Addendum to “A Bifurcation Model of the Quantum Field”

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Abstract

The purpose of this brief Addendum is to elaborate on a couple of relationships introduced in Physica A 165: 399-419 (1990).

Key words: Standard Model parameters, Period doubling bifurcations, fermion generation problem, fermion chirality, CPT theorem, neutrino helicity.

We bring here additional clarifications on two relationships introduced in [1], namely,

1) relation (26) expressing the time-reversal operator applied to fermion doublets,

2) relations (30) and (31) concerning the breaking of chiral symmetry in the neutrino sector.

The reader is also directed to [2], which complements [1] with details on the Feigenbaum route to chaos in unimodal maps.

1) Consider a pair of fields \( \phi(x), \phi(y) \) defined at points separated by a space-like interval

\[
x = (x^0, x) ; \quad y = (y^0, y)
\]

The time-ordered product of fields is customarily represented as [3]

\[
\tau\{\phi(x) \phi(y)\} = \begin{cases} 
\phi(y) \phi(x) ; & y^0 > x^0 \\
\phi(x) \phi(y) ; & y^0 < x^0 
\end{cases}
\]
Reversing the sign of time follows from the action of the time-reversal operator $T$ given by

$$T(\varphi(x)) = \varphi(x, -x^0)$$

$$T(\varphi(y)) = \varphi(y, -y^0)$$

(3a)

(3b)

It is apparent that changing the sign of time swaps the top and bottom rows of (2) as $-y^0 < -x^0$ in the first row and $-y^0 > -x^0$ in the second.

2) In the chiral basis, the Dirac spinor is assembled from a left and right-handed Weyl spinor and reads, in a column vector format,

$$\psi = \left( \begin{array}{c} \psi_L \\ \psi_R \end{array} \right)$$

(4)

Defining the projection operators acting on Dirac spinor enables one to identify the left and right-handed components of (4) as in

$$P_L \psi = \frac{1 - \gamma^5}{2} \psi = \left( \begin{array}{c} \psi_L \\ 0 \end{array} \right)$$

(5a)

$$P_R \psi = \frac{1 + \gamma^5}{2} \psi = \left( \begin{array}{c} 0 \\ \psi_R \end{array} \right)$$

(5b)

As shown in [1], the neutrino-antineutrino branch develops in a triplicated pattern at the first fermionic vertex of the bifurcation diagram ($V^1$) and is described by the trio of states

$$\left( \nu_e, \bar{\nu}_e \right), \left( \nu_\mu, \bar{\nu}_\mu \right), \left( \nu_\tau, \bar{\nu}_\tau \right)$$

(6)
Applying the \( CPT \) operator to the electron-neutrino doublet leads to

\[
(v_e, \bar{v}_e)_L \xrightarrow{c} (v_e, \bar{v}_e)_L \xrightarrow{p} (v_e, \bar{v}_e)_R \xrightarrow{r} (v_e, \bar{v}_e)_R
\]  

(7)

which means that the left-handed and right-handed doublets are physically indistinguishable. In turn, this finding indicates that the only meaningful representation of (4) - (5) is in terms of a broken doublet comprising a pair of symmetrical singlets. In symbolic form, this result can be presented as

\[
v = \begin{pmatrix} v_L \\ v_R \end{pmatrix} = \begin{pmatrix} v_L \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ v_R \end{pmatrix}
\]  

(8)

It follows that neutrinos exist in a single polarized state, either as left-handed fermions or right-handed antifermions, a conclusion validated by all experiments carried out to date. But what happens if the neutrino is a Majorana field instead of a Dirac field? One recalls that a Majorana spinor is still a Dirac spinor whose chiral components are not independent but coupled through the connection [3]

\[
\psi_M = \begin{pmatrix} \psi_L \\ i \sigma^2 \psi^*_L \end{pmatrix}
\]  

(9)

where \( \sigma^2 \) stands for the square of the Pauli matrix. As the only physical distinction between the two components of (9) is the helicity sign, one can formally break (9) into a pair of symmetrical singlets, namely,

\[
\psi_M = \begin{pmatrix} \psi_L \\ i \sigma^2 \psi^*_L \end{pmatrix} = \begin{pmatrix} \psi_L \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ i \sigma^2 \psi^*_L \end{pmatrix}
\]  

(10)
As argued in [1], chiral asymmetry manifest in (8) and (10) carries over to the massive fermion sector including charged leptons and quarks.

References


2. Available at the following sites:

