0$ and $\varepsilon_0 \in \mathbb{R}_+$ such that, for $\varepsilon \leq \varepsilon_0$

$$\left(\sup_{\mu_\varepsilon \in K} \frac{\partial^{|\alpha|} u(\mu_{1,\varepsilon},\ldots,\mu_{i,\varepsilon},\ldots,\mu_{n,\varepsilon})}{(\partial\mu_{1,\varepsilon})^{\alpha_1}\ldots(\partial\mu_{i,\varepsilon})^{\alpha_i}\ldots(\partial\mu_{1,\varepsilon})^{\alpha_n}}\right)_\varepsilon = O((\varepsilon^{-N})_\varepsilon) \tag{1.11}$$

and with the addition and multiplication defined by a natural way by the following formulas:

$$(\tilde{u}_1 + \tilde{u}_2)((\mu_\varepsilon)_\varepsilon) = (\tilde{u}_{1,\varepsilon}(\mu_\varepsilon) + \tilde{u}_{2,\varepsilon}(\mu_\varepsilon))_\varepsilon \tag{1.12}$$

and

$$(\tilde{u}_1 \cdot \tilde{u}_2)((\mu_\varepsilon)_\varepsilon) = (\tilde{u}_{1,\varepsilon}(\mu_\varepsilon) \cdot \tilde{u}_{2,\varepsilon}(\mu_\varepsilon))_\varepsilon \tag{1.13}$$

correspondingly.

(iii) The ideal $\mathcal{N}_\delta^\infty\left(\widetilde{\mathbb{R}}_\delta\right)$ of negligible functions on $\widetilde{\mathbb{R}}_\delta$ is defined in the same way but with the derivatives instead bounded by for all $N > 0$; i.e.,

$$\left(\sup_{\mu_\varepsilon \in K} \frac{\partial^{|\alpha|} u(\mu_{1,\varepsilon},\ldots,\mu_{i,\varepsilon},\ldots,\mu_{n,\varepsilon})}{(\partial\mu_{1,\varepsilon})^{\alpha_1}\ldots(\partial\mu_{i,\varepsilon})^{\alpha_i}\ldots(\partial\mu_{1,\varepsilon})^{\alpha_n}}\right)_\varepsilon = O((\varepsilon^N)_\varepsilon) \qquad (1.14)$$

for all $\varepsilon \leq \varepsilon_0$.

(iv) The point free Colombeau algebra $\mathcal{G}_\delta(\tilde{\mathbb{R}}^n)$ defined as the quotient algebra

$$\mathcal{G}_\delta(\tilde{\mathbb{R}}^n) = \mathcal{E}_\delta^\infty\left(\tilde{\mathbb{R}}_\delta^n\right)/\mathcal{N}_\delta^\infty\left(\widetilde{\mathbb{R}}_\delta\right). \qquad (1.15)$$

The elements of $\mathcal{G}_\delta(\tilde{\mathbb{R}}^n)$ are denoted by the following:

Definition 1.9. Let $(f_\varepsilon(x))_\varepsilon \in \mathcal{G}_\delta(\mathbb{R})$ and $\mathbf{cl}[(\check{x}_\varepsilon)_\varepsilon] \in \widetilde{\mathbb{R}}_\delta$. Assum that $\mathbf{cl}[(f_\varepsilon(\check{x}_\varepsilon))_\varepsilon] \in \widetilde{\mathbb{R}}_\delta$. We will say that Colombeau generalized number $\mathbf{cl}[(f_\varepsilon(\check{x}_\varepsilon))_\varepsilon]$ is a point values of Colombeau generalized function $(f_\varepsilon(x))_\varepsilon$ at point $(\check{x}_\varepsilon)_\varepsilon \in \widetilde{\mathbb{R}}_\delta$.

We briefly recall now the basic supergeneralized Colombeau construction [11]-[14]. Colombeau supergeneralized functions on $\Omega \subseteq \mathbb{R}^n$, where $\dim(\Omega) = n$ are defined as equivalence classes $u = [(u_\varepsilon)_\varepsilon]$ of nets of functions $u_\varepsilon \in C^\infty(\Omega\backslash\Sigma), \varepsilon \in (0,\delta]$ such that any u_ε is a net of functions smooth on $\Omega\backslash\Sigma$ and has a discontinuity on a subset $\Sigma \subset \Omega$, where $\dim(\Sigma) < n$. We assume that for any $\varepsilon \in (0,\delta]$ the derivative $\dfrac{\partial^{|m|}u_\varepsilon}{\partial x_1^{k_1}\ldots\partial x_n^{k_n}}, m = (k_1,\ldots,k_n)$ exists in the sense of the theory of canonical generalized functions and $\dfrac{\partial^{|m|}u_\varepsilon}{\partial x_1^{k_1}\ldots\partial x_n^{k_n}} \in \mathcal{D}'(\Omega)$. The basic idea of generalized *Colombeau's theory of nonlinear supergeneralized functions* [11]-[14] is regularization by sequences (nets) of nonsmooth functions with derivatives in $\mathcal{D}'(\Omega)$ and the use of asymptotic estimates in terms of a regularization parameter ε. Let $(u_\varepsilon)_{\varepsilon\in(0,\delta]}, \delta \leq 1$ with u_ε such that: (i) $u_\varepsilon \in \mathcal{C}^\infty(M\backslash\Sigma)$ and (ii) $L_{\xi_1}\ldots L_{\xi_k}u_\varepsilon \in \mathcal{D}'(M)$, for all $\varepsilon \in (0,\delta]$, where M a separable, smooth orientable Hausdorff manifold of dimension n.

Definition 1.10. The supergeneralized Colombeau's algebra $\widetilde{\mathcal{G}} = \widetilde{\mathcal{G}}(M,\Sigma)$ of supergeneralized functions on M, where $\Sigma \subset M, \dim(M) = n, \dim(\Sigma) < n$, is defined as the quotient:

$$\widetilde{\mathcal{G}}(M,\Sigma) \triangleq \mathcal{E}_M(M,\Sigma)/\mathcal{N}(M,\Sigma) \qquad (1.15)$$

of the space $\mathcal{E}_M(M,\Sigma)$ of sequences of moderate growth modulo the space $\mathcal{N}(M,\Sigma)$ of negligible sequences. More precisely the notions of moderateness resp. negligibility are defined by the following asymptotic estimates (where $\mathfrak{X}(M\backslash\Sigma)$ denoting the space of smooth vector fields on $M\backslash\Sigma$):

$$\begin{gathered}\mathcal{E}_M(M,\Sigma) \triangleq \Big\{(u_\varepsilon)_\varepsilon |\ \forall K(K \subsetneqq M\backslash\Sigma)\,\forall k(k\in\mathbb{N})\,\exists N(N\in\mathbb{N})\\ \forall\xi_1,\ldots,\xi_k(\xi_1,\ldots,\xi_k \in \mathfrak{X}(M\backslash\Sigma))\left[\sup_{p\in K}|L_{\xi_1}\ldots L_{\xi_k}u_\varepsilon(p)| = O(\varepsilon^{-N}), \varepsilon\to 0\right]\ \&\\ \forall K(K\subsetneqq M)\,\forall k(k\in\mathbb{N})\,\exists N(N\in\mathbb{N})\forall(f\in\mathcal{C}^\infty(M))\forall\xi_1,\ldots,\xi_k(\xi_1,\ldots,\xi_k\in\mathfrak{X}(M))\\ \left[\|L^w_{\xi_1}\ldots L^w_{\xi_k}u_\varepsilon\| = \left(\sup_{f\in\mathcal{C}^\infty(M)}\left|L^w_{\xi_1}\ldots L^w_{\xi_k}u_\varepsilon(f)\right|\right) = O(\varepsilon^{-N}), \varepsilon\to 0\right]\Big\},\end{gathered} \qquad (1.16)$$

2.Generalized Einstein's field equations

The general theory of relativity is a nonlinear theory of gravity. The mathematical theory of distributions,on theother hand,is a linear theory that uses avariety of technique swhich cannot be implemented in the nonlinear framework of semi Riemannian geometry. This is also true with regard to canonical Colombeau's nonlinear theory of generalized functions [6]–[8], which, though capable of solving an impressive spectrum of problems associated with the treatment of distributionsin gravitational physics does not always allow a rigorous treatmentof the simultaneously singular and nonlinear field equations of theory.

