

Creation of an infinite Fibonacci Number Sequence Table

(Weblink to the [Infinite Fibonacci Number Sequence Table](#))

- by **Dipl. Ing. (FH) Harry K. Hahn** -
Ettlingen / Germany

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Note: This study is not allowed for commercial use !

Abstract :

A Fibonacci-Number-Sequences-Table was developed, which contains infinite Fibonacci-Sequences. This was achieved with the help of research results from an extensive botanical study. This study examined the phyllotactic patterns (Fibonacci-Sequences) which appear in the tree-species "Pinus mugo" at different altitudes (from 550m up to 2500m) With the increase of altitude above around 2000m the phyllotactic patterns change considerably, the number of patterns (different Fibonacci Sequences) grows from 3 to 12, and the relative frequency of the main Fibonacci Sequence decreases from 88 % to 38 %. The appearance of more Fibonacci-Sequences in the plant clearly is linked to environmental (physical) factors changing with altitude. Especially changes in temperature- / radiation- conditions seem to be the main cause which defines which Fibonacci-Patterns appear in which frequency.

The developed (natural) Fibonacci-Sequence-Table shows interesting spatial dependencies between numbers of different Fibonacci-Sequences, which are connected to each other, by the golden ratio (constant Phi) → see Table An interesting property of the numbers in the main Fibonacci-Sequence F1 seems to be, that these numbers contain all prime numbers as prime factors ! in all other Fibonacci-Sequences $\geq F2$, which are not a multiple of Sequence F1, certain prime factors seem to be missing in the factorized Fibonacci-Numbers (e.g. in Sequences F2, F6 & F8).

With the help of another study (Title: Phase spaces in Special Relativity: Towards eliminating gravitational singularities) a way was found to express (calculate) all natural numbers and their square roots only by using constant Phi (φ) and 1. An algebraic term found by Mr Peter Danenhowe, in his study, made this possible. With the formulas which I found, it seems to be possible to eliminate number systems and base mathematics only on Phi (φ) and 1 (see my 12 conjectures)

Introduction :

In botany Phyllotaxis describes the arrangement of leaves on spiral paths on the stem of a plant. Phyllotactic spirals form a distinctive class of patterns in nature. But the true cause of these phyllotactic spirals, which appear everywhere in nature, still isn't found yet ! The current belief is that the spiral patterns of leaves on the stem of a plant, which can be explained and described by Fibonacci Number Sequences, is controlled by plant hormones like Auxin.

However this can't be the true cause for the precise Fibonacci-spiral-patterns seen on plants ! Because the extensive botanical study carried out by Dr. Iliya Vakarelov clearly shows that the Fibonacci-spiral formation is influenced by environmental conditions, especially temperature and radiation (light).

Therefore the Fibonacci-spiral formation seems to have a fundamental physical cause ! Dr. Vakarelov's study also showed that the phyllotactic-patterns changed cyclic, with six year duration of the cycles. I

I have written an own **hypothesis about the cause of phyllotactic (Fibonacci) patterns :**

see study : → [Microscope Images indicate that Water Clusters are the cause of Phyllotaxis](#), alternative: [Weblink 2](#)

Please also have a look at this study : → [EHT2017 may provide evidence for a Poincare Dodecahedral Space Universe](#)

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1. Extracts from a study produced by Dr. Iliya Iv. Vakarelov, University of Forestry, Bulgaria (1982-1994)

Title : “Changes in phyllotactic pattern structure (Fibonacci Sequences) in *Pinus mugo* due to changes in altitude”

from the book „Symmetry in Plants“ by Roger V. Jean and Denis Barabe, Universities of Quebec and Montreal, Canada (Part I. – Chapter 9 , pages 213 – 229), **weblinks :** [Weblink1](#) (Google Books), [Weblink2](#)

Research Site and methods :

Pinus Mugo grows in high mountainous parts at altitudes up to 2500m forming vast communities. The vertical profile of the research sites for *Pinus mugo* was situated along the northern slopes of the eastern part of the [Rila mountain](#), and test specimens were collected from the following altitudes : 1900, 2200 and 2500 m. Test specimens were also collected from the city of Sofia (at 550 m) where *Pinus mugo* is grown as decorative plant.

