Addendum to "A Bifurcation Model of the Quantum Field"

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Abstract

The purpose of this brief Addendum is to elaborate on a couple of relationships introduced in Physica A 165: 399-419 (1990).

Key words: Standard Model parameters, Period doubling bifurcations, fermion generation problem, fermion chirality, CPT theorem, neutrino helicity.

We bring here additional clarifications on two relationships introduced in [1], namely,

1) relation (26) expressing the time-reversal operator applied to fermion doublets,

2) relations (30) and (31) concerning the breaking of chiral symmetry in the neutrino sector.

The reader is also directed to [2], which complements [1] with details on the Feigenbaum route to chaos in unimodal maps.

1) Consider a pair of fields $\varphi(x), \varphi(y)$ defined at points separated by a space-like interval

$$x = (x^0, \mathbf{x}) \; ; \qquad y = (y^0, \mathbf{y}) \tag{1}$$

The *time-ordered product* of fields is customarily represented as [3]

$$\tau\{\varphi(x)\varphi(y)\} = \begin{cases} \varphi(y)\varphi(x); & y^0 > x^0 \\ \varphi(x)\varphi(y); & y^0 < x^0 \end{cases} \tag{2}$$

Reversing the sign of time follows from the action of the *time-reversal operator* T given by

$$T(\varphi(x)) = \varphi(\mathbf{x}, -x^0) \tag{3a}$$

$$T(\varphi(y)) = \varphi(\mathbf{y}, -y^0) \tag{3b}$$

It is apparent that changing the sign of time swaps the top and bottom rows of (2) as $-y^0 < -x^0$ in the first row and $-y^0 > -x^0$ in the second.

2) In the chiral basis, the Dirac spinor is assembled from a left and right-handed Weyl spinor and reads, in a column vector format,

$$\psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} \tag{4}$$

Defining the projection operators acting on Dirac spinor enables one to identify the left and right-handed components of (4) as in

$$P_L \psi = \frac{1-\gamma_5}{2} \psi = \begin{pmatrix} \psi_L \\ 0 \end{pmatrix} \tag{5a}$$

$$P_R \psi = \frac{1+\gamma^5}{2} \psi = \begin{pmatrix} 0 \\ \psi_R \end{pmatrix} \tag{5b}$$

As shown in [1], the neutrino-antineutrino branch develops in a triplicated pattern at the first fermionic vertex of the bifurcation diagram (V^1) and is described by the trio of states

$$\left(\nu_e, \overline{\nu_e}\right), (\nu_\mu, \overline{\nu_\mu}), (\nu_\tau, \overline{\nu_\tau}) \tag{6}$$

Applying the CPT operator to the electron-neutrino doublet leads to

$$(\nu_e, \overline{\nu_e})_L \xrightarrow{C} (\overline{\nu_e}, \nu_e)_L \xrightarrow{P} (\overline{\nu_e}, \nu_e)_R \xrightarrow{T} (\nu_e, \overline{\nu_e})_R \tag{7}$$

which means that the left-handed and right-handed doublets are physically indistinguishable. In turn, this finding indicates that the only meaningful representation of (4) - (5) is in terms of a *broken doublet* comprising a pair of symmetrical singlets. In symbolic form, this result can be presented as

$$\nu = \begin{pmatrix} \nu_L \\ \overline{\nu_R} \end{pmatrix} = \begin{pmatrix} \nu_L \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ \overline{\nu_R} \end{pmatrix} \tag{8}$$

It follows that neutrinos exist in a single polarized state, either as left-handed fermions or right-handed antifermions, a conclusion validated by all experiments carried out to date. But what happens if the neutrino is a Majorana field instead of a Dirac field? One recalls that a Majorana spinor is *still* a Dirac spinor whose chiral components are not independent but coupled through the connection [3]

$$\psi_M = \begin{pmatrix} \psi_L \\ i\sigma^2\psi_L^* \end{pmatrix} \tag{9}$$

where σ^2 stands for the square of the Pauli matrix. As the only physical distinction between the two components of (9) is the helicity sign, one can formally break (9) into a pair of symmetrical singlets, namely,

$$\psi_M = \begin{pmatrix} \psi_L \\ i\sigma^2\psi_L^* \end{pmatrix} = \begin{pmatrix} \psi_L \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ i\sigma^2\psi_L^* \end{pmatrix} \tag{10}$$

As argued in [1], chiral asymmetry manifest in the right-hand side of either (8) or (10) carries over to the massive fermion sector including charged leptons and quarks. It is for this reason that leptons show up as both $SU(2)$ doublets and $SU(2)$ singlets. Likewise, and by analogy with (8) and (10), a right-handed quark doublet may be thought of as a broken pair of $SU(2)$ singlets, for example,

$$\begin{pmatrix} u \\ d \end{pmatrix}_R = \begin{pmatrix} u_R \\ \ \end{pmatrix} + \begin{pmatrix} \ \\ d_R \end{pmatrix} \tag{11}$$

Unlike lepton singlets, quark singlets are always confined in bound states and do not exist in isolation.

References

1. E. Goldfain, "A Bifurcation Model of the Quantum Field", Physica A 165: 399-419 (1990). A copy of this reference is available at the following sites: https://www.academia.edu/38767540/A_bifurcation_model_of_the_quantum_field https://vixra.org/pdf/1309.0015v1.pdf

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3. M. Maggiore, "A Modern Introduction to Quantum Field Theory", Oxford Univ. Press, 2005.