The generalized action functional for the gravitational field reads [9]-[14]:

$$\left(\int R_\varepsilon \sqrt{-g_\varepsilon}\, d\Omega_\varepsilon\right)_\varepsilon. \tag{2.1}$$

The invariant Colombeau integral (2.1) can be transformed by means of Gauss' theorem to the integral of an expression not containing the second derivatives. Thus Colombeau integral (2.1) can be presented in the following form

$$\left(\int R_\varepsilon \sqrt{-g_\varepsilon}\, d\Omega_\varepsilon\right)_\varepsilon = \left(\int G_\varepsilon \sqrt{-g_\varepsilon}\, d\Omega_\varepsilon\right)_\varepsilon + \left(\int \frac{\partial\left(\sqrt{-g_\varepsilon}\, w_\varepsilon^i\right)}{\partial x^i} d\Omega_\varepsilon\right)_\varepsilon, \tag{2.2}$$

where $(G_\varepsilon)_\varepsilon$ contains only the tensor $(g_{ik,\varepsilon})_\varepsilon$ and its first derivatives, and the integrand of the second integral has the form of a divergence of a certain quantity $(w_\varepsilon^i)_\varepsilon$. According to Gauss' theorem, this second integral can be transformed into an integral over a hypersurface surrounding the four-volume over which the integration is carried out in the other two integrals. When we vary the action, the variation of the second term on the right vanishes, since in the principle of least action, the variations of the field at the limits of the region of integration are zero. Consequently, we may write

$$\delta\left(\int R_\varepsilon \sqrt{-g_\varepsilon}\, d\Omega\right)_\varepsilon = \left(\delta \int R_\varepsilon \sqrt{-g_\varepsilon}\, d\Omega\right)_\varepsilon = \left(\delta \int G_\varepsilon \sqrt{-g_\varepsilon}\, d\Omega\right)_\varepsilon. \tag{2.3}$$

The left side is Colombeau scalar; therefore the expression on the right is also Colombeau scalar (the quantity $(G_\varepsilon)_\varepsilon$ itself is, of course, not Colombeau scalar). The quantity $(G_\varepsilon)_\varepsilon$satisfies the condition imposed above, since it contains only the $(g_{ik,\varepsilon})_\varepsilon$ and its Colombeau derivatives. Thus finally we obtain

$$\delta\mathbf{S}[(g_\varepsilon)_\varepsilon] = -\frac{c^3}{16\pi\kappa}\left(\delta \int G_\varepsilon \sqrt{-g_\varepsilon}\, d\Omega\right)_\varepsilon = -\frac{c^3}{16\pi k}\left(\delta \int R_\varepsilon \sqrt{-g_\varepsilon}\, d\Omega\right)_\varepsilon. \tag{2.4}$$

The constant κ is called the gravitational constant. The dimensions of κ follow from (2.4). Its numerical value is $\kappa = 6.67 \times 10^{-8}\text{sm}^3 \times \text{gr}^{-1} \times \sec^{-2}$.We now proceed to the derivation of the equations of the gravitational field. These equations are obtained from the principle of least action $\delta((S_{m,\varepsilon})_{\varepsilon} + (S_{g_\varepsilon})_{\varepsilon}) = 0_{\widetilde{\mathbb{R}}}$, where $(S_{m,\varepsilon})_{\varepsilon}$ and $(S_{g_\varepsilon})_{\varepsilon}$ are the distributional actions of the gravitational field and matter respectively. We now subject the gravitational Colombeau metric field, that is, the quantities g_{ik}, to variation. Calculating the variation $\delta(S_{g_\varepsilon})_{\varepsilon}$, we get

$$\delta\left(\int R_\varepsilon \sqrt{-g_\varepsilon}\, d\Omega\right)_\varepsilon = \left(\delta \int R_\varepsilon \sqrt{-g_\varepsilon}\, d\Omega\right)_\varepsilon = \left(\delta \int g^{ik}_\varepsilon R_{ik,\varepsilon} \sqrt{-g_\varepsilon}\, d\Omega\right)_\varepsilon =$$
$$\left\{\left(\int R_{ik,\varepsilon}\sqrt{-g_\varepsilon}\,\delta g^{ik}_\varepsilon d\Omega\right)_\varepsilon + \left(\int R_{ik,\varepsilon} g^{ik}_\varepsilon \delta\sqrt{-g_\varepsilon}\, d\Omega\right)_\varepsilon + \left(\int g^{ik}_\varepsilon \sqrt{-g_\varepsilon}\,\delta R_{ik,\varepsilon} d\Omega\right)_\varepsilon\right\} \tag{2.5}$$
$$\int\left\{\left(R_{ik,\varepsilon}\sqrt{-g_\varepsilon}\,\delta g^{ik}_\varepsilon\right)_\varepsilon + \left(R_{ik,\varepsilon} g^{ik}_\varepsilon \delta\sqrt{-g_\varepsilon}\right)_\varepsilon + \left(g^{ik}_\varepsilon\sqrt{-g_\varepsilon}\,\delta R_{ik,\varepsilon}\right)_\varepsilon\right\} d\Omega.$$

Thus, the variation $\mathbf{S}[(g_\varepsilon)_\varepsilon]$ is equal to

$$\mathbf{S}[(g_\varepsilon)_\varepsilon] = -\frac{c^3}{16\pi\kappa}\left(\int\left\{R_{ik,\varepsilon} - \frac{1}{2} g_{ik,\varepsilon} R_\varepsilon\right\}\sqrt{-g_\varepsilon}\,\delta g^{ik}_\varepsilon d\Omega\right)_\varepsilon. \tag{2.6}$$

We note that if we had started from the expression

$$\delta\mathbf{S}_g[(g_\varepsilon)_\varepsilon] = -\frac{c^3}{16\pi\kappa}\left(\delta\int G_\varepsilon \sqrt{-g_\varepsilon}\, d\Omega\right)_\varepsilon \tag{2.7}$$

for the action of the field, then we get

$$\delta\mathbf{S}[(g_\varepsilon)_\varepsilon] =$$
$$-\frac{c^3}{16\pi\kappa}\int \delta(g^{ik}_\varepsilon)_\varepsilon d\Omega \left\{\left(\frac{\partial\left\{G_\varepsilon\sqrt{-g_\varepsilon}\right\}}{\partial g^{ik}_\varepsilon}\right)_\varepsilon - \left(\frac{\partial}{\partial x^l}\frac{\partial\left\{G_\varepsilon\sqrt{-g_\varepsilon}\right\}}{\partial\frac{\partial g^{ik}_\varepsilon}{\partial x^l}}\right)_\varepsilon\right\}. \tag{2.8}$$

Comparing Eq.(2.8) with Eq.(2.6), we get

$$(R_{ik,\varepsilon})_\varepsilon - \frac{1}{2}(g_{ik,\varepsilon}R_\varepsilon)_\varepsilon =$$
$$\left\{\left(\frac{1}{\sqrt{-g_\varepsilon}}\right)_\varepsilon\right\}\left\{\left(\frac{\partial\left\{G_\varepsilon\sqrt{-g_\varepsilon}\right\}}{\partial g^{ik}_\varepsilon}\right)_\varepsilon - \left(\frac{\partial}{\partial x^l}\frac{\partial\left\{G_\varepsilon\sqrt{-g_\varepsilon}\right\}}{\partial\frac{\partial g^{ik}_\varepsilon}{\partial x^l}}\right)_\varepsilon\right\}. \quad . \tag{1.9.13}$$

For the variation of the action of the matter we can write

$$(\delta\mathbf{S}_{m,\varepsilon})_\varepsilon = \frac{1}{2c}\left(\int T_{ik,\varepsilon}\sqrt{-g_\varepsilon}\,\delta g^{ik}_\varepsilon d\Omega\right)_\varepsilon, \tag{2.9}$$

where $(T_{ik,\varepsilon})_\varepsilon \in \mathcal{G}(\mathbb{R}^4)$ is the generalized energy-momentum tensor of the matter fields.

Thus, from the principle of least action

$$\delta\{\mathbf{S}[(g_\varepsilon)_\varepsilon] + (\mathbf{S}_{m,\varepsilon})_\varepsilon\} = 0_{\widetilde{\mathbb{R}}} \tag{2.10}$$

one obtains

$$-\frac{c^3}{16\pi\kappa}\left(\int\left\{R_{ik,\varepsilon} - \frac{1}{2}g_{ik,\varepsilon}R_\varepsilon - \frac{8\pi\kappa}{c^4}T_{ik,\varepsilon}\right\}\sqrt{-g_\varepsilon}\,\delta g^{ik}_\varepsilon d\Omega\right)_\varepsilon = 0_{\widetilde{\mathbb{R}}}. \tag{2.11}$$

From Eq.(2.11), since of the arbitrariness of the $(\delta g^{ik}_\varepsilon)_\varepsilon \in \mathcal{G}(\mathbb{R}^4)$ finally we get

$$(R_{ik,\varepsilon})_\varepsilon - \frac{1}{2}(g_{ik,\varepsilon}R_\varepsilon)_\varepsilon = \frac{8\pi\kappa}{c^4}(T_{ik,\varepsilon})_\varepsilon \tag{2.10}$$

or, in mixed components,

$$(R^k_{i,\varepsilon})_\varepsilon - \frac{1}{2}\delta^k_i (R_\varepsilon)_\varepsilon = \frac{8\pi\kappa}{c^4}(T^k_{i,\varepsilon})_\varepsilon. \quad (2.13)$$

They are called the generalized Einstein equations.

Contracting (2.13) on the indices i and k, we get

$$(R_\varepsilon)_\varepsilon = -\frac{8\pi\kappa}{c^4}(T^i_{i,\varepsilon})_\varepsilon = -\frac{8\pi\kappa}{c^4}(T_\varepsilon)_\varepsilon. \quad (2.14)$$

Therefore the generalized Einstein equations of the field can also be written in the form

[6]-[7]

$$(R_{ik,\varepsilon})_\varepsilon = \frac{8\pi\kappa}{c^4}\left\{(T_{ik,\varepsilon})_\varepsilon - \frac{1}{2}(g_{ik,\varepsilon}T_\varepsilon)_\varepsilon\right\}. \quad (2.15)$$

Note that the generalized Einstein equations of the gravitational field are nonlinear Colombeau type equations.

3.The current paradigm for Active Galactic Nuclei.High energy emission from galactic jets.

The current paradigm for AGN and quasars is that their radio emission is explained by synchrotron radiation from relativistic electrons that are Doppler boosted through bulk motion [15]-[18].

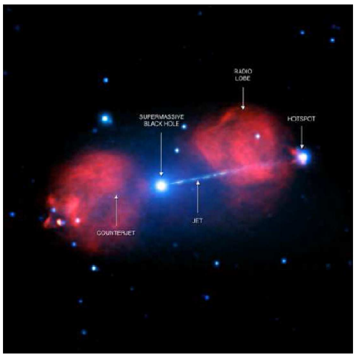


Fig.1.Jet from Black Hole in a Galaxy Pictor A
The active galaxy Pictor A lies nearly 500 million
light-years from Earth and contains a
supermassive black hole at its centre.
This is a composite radio and X-ray image.

Fig.1. Accretion of gas onto the supermassive Kerr black holes lurking at the center of active galactic nuclei (AGN) gives rise to powerful relativistic jets. However in this model, the intrinsic brightness temperatures cannot exceed 10^{11} to 10^{12} K. Typical Doppler boosting is expected to be able to raise this temperature by a factor of 10.The observed brightness temperature of the most compact structures in BL Lac, constrained by baselines longer than $5.3G\lambda$, must indeed exceed 2×10^{13}K and can reach as high as $\sim 3 \times 10^{14}$K. As well known, these visibilities correspond to the structural scales of $30 - 40\ \mu as$ oriented along position angles of $25^\circ - 30^\circ$. These values are indeed close to the width of the inner jet and the normal to its direction.The

observed, $T_{b,obs}$, and intrinsic, $T_{b,int}$, brightness temperatures are related by [19]

$$T_{b,obs} = \delta(1+z)^{-1}T_{b,int} \quad (3.1)$$

where where $\delta = (1-\beta^2)^{1/2}(1-\beta cos\varphi)^{-1}$ is the Doppler factor, β is the jet bulk velocity in

units of the speed of light, φ is the jet viewing angle, and z is the redshift of the source. Variability argument and kinematical analyses yield consistent value of $\delta = 7.2$. The estimeted by Eq.2.1 a lower limit of the intrinsic brightness temperature in the core component of our Radio Astron observations of $T_{b,int} > 2.910^{12}$ K [19]. It is commonly considered that inverse Compton losses limit the intrinsic brightness temperature for incoherent synchrotron sources, such as AGN, to about 10^{12}K [19].In case of a strong flare, the "Compton catastrophe" is calculated to take about one day to drive the brightness temperature below 10^{12}K [19]. The estimated lower limit for the intrinsic brightness temperature of the core in the Radio Astron image of $T_{b,int} > 2.910^{12}$K is therefore more than an order of magnitude larger than the equipartition brightness temperature limit established in [19] and at least several times larger than the limit established by inverse Compton cooling.

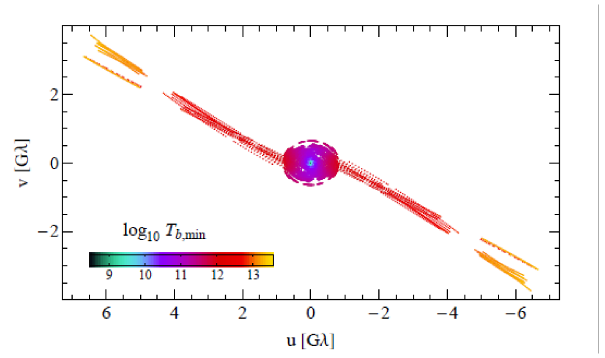


Fig.2.2.Fourier coverage (uv-coverage) of the fringe fitted data (i.e.,reliable fringe detections) of the Radio Astron observations of BL Lac on 2013 November 10-11at 22 GHz.
Color marks the lower limit of observed brightness temperature obtained from visibility amplitudes. Adopted from [19].

Remark 2.1.Note that if the estimate of the maximum brightness temperature given in [], is closer to actual values, it would imply $T_{b;int} = 5 \times 10^{13}$K. This is difficult to reconcile with current incoherent synchrotron emission models from relativistic electrons,requiring alternative models such as emission from relativistic protons.
Remark 2.2. However the proton, as we know, is 1836 times heavier than an electron and absolutely huge energy is required to accelerated it to sublight speed. We argue that these alternative models such as emission from relativistic protons can be suported by semiclassical gravity effect finds its roots in the singular behavior of quantum fields on curved distributional spacetimes presented by rotating gravitational singularities [1],[6].

3.The Colombeau distributional Kerr spacetime in Boyer- Lindquist form.

The classical Kerr metric in Boyer-Lindquist form reads

$$ds^2 = -\Xi(r,\theta)dt^2 - \frac{4mra\sin^2\theta}{\rho^2}dtd\phi + \frac{\rho^2}{\Delta_a}dr^2 + \rho^2 d\theta^2 + \left(r^2 + a^2 + \frac{2mra^2\sin^2\theta}{\rho^2}\right)\sin^2\theta d\phi^2, \tag{3.1}$$

where $\rho^2 = \rho^2(r) = r^2 + a^2\cos^2\theta, \Delta_a = \Delta_a(r) = r^2 - 2mr + a^2$, $\Xi(r,\theta) = (r^2 - 2mr + a^2\cos^2\theta)/\rho^2$.
Note that

$$\Xi(r,\theta) = \frac{r^2 - 2mr + a^2\cos^2\theta}{\rho^2} = \frac{(r - r_{E_+}(\theta))(r - r_{E_-}(\theta))}{\rho^2}, \tag{3.2}$$

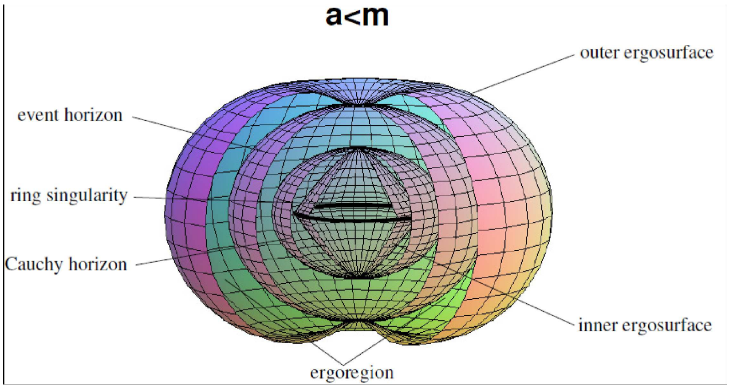
where $r_{E_\pm}(\theta) = m \pm \sqrt{m^2 - a^2\cos^2\theta}$ and

$$\Delta_a = r^2 - 2mr + a^2 = (r - r_+)(r - r_-) \tag{3.3}$$

where $r_\pm(a) = m \pm \sqrt{m^2 - a^2}$.
Let $\mathbf{K}(r,\theta)$ be a submanifold given by equation $\phi = \text{const}$, then the metric (3.1) restricted on submanifold $\mathbf{K}(r,\theta)$ reads

$$ds^2 = -\Xi(r,\theta)dt^2 + \frac{\rho^2}{\Delta_a}dr^2 + \rho^2 d\theta^2. \tag{3.4}$$

Note that: (i) the metric (3.4) is degenerates on outer ergosurface: $r = r_{E_+}(\theta)$ and inner ergosurface $r = r_{E_-}(\theta)$, (ii) the metric (3.4) is singular on horizon $r = r_+$, (iii) the metric (3.4) is singular on submanifold given by equation $r = r_-$.



Pic.4.1.1.Ergosurface,horizon,and singularity for slow Kerr black hole.