The research was carried out over a period of 12 years (except of altitude 550m here research was carried out only around 6 years). The initiation of leaf primordia in the bud (meristem) occurs at the end of the growing period. The apical meristem of *Pinus mugo* starts this process around the beginning of mid of August and ends in autumn when the air temperature goes below a certain point.



Fig : *Pinus mugo*

The interesting results of the study :

(3) With the increase of altitude from 1900m to 2500m the phyllotactic pattern structure of “*Pinus mugo*” twigs changes considerably, the number of patterns (different [Fibonacci Sequences](#)) grows from 3 to 12, and the relative frequency of the main sequence decreases from 88 % to 38 %.

At the upper boundary of *Pinus mugo* natural distribution – at about 2500m, the variation of phyllotactic twig pattern structure (entropy) becomes cyclic, with six year duration of the cycles.

(5) The changes in temperature during the period of phyllotactic pattern formation of *Pinus mugo* twigs determine about 48 % of the changes in pattern structure, the latter lagging behind with one or two years.

It is obvious that when the altitude increases, the number of phyllotactic patterns (Fibonacci-Sequences) of the vegetative organs of *Pinus mugo* also increases above a given altitude. → see Table below !

Sequence No.	FIBONACCI-Sequences present in given altitude	Altitude in (m)								Total			
		550		1900		2200		2500					
		Frequency	Relative Frequency	Frequency	Relative Frequency	Frequency	Relative Frequency	Frequency	Relative Frequency	Frequency	Relative Frequency		
F1	<1,2,3,5,8,13,...>	231	0.902	431	0.885	619	F1	0.812	246	F1	0.381	1527	0.710
F3	2<1,2,3,5,8,13,...>	16	0.063	34	0.070	35	F3	0.046	111	F3	0.172	196	0.092
F2	<1,3,4,7,11,18,...>	3	0.012	22	0.045	49	F2	0.064	86	F2	0.133	160	0.074
F4	3<1,2,3,5,13,...>	6	0.023	-	-	29	F4	0.038	98	F4	0.152	133	0.062
F8	<2,5,7,12,19,31,...>	-	-	-	-	10		0.013	50		0.077	60	0.028
F11	<3,7,10,17,27,44,...>	-	-	-	-	5		0.007	18		0.028	23	0.011
F6	<1,4,5,9,14,23,...>	-	-	-	-	1		0.001	8		0.012	9	0.004
F9	2<1,3,4,7,11,18,...>	-	-	-	-	4		0.005	7		0.011	11	0.005
(?) F6	<1,7,8,15,23,38,...>	-	-	-	-	2		0.003	7		0.011	9	0.004
F5	4<1,2,3,5,8,13,...>	-	-	-	-	8		0.011	9		0.013	17	0.008
(?) F13	<1,6,7,13,20,33,...>	-	-	-	-	-		-	3		0.005	3	0.001
F10	<2,7,9,16,25,41,...>	-	-	-	-	-		-	3		0.005	3	0.001

Note : The number of Fibonacci-Sequences is increasing with altitude!

Table 1 : Data on the frequency and relative frequency of the different phyllotactic patterns for *Pinus mugo* twigs at different altitudes. Specimen formed during the period 1982-1994 have been tested for all sites except for the one at 550 m where the period covers the years 1989 – 1993.

1.1 Different Temperatures at different altitudes caused changes in Phyllotactic-pattern-variation

Different temperatures at the research sites at different altitudes (550 – 2500 m), during the period of **phyllotactic-pattern** formation, caused the changes in variability of the found phyllotactic patterns.

