Lattice Model of a MAR-Cosmos

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Abstract

The concept of matter-antimatter repulsion (MAR) [1] suggest that the evolution of the cosmos consists of an ongoing process of phase separation of matter and antimatter leading to ever-growing clusters of each type. To illustrate this process we have devised a lattice model in which densities of the two species are located at the points of a cubic lattice. The results are presented in two videos that show the time-evolution of the model. Final states are reasonably comparable with galaxy distributions in the Universe.

The Model

Our model is defined on a cubic lattice, with periodic boundary conditions of \( N \) steps in each dimension. On the lattice points we define a matter and an antimatter density \( m \) and \( a \), each normalized to unit average per lattice point. Interaction of two unit densities at a distance \( x \) is via a Coulomb-type potential \( V(x) \) of the form

\[
V(x) = \pm \frac{1}{\sqrt{1 + x^2}},
\]

with + and – signs associated with matter and antimatter, respectively (see Fig. 1).

Time evolution occurs in steps, and a further ingredient of the model is the restriction, that density shifts for a single time step are restricted to nearest-neighbor bonds. Furthermore, density shifts during a single step are proportional to the actual density at a lattice point an take place away from the point and exclusively to neighbors of higher potential for matter density. Antimatter moves exclusively to lower-potential neighbors. The shift rate is proportional to the potential difference. This behavior corresponds to a diffusive type of dynamics, without inertial behavior.

Furthermore, an additional nearest-neighbor term is added of the form

\[
s_{A-B} = (d_A - d_B)(d_A + d_B),
\]

which describes the shift of density \( d \) (or \( a \)) from point \( A \) to point \( B \). This term opposes concentration of masses an is considered a description of the fact, that concentration goes along with increased kinetic energy. The relative strength of this term as compared to the forced shift is a parameter of the model, which we call \( R \), to indicate a resistance to straight gravitational contraction.

Time evolution takes place in steps which consist of several parts:

1. Calculation of the potential at lattice points
2. Calculation of density shifts along nearest-neighbor lattice bonds which are the only allowed shift geometries

Fig 1. Potential function $V$ (eq. 1) of a unit mass vs. distance [lattice spacings]

**Potential Calculation**

It may be possible to perform an exact calculation of the potential in analogy to Madelung's evaluation of lattice energies for ionic crystals [2]. However, we are not aware of such a treatment, nor have we spent any effort in finding one. Thus, the potential is calculated approximately. For a point in the initial cube, contributions from densities on points in this cube are calculated exactly. This is the lowest level 0. Level 1, which is restricted to lattices of even size on each cell axis, takes, in addition, exact evaluations from points in a surrounding half-shell, while level 2 includes points from a full first shell, see Fig. 2. Contributions from cubes of farther shells are accounted for in dipole approximation by dipoles positioned at the centers of cubes up to the seventh shell. Farther away shells as well as contributions from higher multipoles are omitted. These approximations break the exact periodicity of the potentials and lead to small spurious effects, most pronounced near the faces of the cube.

Fig 2. Cell (filled) and ranges of exact potential evaluation for Level 1 (inner 'square') and Level 2 (outer 'square')
Evaluation of the potential requires the largest calculation effort because the number of function evaluations grows with the square of the number of lattice points and hence with the sixth power of the lattice size. In addition, level 1 and level 2 require 8 and 27 times more calculation time, respectively, than level 0.

**Density Shifts**

As mentioned, time evolution is calculated in steps of finite size in which density is shifted by finite amounts along lattice bonds. A fraction (proportional to the potential difference) of a density at a point is moved along those bonds for which an energy loss results. This forced shift corresponds to a movement caused by Newton’s force. However, as mentioned, the character is of diffusive nature as may be appropriate for densities as opposed to mass points.

At first, the shifts along all bonds of each species are calculated, then a step size is determined, which multiplies these shifts such that no negative density values result. Step sizes, i.e. time intervals, decrease with increasing density variations.

**Numerical Calculations**

Systematic calculations were performed with a cubic unit cell of 24 lattice spacings and a level 1 approximation, which led to a calculation time of some three minutes for a single potential evaluation. For a typical run some 1000 such evaluations were taken.

Each run starts from a random distribution of matter and antimatter. For systematic studies this initial random distribution was always the same, in order to eliminate possible influences of variations of the initial state.

Clearly, this finite-size model will eventually end in a situation with two blobs, one with matter one with antimatter, in a space-centered arrangement. For reasons of computation time this situation could only be reached for small model sizes (<16), one blob being positioned on a face of the model, the other on the four non-adjacent edges. This special arrangement must originate in the approximations of the potential evaluation. For an exact evaluation one would expect the two blobs in arbitrary position with a mutual space-centered arrangement.

**Characterization of Density Distributions**

For direct, illustrative, representations of a run several representations are possible. A dotted representation (starry night) attributes to each lattice cell a number of dots (different colors for matter and antimatter), proportional to the density of each species in that cell. Further representations use contouring of the density by various criteria. A useful choice is to fix the contour level such that the corresponding surface includes a given fraction of the total mass. A further choice is to select a level, such that the contour surface includes a constant fraction of the volume [2]. This latter choice may include large regions of low density.

Density distributions are advantageously described by power spectra of the Fourier components, of wave number k in the Brillouin zone. As an illustration Fig. 3 shows such power spectra at different times for a typical run. As a function of the absolute value of the k-vector the randomized starting configuration yields the typical quadratic behavior at small k-values. As time goes on, high-frequency components disappear and low-frequency components start to grow. For zero resistance parameter R the spectrum keeps (decreasing) higher frequency components, corresponding to sharp density peaks. With increasing R the higher frequency components become less important as compared to the ever
more strong lowest-frequency components. This indicates an increasingly smooth shape of the density clusters. As expected there is no significant difference in behavior of matter- and antimatter densities. However, the best illustration is a presentation of the development of matter- and anti-matter densities as a function of time. The starting configuration is a random distribution of each type of matter. We have produces two corresponding videos which differ by the method of representation of the density. The first video [3] chooses in each cell element a set of points, randomly distributed, the number of which is proportional to the corresponding densities. The color of the points differ for the two densities. This representation may be compare with a view to a starry night sky. The second video [4] utilizes contour representations, with the two types of contours chosen such that they contain half the amount of total matter of each type. The separation process is nicely illustrated. The late, separated states can quite plausibly be compared to distributions of galaxies in the actual Universe. The regions of visible galaxies must be identified with the regions of either type of density, while the the other type fills regions that must be identified with void-regions in the observed Universe.

Fig 3. Power spectra of the density of a simulation run of a 48-lattice model. The parameter R has a value of 8%. On the x-axis the absolute values of k-vectors are given in units of the largest possible value. black: initial random state, blue: at time 149 (in arbitrary units), red: at time 1998, green: at time 2999. The decay of the random distribution happens on a time scale much shorter than the build up of large-scale structures by a factor of the order of the cell size (48). As time goes on the small-k peak keeps growing to very high values (not shown).
References and Videos

3. Starry Night representation, MPEG video from personal web site [5]
4. Contour Plot in stereo, MPEG video from personal web site [5]
5. Personal Web Site