We introduce now the following regularized above (below) ergosurface $r = r_{E_+}(\theta)$ quantities

$$\Xi_\varepsilon^+(r,\theta) = \frac{(r - r_{E_-}(\theta))\sqrt{(r - r_{E_+}(\theta))^2 + \varepsilon^2}}{\rho^2(r)}, \tag{3.5}$$

$\Xi_\varepsilon^-(r,\theta) = -\Xi_\varepsilon^+(r,\theta)$ and regularized above (below) horizon quantities

$$\Delta_{a,\varepsilon}^+ = (r - r_-(a))\sqrt{(r - r_+(a))^2 + \varepsilon^2}. \tag{3.6}$$

Thus Colombeau generalized metric corresponding to classical Kerr metric (3.1) reads

$$(ds_{\varepsilon}^{\pm 2})_{\varepsilon} = -[(\Xi_{\varepsilon}^{\pm}(r_{\varepsilon},\theta))_{\varepsilon}]dt^2 - 4ma\sin^2\theta\left[\left(\frac{r_{\varepsilon}}{\rho_{\varepsilon}^2}\right)_{\varepsilon}\right]dtd\phi +$$
$$\frac{\rho_{\varepsilon}^2}{(\Delta_{a,\varepsilon}(r_{\varepsilon}))_{\varepsilon}}[(dr_{\varepsilon}^2)_{\varepsilon}] + [(\rho_{\varepsilon}^2)_{\varepsilon}]d\theta^2 + \left(r_{\varepsilon}^2 + a^2 + \frac{2mr_{\varepsilon}a^2\sin^2\theta}{\rho_{\varepsilon}^2}\right)_{\varepsilon}\sin^2\theta d\phi^2. \quad (3.7)$$

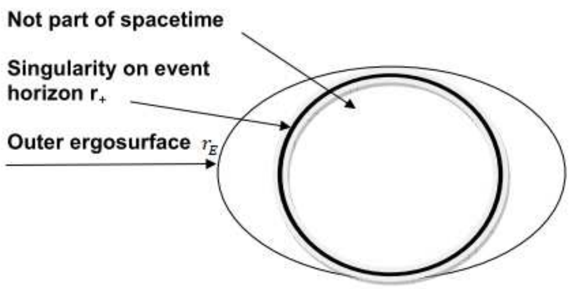


Fig.3.1.Rotatin gravitational singularity.

Let $(\mathbf{R}^{a<m}(r_{\varepsilon},\varepsilon))_{\varepsilon}$ be Colombeau generalized curvature scalar corresponding to the metric (3.7) with $a < m$.By straightforward calculation from Eq.(3.7) one obtains that main singular part $\mathbf{sing}[(\mathbf{R}^{a<m}(r_{\varepsilon},\theta,\varepsilon))_{\varepsilon}]$ of the Colombeau generalized curvature scalar $(\mathbf{R}^{a<m}(r_{\varepsilon},\theta,\varepsilon))_{\varepsilon}$ with $(r_{\varepsilon} - r_{E_+}(\theta))_{\varepsilon} \approx_{\widetilde{\mathbb{R}}} 0_{\widetilde{\mathbb{R}}}$, corresponding to the metric (3.7) reads

$$\mathbf{sing}[(\mathbf{R}^{a<m}(r_{\varepsilon},\theta,\varepsilon))_{\varepsilon}] =_{\widetilde{\mathbb{R}}} -\left(\frac{r_{\varepsilon} - r_{E_-}(\theta)}{r_{\varepsilon}^2 + a^2\cos^2\theta}\frac{\varepsilon^2}{\Delta_{\varepsilon}(r_{\varepsilon})\left[(r_{\varepsilon} - r_{E_+}(\theta))^2 + \varepsilon^2\right]^{3/2}}\right)_{\varepsilon} \quad (3.8)$$

where

$$\Delta_{\varepsilon}(r_{\varepsilon}) = (\Xi_{\varepsilon}^{\pm}(r,\theta))\frac{\rho^2(r_{\varepsilon})}{(\Delta_{a,\varepsilon}(r))_{\varepsilon}} + \frac{8mr_{\varepsilon}a\sin^2\theta}{r_{\varepsilon}^2 + a^2\cos^2\theta}. \quad (3.9)$$

From Eq.(3.8)-Eq.(3.9) on outer ergosurface $(r_{\varepsilon})_{\varepsilon} = r_{E_+}(\theta)$ we obtain

$$\mathbf{sing}[(\mathbf{R}^{a<m}(r_{E_+}(\theta),\varepsilon))_{\varepsilon}] =_{\widetilde{\mathbb{R}}} \frac{r_{E_+}(\theta) - r_{E_-}(\theta)}{8mar_{E_+}(\theta)\sin^2\theta}(\varepsilon^{-1})_{\varepsilon} =_{\widetilde{\mathbb{R}}}$$
$$\frac{\sqrt{m^2 - a^2}}{4mar_{E_+}(\theta)\sin^2\theta}(\varepsilon^{-1})_{\varepsilon} =_{\widetilde{\mathbb{R}}} c_1(m,a,\theta)(\varepsilon^{-1})_{\varepsilon}. \quad (3.10)$$

Note that main singular part $\mathbf{sing}[(\mathbf{R}^{a<m}(r_{\varepsilon},\theta,\varepsilon))_{\varepsilon}]$ of the Colombeau generalized curvature scalar $(\mathbf{R}^{a<m}(r_{\varepsilon},\theta,\varepsilon))_{\varepsilon}$ with $(r_{\varepsilon} - r_+)_{\varepsilon} \approx_{\widetilde{\mathbb{R}}} 0_{\widetilde{\mathbb{R}}}$, corresponding to the metric (3.7) reads

$$\mathbf{sing}[(\mathbf{R}^{a<m}(r_{\varepsilon},\theta,\varepsilon))_{\varepsilon}] =_{\widetilde{\mathbb{R}}} \quad (3.11)$$

Let $\left(\mathbf{R}^{\mu\nu(a<m)}(r_\varepsilon,\varepsilon)\mathbf{R}_{\mu\nu}^{(a<m)}(r_\varepsilon,\varepsilon)\right)_\varepsilon$ be Colombeau generalized quadratic scalar $(\mathbf{R}^{\mu\nu}(r_\varepsilon,\varepsilon)\mathbf{R}_{\mu\nu}(r_\varepsilon,\varepsilon))_\varepsilon$ corresponding to the metric (3.7) with $a < m$. From Eq.(3.7) one obtains that main singular part $\mathbf{sing}\left[\left(\mathbf{R}^{\mu\nu(a<m)}(r_\varepsilon,\varepsilon)\mathbf{R}_{\mu\nu}^{(a<m)}(r_\varepsilon,\varepsilon)\right)_\varepsilon\right]$ of the Colombeau generalized quadratic scalar $\left(\mathbf{R}^{\mu\nu(a<m)}(r_\varepsilon,\varepsilon)\mathbf{R}_{\mu\nu}^{(a<m)}(r_\varepsilon,\varepsilon)\right)_\varepsilon$ reads

$$\mathbf{sing}\left[\left(\mathbf{R}^{\mu\nu(a<m)}(r_\varepsilon,\varepsilon)\mathbf{R}_{\mu\nu}^{(a<m)}(r_\varepsilon,\varepsilon)\right)_\varepsilon\right] =_{\widetilde{\mathbb{R}}} \left(\frac{\varepsilon^4}{4(r_{E_+}(\theta))^4\left[\varepsilon^2+(r_\varepsilon-2m)^2\right]^3}\right)_\varepsilon. \quad (3.11)$$

Let $\left(\mathbf{R}^{\rho\sigma\mu\nu(a<m)}(r_\varepsilon,\varepsilon)\mathbf{R}_{\rho\sigma\mu\nu}^{(a<m)}(r_\varepsilon,\varepsilon)\right)_\varepsilon$ be Colombeau generalized quadratic scalar $(\mathbf{R}^{\rho\sigma\mu\nu}(r_\varepsilon,\varepsilon)\mathbf{R}_{\rho\sigma\mu\nu}(r_\varepsilon,\varepsilon))_\varepsilon$ corresponding to the metric (3.7) with $a < m$. From Eq.(3.7) one obtains that main singular part $\mathbf{sing}\left[\left(\mathbf{R}^{\rho\sigma\mu\nu(a<m)}(r_\varepsilon,\varepsilon)\mathbf{R}_{\rho\sigma\mu\nu}^{(a<m)}(r_\varepsilon,\varepsilon)\right)_\varepsilon\right]$

4. Distributional Kerr spacetime induced vacuum dominance. Classical distributional background.

Let us consider Colombeau generalized quantity $(W_\varepsilon^\pm)_\varepsilon$, called the effective action for the quantum matter fields in curved distributional spcetime, which, when functionally differentiated, yields [9]

$$\left(\frac{2}{(-g(\varepsilon))^{\frac{1}{2}}}\frac{\delta W_\varepsilon^\pm}{\delta g^{\mu\nu}(\varepsilon)}\right)_\varepsilon = (\langle\mathbf{T}_{\mu\nu}^\pm(\varepsilon)\rangle)_\varepsilon. \quad (4.1)$$