The number of found patterns (different **Fibonacci Sequences**) increased with altitude. But because „temperature at different altitudes“ is a complex subject, **we must understand „temperature & radiation at different altitudes“ precisely**, to understand the causes of pattern variability ! **→ see also my study : Weblink 1**

Some fundamental facts about „Temperature“ :

The temperature (thermal energy) of a solid body (e.g. a plant) is associated primarily with the vibrations of it's molecules. Heat transfer to the plant happens through thermal conduction or thermal radiation. **Here especially heat transfer through thermal radiation to the plant must be examined more closely !** This is the transfer of energy by means of electromagnetic waves (photons). Especially **Infrared-Radiation** is important for the heat transfer to the plant

Infrared radiation lies energetically in the area of the rotation niveaus of small molecules and in the area of the oscillation niveaus of molecule bindings. That means the absorption of infrared light (infrared radiation) leads to an **vibration excitation of the molecule bindings** and of the matter in the plant in general, or in other words to an increase of the heat energy (temperature) of the plant. The energetic **Near-Infrared-Radiation (IR-A/B)**, with approximately **0.7 to 3 µm** wavelength can excite **overtone or harmonic vibrations** in matter (in the plant molecules /plant structure)

1.2 Radiation is different at different altitudes

The temperature (thermal energy) of the plant increases or decreases by absorbing (see **Spectroscopy**) or by emitting radiation, or through thermal conduction.

Especially **Near-Infrared-Radiation** with wave-lengths of **0.7 to 3 µm** is absorbed by the water molecules of the plant and is responsible for the temperature of the plant. The **distribution of Infrared-Radiation in the atmosphere is different in different altitudes**, as the diagram on the right clearly shows. The sun's **IR-A/B-radiation** with **1 to 3 µm** wave-length is absorbed by H₂O, CO₂ and other atmospheric gas, more and more **on it's way from 10 km altitude to sealevel**. But also IR-C and Far-IR radiation with **3-50 µm** gets absorbed more &

Another important result of Dr. Vakarelov's study :

Additional Dr. Vakarelov's study showed that the **phyllotactic pattern variability (Fibonacci Sequence-variability)** changed over the years ! The study also showed that **the variability of the phyllotactic patterns in high altitude changed cyclic, with six year duration of the cycles.**

Figure 3 : The diagram on the righthand side shows the variability of entropy (variability of **Fibonacci Sequences**) with respect to altitude for „Pinus Mugo“ twigs. It is obvious that **at 2500 m** the curve shows a clear **cyclic process**, while at 2200 m the cyclic process is less significant, and at 1900 m nonexistent. **The cyclic process has a period of ~6 Years.**

1.3 Phyllotactic-pattern-variability seems to vary with the sunspot-cycle

Figure 4 : The next diagram on the right shows how **sunspot-numbers, cosmic ray flux, X-ray's and proton flux** changes with the 11 to 12 year sunspot-cycle. A weak correlation between **phyllotactic-pattern-variability** and cosmic ray flux is noticeable.

How does the radiation in the atmosphere change with the sunspot-cycle ? :

Solar X-ray radiation and **Ultraviolet radiation** (especially extreme UV (EUV) with **10 to 124 nm** wavelength varies markedly over the sunspot-cycle (**UV-B at 300 nm** (by up to 400% !). This radiation has a big impact on Earth's upper atmosphere. Increased X-ray & UV-radiation leads to **heating of the Ionosphere**. The ionisation of the Ionosphere also affects the propagation of **radio-waves**. Especially the **HF-radio spectrum** (3-30 MHz), but also the MF- & VHF-radio-spectrum is effected (MF=300kHz-3MHz & VHF=30-100 MHz). 30 MHz corresponds to 10 m wave-length.

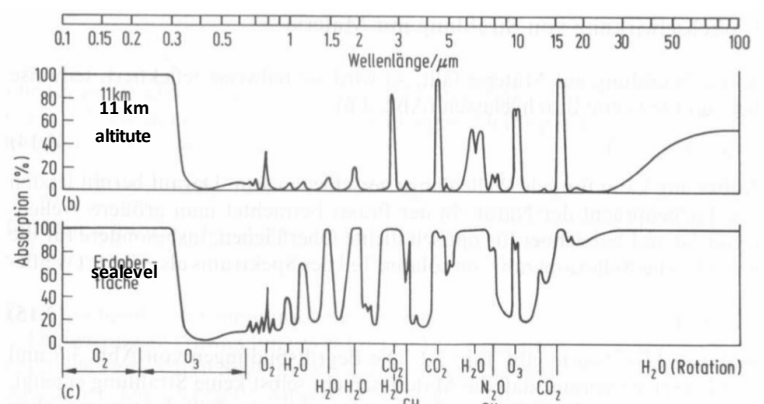


Fig. 2 : Distribution of radiation in the atmosphere, at 11 km altitude and at sealevel. It is obvious that at higher altitude the variation of radiation with different wave lengths is higher than at sea level

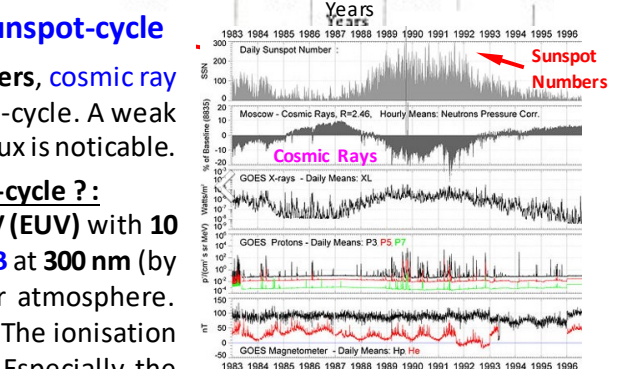
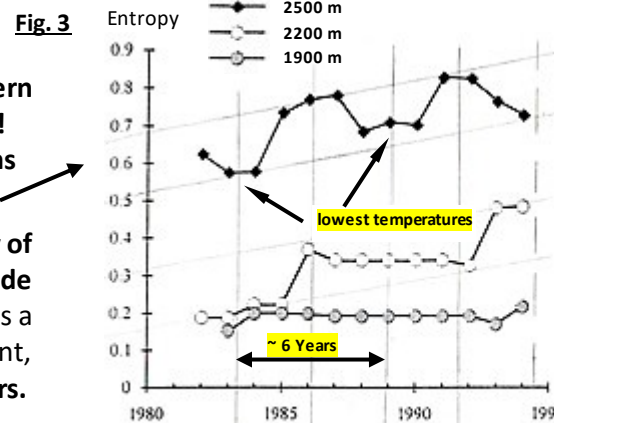


Fig. 4 : see: **Sun-Climate-Connections**

2 From the Fibonacci-Sequences shown by *Pinus mugo* at 2500m an infinite Fibonacci-Table was developed :
 There are clear spatial interdependencies noticeable between the different Fibonacci-Sequences, which are connected by the golden ratio ϕ . There is a complex network visible between the numbers of all Sequences. This table of Fibonacci-Number Sequences can be extended towards infinity and all natural numbers are contained in the lower half only once!

For 3 numbers A, B and C in the below shown arrangement, which belong to the same 3 (or 2) different Fibonacci-Sequences, the following rule is true :

The ratio of the difference (C-A) indicated by a "red line", to the difference (B-C) indicated by a "black line" is approaching the golden ratio ϕ for the further progressing Fibonacci-Number Sequences towards infinity (downwards in the table).

„Main Bow-Structures“ are also linked by the „golden ratio“ ϕ !

$$\lim \frac{C - A}{B - C} = \phi \quad \text{for } A, B, C \rightarrow \infty$$

FIBONACCI – Number Sequences No. 1 to 14 (F1 - F14) → see extended table in the Appendix !