Proceeding in standard manner we get [9]

$$(W_\varepsilon^\pm)_\varepsilon = \frac{i}{2}\left[\left(\int d^n x_\varepsilon[-g^\pm(x_\varepsilon,\varepsilon)]^{\frac{1}{2}}\right)_\varepsilon\right]\left(\lim_{x_\varepsilon\to x'_\varepsilon}\int_{m^2}^{\infty} G_\varepsilon^\pm(x_\varepsilon,x'_\varepsilon;m^2)dm^2\right)_\varepsilon. \quad (4.2)$$

Interchanging now the order of integration and taking the limit $x \to x'$ one obtains

$$(W_\varepsilon^\pm)_\varepsilon = \frac{i}{2}\left(\int_{m^2}^{\infty} dm^2\int d^n x_\varepsilon[-g^\pm(x_\varepsilon,\varepsilon)]^{\frac{1}{2}} G_\varepsilon^\pm(x_\varepsilon,x_\varepsilon;m^2)\right)_\varepsilon. \quad (4.3)$$

Colombeau generalized quantity $(W_\varepsilon^\pm)_\varepsilon$ is colled as the one-loop effective action. In the case of fermion effective actions, there would be a remaining trace over spinorial indices. From Eq.(4.3) we may define an effective Lagrangian density $\left(L_{\varepsilon;\mathbf{eff}}^\pm(x_\varepsilon)\right)_\varepsilon$ by

$$(W_\varepsilon^\pm)_\varepsilon = \left(\int d^n x_\varepsilon[-g^\pm(x_\varepsilon,\varepsilon)]^{\frac{1}{2}} L_{\varepsilon;\mathbf{eff}}^\pm(x_\varepsilon)\right)_\varepsilon \quad (4.4)$$

whence finally we get

$$(L_\varepsilon^\pm(x_\varepsilon))_\varepsilon = \left([-g^\pm(x_\varepsilon,\varepsilon)]^{\frac{1}{2}}\mathcal{L}_{\varepsilon;\mathbf{eff}}^\pm(x_\varepsilon)\right)_\varepsilon = \frac{i}{2}\left(\lim_{x\to x'}\int_{m^2}^{\infty} dm^2 G_\varepsilon^\pm(x_\varepsilon,x'_\varepsilon;m^2)\right)_\varepsilon. \quad (4.5)$$

Note that $(L_\varepsilon^\pm(x_\varepsilon))_\varepsilon$ diverges at the lower end of the s integral because the $(\sigma_\varepsilon)_\varepsilon/2s$ $((\sigma_\varepsilon)_\varepsilon = (\sigma(x_\varepsilon,x'_\varepsilon))_\varepsilon)$ damping factor in the exponent vanishes in the limit $x_\varepsilon \to x'_\varepsilon$. (Convergence at the upper end is guaranteed by the $-i\epsilon$ that is implicitly added to m^2 in the De Witt-Schwinger representation of $(L_\varepsilon^\pm(x_\varepsilon))_\varepsilon$. In four dimensions, the potentially divergent terms in the De Witt-Schwinger expansion of $(L_\varepsilon^\pm(x_\varepsilon))_\varepsilon$ are

$$\left(L^{\pm}_{\varepsilon;\mathrm{div}}(x_\varepsilon)\right)_\varepsilon =$$
$$-(32\pi^2)^{-1}\left(\lim_{x\to x'}\left[\left(\Delta_{\pm}^{1/2}(x_\varepsilon,x'_\varepsilon;\varepsilon)\right)_\varepsilon\right]\int_0^\infty \frac{ds}{s^3}\exp\left[-im^2 s+\frac{\sigma(x_\varepsilon,x'_\varepsilon)}{2is}\right]\times\right. \tag{4.6}$$
$$\left.\times\left[a_0^{\pm}(x_\varepsilon,x'_\varepsilon;\varepsilon)+isa_1^{\pm}(x_\varepsilon,x'_\varepsilon;\varepsilon)+(is)^2a_2^{\pm}(x_\varepsilon,x'_\varepsilon;\varepsilon)\right]\right)_\varepsilon$$

where the coefficients $(a_0^{\pm}(x_\varepsilon,x'_\varepsilon;\varepsilon))_\varepsilon$, $(a_1^{\pm}(x_\varepsilon,x'_\varepsilon;\varepsilon))_\varepsilon$ and $(a_2^{\pm}(x_\varepsilon,x'_\varepsilon;\varepsilon))_\varepsilon$ are given by the equation

$$(a_1^{\pm}(x_\varepsilon,x'_\varepsilon;\varepsilon))_\varepsilon =$$
$$\left(\frac{1}{6}-\xi\right)(\mathbf{R}^{\pm}(\varepsilon))_\varepsilon-\frac{i}{2}\left(\frac{1}{6}-\xi\right)[(\mathbf{R}^{\pm}_{;\alpha}(\varepsilon))_\varepsilon](y^\alpha_\varepsilon)_\varepsilon-\frac{1}{3}\left[(a^{\pm}_{\alpha\beta}(\varepsilon))_\varepsilon\right]\left(y^\alpha_\varepsilon y^\beta_\varepsilon\right)_\varepsilon \tag{4.7}$$
$$(a_2^{\pm}(x_\varepsilon,x'_\varepsilon;\varepsilon))_\varepsilon=\frac{1}{2}\left(\frac{1}{6}-\xi\right)(\mathbf{R}^{\pm 2}(\varepsilon))_\varepsilon+\frac{1}{3}\left(a^{\pm\lambda}_{\ \lambda}(\varepsilon)\right)_\varepsilon$$

with all geometric quantities on the right-hand side of Eq.(4.7) evaluated at $(x'_\varepsilon)_\varepsilon \in \widetilde{\mathbb{R}}_\delta$.
The remaining terms in this asymptotic expansion, involving $a_3^{\pm}$ and higher, are finite in the limit $x_\varepsilon \to x'_\varepsilon$.
Let us determine now the precise form of the geometrical $(L^{\pm}_{\varepsilon;\mathrm{div}}(x_\varepsilon))_\varepsilon$ terms, to compare them with the distributional generalization of the gravitational Lagrangian that appears in Eq.(2.1). This is a delicate matter because (4.6) is, of course, infinite. What we require is to display the divergent terms in the form $\infty\times$ [geometrical object]. This can be done in a variety of ways. For example, in n dimensions, the asymptotic (adiabatic) expansion of $\left(L^{\pm}_{\varepsilon;\mathbf{eff}}(x_\varepsilon)\right)_\varepsilon$ is

$$\left(L^{\pm}_{\varepsilon;\mathbf{eff}}(x_\varepsilon)\right)_\varepsilon \asymp$$
$$2^{-1}(4\pi)^{-n/2}\left(\lim_{x\to x'}\left[\left(\Delta_{\pm}^{1/2}(x_\varepsilon,x'_\varepsilon;\varepsilon)\right)_\varepsilon\right]\sum_{j=0}^{\infty}a_j(x_\varepsilon,x'_\varepsilon;\varepsilon)\times\right. \tag{4.8}$$
$$\left.\times\int_0^\infty ids(is)^{j-1-n/2}\exp\left[-im^2s+\frac{\sigma(x_\varepsilon,x'_\varepsilon)}{2is}\right]\right)_\varepsilon$$

of which the first $n/2+1$ terms are divergent as $\sigma_\varepsilon\to 0$. If n is treated as a variable which can be analytically continued throughout the complex plane, then we may take the $x_\varepsilon \to x'_\varepsilon$ limit

$$\left(L^{\pm}_{\varepsilon;\mathbf{eff}}(x_\varepsilon)\right)_\varepsilon \asymp 2^{-1}(4\pi)^{-n/2}\left(\sum_{j=0}^{\infty}a_j(x_\varepsilon;\varepsilon)\int_0^\infty ids(is)^{j-1-n/2}\exp[-im^2s]\right)_\varepsilon =$$
$$2^{-1}(4\pi)^{-n/2}\sum_{j=0}^{\infty}(m^2)^{n/2-j}\Gamma\left(j-\frac{n}{2}\right)(a_j(x_\varepsilon;\varepsilon))_\varepsilon, \tag{4.9}$$
$$(a_j(x_\varepsilon;\varepsilon))_\varepsilon=(a_j(x_\varepsilon,x_\varepsilon;\varepsilon))_\varepsilon.$$

From Eq.(4.9) follows we shall wish to retain the units of $\left(L^{\pm}_{\varepsilon;\mathbf{eff}}(x_\varepsilon)\right)_\varepsilon$ as (length)$^{-4}$, even when $n\neq 4$. It is therefore necessary to introduce an arbitrary mass scale μ and to rewrite Eq.(4.9) as

$$\left(L^{\pm}_{\varepsilon;\mathbf{eff}}(x_\varepsilon)\right)_\varepsilon \asymp 2^{-1}(4\pi)^{-n/2}\left(\frac{m}{\mu}\right)^{n-4}\left(\sum_{j=0}^{\infty} a_j(x_\varepsilon;\varepsilon)(m^2)^{4-2j}\Gamma\left(j-\frac{n}{2}\right)\right)_\varepsilon . \qquad (4.10)$$

If $n \to 4,$ the first three terms of Eq.(4.10) diverge because of poles in the Γ- functions:

$$\begin{aligned}\Gamma\left(-\frac{n}{4}\right) &= \frac{4}{n(n-2)}\left(\frac{2}{4-n}-\gamma\right)+O(n-4),\\ \Gamma\left(1-\frac{n}{2}\right) &= \frac{4}{(2-n)}\left(\frac{2}{4-n}-\gamma\right)+O(n-4),\\ \Gamma\left(2-\frac{n}{2}\right) &= \frac{2}{4-n}-\gamma+O(n-4).\end{aligned} \qquad (4.11)$$

Denoting these first three terms by $(L^{\pm}_{\varepsilon;\mathbf{div}}(x_\varepsilon))_\varepsilon$, we have

$$\begin{aligned}(L^{\pm}_{\varepsilon;\mathbf{div}}(x_\varepsilon))_\varepsilon &= (4\pi)^{-n/2}\left\{\frac{1}{n-4}+\frac{1}{2}\left[\gamma+\ln\left(\frac{m^2}{\mu^2}\right)\right]\right\}\times\\ &\left(\left[\frac{4m^4 a_0(x_\varepsilon;\varepsilon)}{n(n-2)}-\frac{2m^2 a_1(x_\varepsilon;\varepsilon)}{n-2}+a_2(x_\varepsilon;\varepsilon)\right]\right)_\varepsilon .\end{aligned} \qquad (4.12)$$

The functions $(a_0(x_\varepsilon;\varepsilon))_\varepsilon, (a_1(x_\varepsilon;\varepsilon))_\varepsilon$ and $(a_2(x_\varepsilon;\varepsilon))_\varepsilon$ are given by taking the coincidence limits of (4.7)

$$\begin{aligned}(a^{\pm}_0(x_\varepsilon;\varepsilon))_\varepsilon &= 1, (a^{\pm}_1(x_\varepsilon;\varepsilon))_\varepsilon = \left(\frac{1}{6}-\xi\right)(\mathbf{R}^{\pm}(\varepsilon))_\varepsilon,\\ (a^{\pm}_2(x_\varepsilon;\varepsilon))_\varepsilon &= \frac{1}{180}(\mathbf{R}^{\pm}_{\alpha\beta\gamma\delta}(x_\varepsilon,\varepsilon)\mathbf{R}^{\pm\alpha\beta\gamma\delta}(x_\varepsilon,\varepsilon))_\varepsilon - \frac{1}{180}(\mathbf{R}^{\pm\alpha\beta}(x_\varepsilon,\varepsilon)\mathbf{R}^{\pm}_{\alpha\beta}(x_\varepsilon,\varepsilon))_\varepsilon -\\ &-\frac{1}{6}\left(\frac{1}{5}-\xi\right)(\Box_{\varepsilon,x}\mathbf{R}^{\pm}(x_\varepsilon,\varepsilon))_\varepsilon + \frac{1}{2}\left(\frac{1}{6}-\xi\right)(\mathbf{R}^{\pm 2}(x_\varepsilon,\varepsilon))_\varepsilon .\end{aligned} \qquad (4.13)$$

Finally one obtains [9]

$$(L^{\pm}_{\varepsilon;\mathbf{ren}}(x_\varepsilon))_\varepsilon \asymp -\frac{1}{64\pi^2}\left(\int_0^\infty ids\ln(is)\frac{\partial^3}{\partial(is)^3}\left[\mathcal{F}^{\pm}_{\varepsilon}(x_\varepsilon,x_\varepsilon;is)e^{-ism^2}\right]\right)_\varepsilon . \qquad (4.14)$$

All the higher order $(j > 2)$ terms in the DeWitt-Schwinger expansion of the effective Lagrangian (4.14) are infrared divergent at $n = 4$ as $m \to 0,$ we can still use this expansion to yield the ultraviolet divergent terms arising from $j = 0,1,$ and 2 in the four-dimensional case. We may put $m = 0$ immediately in the $j = 0$ and 1 terms in the expansion, because they are of positive power for $n \sim 4.$ These terms therefore vanish. The only nonvanishing potentially ultraviolet divergent term is therefore $j = 2$:

$$2^{-1}(4\pi)^{-n/2}\left(\frac{m}{\mu}\right)^{n-4} a_2(x_\varepsilon,\varepsilon)\Gamma\left(2-\frac{n}{2}\right), \qquad (4.15)$$

which must be handled carefully. Substituting for $(a_2(x_\varepsilon;\varepsilon))_\varepsilon$ with $\xi = \xi(n)$ from (4.13), and rearranging terms, we may write the divergent term in the effective action arising from (4.14) as follows

$$(W^{\pm}_{\varepsilon,\mathbf{div}})_{\varepsilon} = 2^{-1}(4\pi)^{-n/2}\left(\frac{m}{\mu}\right)^{n-4}\Gamma\left(2-\frac{n}{2}\right)\left(\int d^n x_{\varepsilon}[-g^{\pm}(x_{\varepsilon},\varepsilon)]^{\frac{1}{2}}a_2(x_{\varepsilon},\varepsilon)\right)_{\varepsilon} = 2^{-1}(4\pi)^{-n/2}\left(\frac{m}{\mu}\right)^{n-4}\Gamma\left(2-\frac{n}{2}\right)\times \left(\int d^n x_{\varepsilon}[-g^{\pm}(x_{\varepsilon},\varepsilon)]^{\frac{1}{2}}\left[\widetilde{\alpha}\mathsf{F}^{\pm}_{\varepsilon}(x_{\varepsilon})+\widetilde{\beta}G^{\pm}_{\varepsilon}(x_{\varepsilon})\right]\right)_{\varepsilon} + O(n-4), \tag{4.16}$$

where

$$\begin{gathered}(\mathsf{F}^{\pm}_{\varepsilon}(x_{\varepsilon}))_{\varepsilon} = (\mathbf{R}^{\pm\alpha\beta\gamma\delta}(x_{\varepsilon},\varepsilon)\mathbf{R}^{\pm}_{\alpha\beta\gamma\delta}(x_{\varepsilon},\varepsilon))_{\varepsilon} - 2(\mathbf{R}^{\pm\alpha\beta}(x_{\varepsilon},\varepsilon)\mathbf{R}^{\pm}_{\alpha\beta}(x_{\varepsilon},\varepsilon))_{\varepsilon} + \frac{1}{3}(\mathbf{R}^{\pm 2}(x_{\varepsilon},\varepsilon))_{\varepsilon},\\ (G^{\pm}_{\varepsilon}(x))_{\varepsilon} = (\mathbf{R}^{\pm\alpha\beta\gamma\delta}(x_{\varepsilon},\varepsilon)\mathbf{R}^{\pm}_{\alpha\beta\gamma\delta}(x_{\varepsilon},\varepsilon))_{\varepsilon},\\ \widetilde{\alpha} = \frac{1}{120}, \widetilde{\beta} = -\frac{1}{360}.\end{gathered} \tag{4.17}$$

Finally we obtain [9]

$$(\langle T^{\mu}_{\mu}(x_{\varepsilon},\varepsilon)\rangle_{\mathbf{ren}})_{\varepsilon} = -(1/2880\pi^2)\left[\widetilde{\alpha}\left(\mathsf{F}_{\varepsilon}(x_{\varepsilon}) - \tfrac{2}{3}\square_{\varepsilon,x}\mathbf{R}^{\pm}(x_{\varepsilon},\varepsilon)\right)_{\varepsilon} + \widetilde{\beta}(G^{\pm}_{\varepsilon}(x_{\varepsilon}))_{\varepsilon}\right] = -(1/2880\pi^2)\times \left[(\mathbf{R}^{\pm}_{\alpha\beta\gamma\delta}(x_{\varepsilon},\varepsilon)\mathbf{R}^{\pm\alpha\beta\gamma\delta}(x_{\varepsilon},\varepsilon))_{\varepsilon} - (\mathbf{R}^{\pm}_{\alpha\beta}(x_{\varepsilon},\varepsilon)\mathbf{R}^{\pm\alpha\beta}(x_{\varepsilon},\varepsilon))_{\varepsilon} - (\square_{\varepsilon,x}\mathbf{R}^{\pm}(x_{\varepsilon},\varepsilon))_{\varepsilon}\right]. \tag{4.18}$$

In order to obtain finite result from Eq.(4.18) we have applied loop quantum gravity approuch [9]-[10]. Thus final result in general case reads

$$\langle T^{\mu}_{\mu}(x)\rangle_{\mathbf{ren}} \triangleq \langle T^{\mu}_{\mu}(x,\Delta)\rangle_{\mathbf{ren}} = -(1/2880\pi^2)[\mathbf{R}^{\pm}_{\alpha\beta\gamma\delta}(x,\Delta)\mathbf{R}^{\pm\alpha\beta\gamma\delta}(x,\Delta) - \mathbf{R}^{\pm}_{\alpha\beta}(x,\Delta)\mathbf{R}^{\pm\alpha\beta}(x,\Delta) - \square_x\mathbf{R}^{\pm}(x,\Delta)] \tag{4.19}$$

where $\Delta \sim \ell_{\mathbf{Planck}}$.

5.Quantum distributional background.

In section 4 above we have considered the calculation on a classical distributional background goes.However, the quantum distributional background introduces a principal difference. The main difference between considering a quantum field on a distributional quantum space-time as opposed to a classical space-time is that the field equations become "discretized" and the divergences naturally regulated as was considered in [1],[10].
As an appropriate simple exmple we consider now the Schwarzschild spacetime in $d = 2$.The Schwarzschild metric in $d = 1+1$ its original singular form reads:

$$ds^2 = -\left(1-\frac{2m}{r}\right)dt^2 + \left(1-\frac{2m}{r}\right)^{-1}dr^2. \tag{5.1}$$

The metric (5.1) is degenerates and singular on Schwarzschild horizon $r = 2m$. Following [12]-[14] using the canonical nonsmooth regularization we embed the metric coefficients into Colombeau algebra $\widetilde{\mathcal{G}}_{\delta}(\widetilde{\mathbb{R}}^2,\{r=2m\})$.Thus we have to replace the nonclassical singular metric (5.1) by the Colombeau generalized metric above horizon $r \geq 2m$

$$(ds^{+2}_{\varepsilon})_{\varepsilon} = -[(g^{+}_{\varepsilon})_{\varepsilon}](dt^2_{\varepsilon})_{\varepsilon} + [(g^{+}_{\varepsilon})_{\varepsilon}]^{-1}(dr^2_{\varepsilon})_{\varepsilon}. \tag{5.2}$$

and below horizon $(r_{\varepsilon})_{\varepsilon} < 2m$

$$(ds_{\varepsilon}^{-2})_{\varepsilon} = -[(g_{\varepsilon}^{-})_{\varepsilon}](dt_{\varepsilon}^{2})_{\varepsilon} + [(g_{\varepsilon}^{-})_{\varepsilon}]^{-1}(dr_{\varepsilon}^{2})_{\varepsilon}. \quad (5.3)$$

correspondingly,where

$$(g_{\varepsilon}^{\pm})_{\varepsilon} = \pm r^{-1}\left(\sqrt{(r_{\varepsilon}-2m)^2+\varepsilon^2}\right)_{\varepsilon}. \quad (5.4)$$

By straightforward calculation from Eq.(5.2)-Eq.(5.4) one obtains that main singular part $\mathbf{sing}[(\mathbf{R}^{\pm}(r_{\varepsilon},\varepsilon))_{\varepsilon}]$ of the Colombeau generalized curvature scalar $(\mathbf{R}^{\pm}(r_{\varepsilon},\varepsilon))_{\varepsilon}$ corresponding to the metric tensor (5.4) reads [1]:

$$\mathbf{sing}[(\mathbf{R}^{\pm}(r_{\varepsilon},\varepsilon))_{\varepsilon}] = \left(\frac{\varepsilon^2}{2m\left[(r_{\varepsilon}-2m)^2+\varepsilon^2\right]^{3/2}}\right)_{\varepsilon}. \quad (5.5)$$

By straightforward calculation from Eq.(5.2)-Eq.(5.4) one obtains that main singular part $\mathbf{sing}\left[(\mathbf{R}^{\pm\mu\nu}(r_{\varepsilon},\varepsilon)\mathbf{R}_{\mu\nu}^{\pm}(r_{\varepsilon},\varepsilon))_{\varepsilon}\right]$ of the Colombeau generalized quadratic scalar $(\mathbf{R}^{\pm\mu\nu}(r_{\varepsilon},\varepsilon)\mathbf{R}_{\mu\nu}^{\pm}(r_{\varepsilon},\varepsilon))_{\varepsilon}$ corresponding to the metric tensor (5.4) reads [1]:

$$\mathbf{sing}\left[(\mathbf{R}^{\pm\mu\nu}(r_{\varepsilon},\varepsilon)\mathbf{R}_{\mu\nu}^{\pm}(r_{\varepsilon},\varepsilon))_{\varepsilon}\right] = \left(\frac{\varepsilon^4}{4m^2\left[(r_{\varepsilon}-2m)^2+\varepsilon^2\right]^{3}}\right)_{\varepsilon}. \quad (5.6)$$

By straightforward calculation from Eq.(5.2)-Eq.(5.4) one obtains that main singular part $\mathbf{sing}\left[(\mathbf{R}^{\pm\rho\sigma\mu\nu}(r_{\varepsilon},\varepsilon)\mathbf{R}_{\rho\sigma\mu\nu}^{\pm}(r_{\varepsilon},\varepsilon))_{\varepsilon}\right]$ of the Colombeau generalized quadratic scalar $(\mathbf{R}^{\pm\rho\sigma\mu\nu}(r_{\varepsilon},\varepsilon)\mathbf{R}_{\rho\sigma\mu\nu}^{\pm}(r_{\varepsilon},\varepsilon))_{\varepsilon}$ corresponding to the metric tensor (5.4) reads [1]:

$$\mathbf{sing}\left[(\mathbf{R}^{\pm\rho\sigma\mu\nu}(r_{\varepsilon},\varepsilon)\mathbf{R}_{\rho\sigma\mu\nu}^{\pm}(r_{\varepsilon},\varepsilon))_{\varepsilon}\right] = \left(\frac{\varepsilon^4}{4m^2\left[(r_{\varepsilon}-2m)^2+\varepsilon^2\right]^{3}}\right)_{\varepsilon}. \quad (5.7)$$

We will consider now the stress $(\langle\mathbf{T}_{\mu\nu}^{\pm}(\varepsilon)\rangle)_{\varepsilon}$ tensor corresponding to the metric (5.2)-(5.4). $(\langle\mathbf{T}_{\mu\nu}^{\pm}(\varepsilon)\rangle)_{\varepsilon}$ reads

$$(\langle\mathbf{T}_{\mu\nu}^{\pm}(\varepsilon)\rangle)_{\varepsilon} = \frac{2}{\sqrt{-(g_{\varepsilon}^{\pm})_{\varepsilon}}}\left(\frac{\delta W_{\varepsilon}^{\pm}[g_{\varepsilon}^{\pm}]}{\delta g_{\varepsilon}^{\mu\nu}}\right)_{\varepsilon}, \quad (5.8)$$

where

$$(\delta W_{\varepsilon}^{\pm}[g_{\varepsilon}^{\pm}])_{\varepsilon} = -\frac{i}{2}\mathbf{Tr}(\ln[(-G_F^{\pm}(g_{\varepsilon}^{\pm},\varepsilon))_{\varepsilon}]) \quad (5.9)$$

and where $(G_F^{\pm}(x_{\varepsilon}-x'_{\varepsilon},\varepsilon,m^2;g_{\varepsilon}^{\pm}))_{\varepsilon}$ the Feynman propagator for the massive generalized
scalar field,

$$((\nabla_{\mu}\nabla^{\nu}+m^2+\xi R(\varepsilon))G_F^{\pm}(x_{\varepsilon},x'_{\varepsilon},\varepsilon,m^2;g_{\varepsilon}^{\pm}))_{\varepsilon} = \left((g_{\varepsilon}^{\pm}(x_{\varepsilon}))^{-1/2}\delta(x_{\varepsilon}-x'_{\varepsilon})\right)_{\varepsilon}. \quad (5.10)$$

Since we are interested in the infinite small distance behavior of the Green's function we have expand the metric in Riemann normal coordinates around a point $(x'_{\varepsilon})_{\varepsilon}$, we can expand the metric tensor (5.4) as [9]

$$(g_{\mu\nu}^{\pm}(y_{\varepsilon},\varepsilon))_{\varepsilon} = \eta_{\mu\nu} - \tfrac{1}{3}\left(R_{\mu\alpha\nu\beta}^{\pm}(\varepsilon)y_{\varepsilon}^{\alpha}y_{\varepsilon}^{\beta}\right)_{\varepsilon} - \tfrac{1}{3}\left(R_{\mu\alpha\nu\beta,\gamma}^{\pm}(\varepsilon)y_{\varepsilon}^{\alpha}y_{\varepsilon}^{\beta}y_{\varepsilon}^{\gamma}\right)_{\varepsilon} + \ldots \quad (5.11)$$

with $(y_{\varepsilon})_{\varepsilon} = (x_{\varepsilon})_{\varepsilon} - (x'_{\varepsilon})_{\varepsilon}$,and therefore we can expand the propagator in momentum space as,

$$(G_F^{\pm}(k_{\varepsilon},m^2,\varepsilon))_{\varepsilon} = ((k_{\varepsilon}^2)_{\varepsilon}-m^2)^{-1} - \left(\tfrac{1}{6}-\xi\right)((k_{\varepsilon}^2)_{\varepsilon}-m^2)^{-2}(R_{\mu\alpha\nu\beta}^{\pm}(\varepsilon))_{\varepsilon} + \ldots \quad (5.12)$$

We now rescale the propagator $\left(\overline{G}_F^{\pm}(k_{\varepsilon},m^2,\varepsilon)\right)_{\varepsilon} = ((-g_{\varepsilon}^{\pm})^{1/4}G_F^{\pm}(k_{\varepsilon},m^2,\varepsilon))_{\varepsilon}$, and recall that we are in spherical symmetry so only the radial and time coordinates are involved, we get,

$$\begin{aligned}&\left(\overline{G}_F^{\pm}(x_\varepsilon,x'_\varepsilon,\varepsilon,m^2;g_\varepsilon^{\pm})\right)_\varepsilon = \\ &(2\pi)^{-2}\left(\int_{\tilde{\mathbb{R}}_\delta\times\tilde{\mathbb{R}}_\delta} d^2k_\varepsilon \exp[i(-k_{0,\varepsilon}y_\varepsilon^0 - k_{1,\varepsilon}y_\varepsilon^1)]\times\right. \\ &\left(\left[1+a_1^{\pm}(x_\varepsilon,x'_\varepsilon,\varepsilon)\left(-\frac{\partial}{\partial m^2}\right)+a_2^{\pm}(x_\varepsilon,x'_\varepsilon,\varepsilon)\left(-\frac{\partial}{\partial m^2}\right)^2\right]\frac{1}{k_{0,\varepsilon}^2-k_{1,\varepsilon}^2-m^2}\right)_\varepsilon,\end{aligned} \tag{5.13}$$

where

$$(a_1^{\pm}(x_\varepsilon,x'_\varepsilon,\varepsilon))_\varepsilon = \left(\tfrac{1}{6}-\xi\right)(R^{\pm}(\varepsilon))_\varepsilon - \tfrac{1}{2}\left(\tfrac{1}{6}-\xi\right)(R^{\pm}_{,\alpha}(\varepsilon)y_\varepsilon^\alpha)_\varepsilon - \tfrac{1}{3}\left(a^{\pm}_{\alpha\beta}(\varepsilon)y_\varepsilon^\alpha y_\varepsilon^\beta\right)_\varepsilon. \tag{5.14}$$

The Colombeau quantity $\left(a^{\pm}_{\alpha\beta}(\varepsilon)\right)_\varepsilon$ is a geometric expression involving linear and quadratic terms in the distributional scalar curvature, distributional Ricci and distributional Riemann tensor.In $d = 3+1$ case the term involving $\left(a^{\pm}_{\alpha\beta}(\varepsilon)\right)_\varepsilon$ also leads to divergent corrections that need to be compensated introducing counterterms quadratic in the curvature. In spherical symmetry we does not need to consider such term. In order to compute the Green's function we use the identity

$$((k_\varepsilon^2)_\varepsilon - m^2)^{-1} = -i\int_0^\infty ds \exp[is((k_\varepsilon^2)_\varepsilon - m^2)]. \tag{5.15}$$

From Eq.(5.13) and Eq.(5.15) by integrate on $(k_\varepsilon)_\varepsilon$ we obtain

$$\begin{aligned}\left(\overline{G}_F^{\pm}(x_\varepsilon,x'_\varepsilon,\varepsilon,m^2;g_\varepsilon^{\pm})\right)_\varepsilon &= -\frac{i}{4\pi}\int_0^\infty \frac{ds}{s}\exp\left[-im^2s+\frac{(\sigma_\varepsilon)_\varepsilon}{2is}\right]\times \\ &\left[1+is(a_1^{\pm}(x_\varepsilon,x'_\varepsilon,\varepsilon))_\varepsilon+(is)^2(a_2^{\pm}(x_\varepsilon,x'_\varepsilon,\varepsilon))_\varepsilon\right].\end{aligned} \tag{5.16}$$

Here $(\sigma_\varepsilon)_\varepsilon$ is related to the geodesic distance squared between $(x_\varepsilon)_\varepsilon$ and $(x'_\varepsilon)_\varepsilon, (\sigma_\varepsilon)_\varepsilon = (y_\varepsilon^2)_\varepsilon/2$. The condition of the quantization of the areas of symmetry leads to an effective quantization of the radial coordinate with $(r_{i,\varepsilon}^2)_\varepsilon = \ell^2_{\mathbf{Planck}}(k_{i,\varepsilon})_\varepsilon$ where i the label of the vertex of the spin network associated with the radial position $(r_{i,\varepsilon})_\varepsilon$. We will consider the simplest case of a spin network that is equispaced in normal coordinates with lattice spacing Δ. This imposes a cutoff in the radial integral in $k_{1,\varepsilon}$ of $2\pi/\Delta$ as is common on a lattice. Then the Green's function reads,

$$\begin{aligned}&\left(\overline{G}_\Delta^{\pm}(x_\varepsilon,x'_\varepsilon,\varepsilon,m^2;g_\varepsilon^{\pm})\right)_\varepsilon = \\ &-\frac{i}{8\pi}\int_0^\infty \frac{ds}{s}\exp\left(-im^2s+\frac{(\sigma_\varepsilon)_\varepsilon}{2is}\right)\left[\operatorname{erf}\left(\frac{\sqrt{i}}{2}\left(\frac{4\pi s-\Delta(y_\varepsilon^1)_\varepsilon}{\Delta\sqrt{s}}\right)\right)\right. \\ &\left.-\operatorname{erf}\left(\frac{-\sqrt{i}}{2}\left(\frac{4\pi s+\Delta(y_\varepsilon^1)_\varepsilon}{\Delta\sqrt{s}}\right)\right)\right].\end{aligned} \tag{5.17}$$

Thus the effective action is finite and takes the form

$$(W_\varepsilon^{\pm}(x_\varepsilon))_\varepsilon = \frac{i}{2}\left(\int dx_\varepsilon^0\int dr_\varepsilon\sqrt{-g_\varepsilon^{(2)}}\right)_\varepsilon\left(\lim_{x_\varepsilon\to x'}\int_{m^2}^\infty \overline{G}_\Delta^{\pm}(x_\varepsilon,x'_\varepsilon,\varepsilon,m^2;g_\varepsilon^{\pm})dm^2\right) \tag{5.18}$$

From Eq.(5.18) we can identify the effective Lagrangian where we study the divergence,

$$\left(L^{\pm\mathbf{div}}_{\mathbf{effective}}(x_\varepsilon,\varepsilon)\right)_\varepsilon = -\frac{i}{8\pi}\int_0^\infty \frac{ds}{s^2}\exp(-im^2 s)\mathrm{erf}\left(\left(\frac{2\pi\sqrt{is}}{\Delta}\right)\right)\times \left[1+is(a_1^{\pm}(x_\varepsilon,\varepsilon))_\varepsilon+(is)^2(a_2^{\pm}(x_\varepsilon,\varepsilon))_\varepsilon\right] = \quad (5.19)$$

From Eq.(5.19) we obtain

$$\left(L^{\pm\mathbf{div}}_{\mathbf{effective}}(x_\varepsilon,\varepsilon)\right)_\varepsilon = -\frac{i}{8\pi}\int_0^\infty \frac{ds}{s^2}\exp(-im^2 s)\left[1+is(a_1^{\pm}(x_\varepsilon,\varepsilon))_\varepsilon+(is)^2(a_2^{\pm}(x_\varepsilon,\varepsilon))_\varepsilon\right]. \quad (5.20)$$

For the particular background quantum state we chose with an equispaced lattice with invariant distance among vertices of the spin network given by Δ, the first two terms in the expansion in powers of (is) would lead to divergent contributions in the limit $\Delta \to 0$. For a finite, sub-Planckian Δ they are very large. They can were considered as fundamental physical effect arises from quantum distributional background

Conclusion

In this article, we argue that the canonical interpretation of the Kerr spacetime in contemporary general relativity is wrong and that revision is needed. We studied the Kerr solution using Colombeau distributional geometry, thus without leaving singular Boyer- Lindquist coordinates We argue that the Kerr solution is impossible to treat classically but it can only be treated by using an embedding of the classical Kerr metric tensor into appropriate Colombeau algebra supergeneralized functions $\widetilde{\mathcal{G}}_\delta(\tilde{\mathbb{R}}^4,\widetilde{\Sigma})$. This meant that the classical Kerr spacetime could be extended up to the distributional semi-Riemannian spacetime, since the classical Levi-Civita connection is not available for the whole Kerr spacetime in singular Boyer- Lindquist coordinates.

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