Row No.	F1 Fibonacci-Base-Sequence	F2 Lucas-Sequence	F3 Fibonacci-Sequence (x 2)	F4 Fibonacci-Sequence (x 3)	F5 Fibonacci-Sequence (x 4)	F6	F7	F8	F9 Lucas-Sequence (x 2)	F10	F11	F12	F13 Lucas-Sequence (x 3)	F14
1	1	1	2	3	4	1	1							
2	2	3	4	5	6	2	2	2	2	2	3	3	3	4
3	3	4	5	6	7	3	3	3	3	3	4	4	4	5
4	4	5	6	7	8	4	4	4	4	4	5	5	5	6
5	5	6	7	8	9	5	5	5	5	5	6	6	6	7
6	6	7	8	9	10	6	6	6	6	6	7	7	7	8
7	7	8	9	10	11	7	7	7	7	7	8	8	8	9
8	8	9	10	11	12	8	8	8	8	8	9	9	9	10
9	9	10	11	12	13	9	9	9	9	9	10	10	10	11
10	10	11	12	13	14	10	10	10	10	10	11	11	11	12
11	11	12	13	14	15	11	11	11	11	11	12	12	12	13
12	12	13	14	15	16	12	12	12	12	12	13	13	13	14
13	13	14	15	16	17	13	13	13	13	13	14	14	14	15
14	14	15	16	17	18	14	14	14	14	14	15	15	15	16
15	15	16	17	18	19	15	15	15	15	15	16	16	16	17
16	16	17	18	19	20	16	16	16	16	16	17	17	17	18
17	17	18	19	20	21	17	17	17	17	17	18	18	18	19
18	18	19	20	21	22	18	18	18	18	18	19	19	19	20
19	19	20	21	22	23	19	19	19	19	19	20	20	20	21
20	20	21	22	23	24	20	20	20	20	20	21	21	21	22
21	21	22	23	24	25	21	21	21	21	21	22	22	22	23
22	22	23	24	25	26	22	22	22	22	22	23	23	23	24
23	23	24	25	26	27	23	23	23	23	23	24	24	24	25
24	24	25	26	27	28	24	24	24	24	24	25	25	25	26
25	25	26	27	28	29	25	25	25	25	25	26	26	26	27
26	26	27	28	29	30	26	26	26	26	26	27	27	27	28
27	27	28	29	30	31	27	27	27	27	27	28	28	28	29
28	28	29	30	31	32	28	28	28	28	28	29	29	29	30
29	29	30	31	32	33	29	29	29	29	29	30	30	30	31
30	30	31	32	33	34	30	30	30	30	30	31	31	31	32
31	31	32	33	34	35	31	31	31	31	31	32	32	32	33
32	32	33	34	35	36	32	32	32	32	32	33	33	33	34
33	33	34	35	36	37	33	33	33	33	33	34	34	34	35
34	34	35	36	37	38	34	34	34	34	34	35	35	35	36
35	35	36	37	38	39	35	35	35	35	35	36	36	36	37
36	36	37	38	39	40	36	36	36	36	36	37	37	37	38
37	37	38	39	40	41	37	37	37	37	37	38	38	38	39
38	38	39	40	41	42	38	38	38	38	38	39	39	39	40
39	39	40	41	42	43	39	39	39	39	39	40	40	40	41
40	40	41	42	43	44	40	40	40	40	40	41	41	41	42
41	41	42	43	44	45	41	41	41	41	41	42	42	42	43
42	42	43	44	45	46	42	42	42	42	42	43	43	43	44
43	43	44	45	46	47	43	43	43	43	43	44	44	44	45
44	44	45	46	47	48	44	44	44	44	44	45	45	45	46
45	45	46	47	48	49	45	45	45	45	45	46	46	46	47
46	46	47	48	49	50	46	46	46	46	46	47	47	47	48
47	47	48	49	50	51	47	47	47	47	47	48	48	48	49
48	48	49	50	51	52	48	48	48	48	48	49	49	49	50
49	49	50	51	52	53	49	49	49	49	49	50	50	50	51
50	50	51	52	53	54	50	50	50	50	50	51	51	51	52
51	51	52	53	54	55	51	51	51	51	51	52	52	52	53
52	52	53	54	55	56	52	52	52	52	52	53	53	53	54
53	53	54	55	56	57	53	53	53	53	53	54	54	54	55
54	54	55	56	57	58	54	54	54	54	54	55	55	55	56
55	55	56	57	58	59	55	55	55	55	55	56	56	56	57
56	56	57	58	59	60	56	56	56	56	56	57	57	57	58
57	57	58	59	60		57	57	57	57	57	58	58	58	59
58	58	59	60			58	58	58	58	58	59	59	59	60
59	59	60				59	59	59	59	59	60	60	60	
60	60					60	60	60	60	60				

Note : Below this line all natural numbers are contained in the Fibonacci Sequences just once !

Fibonacci-Sequence (top main bows)

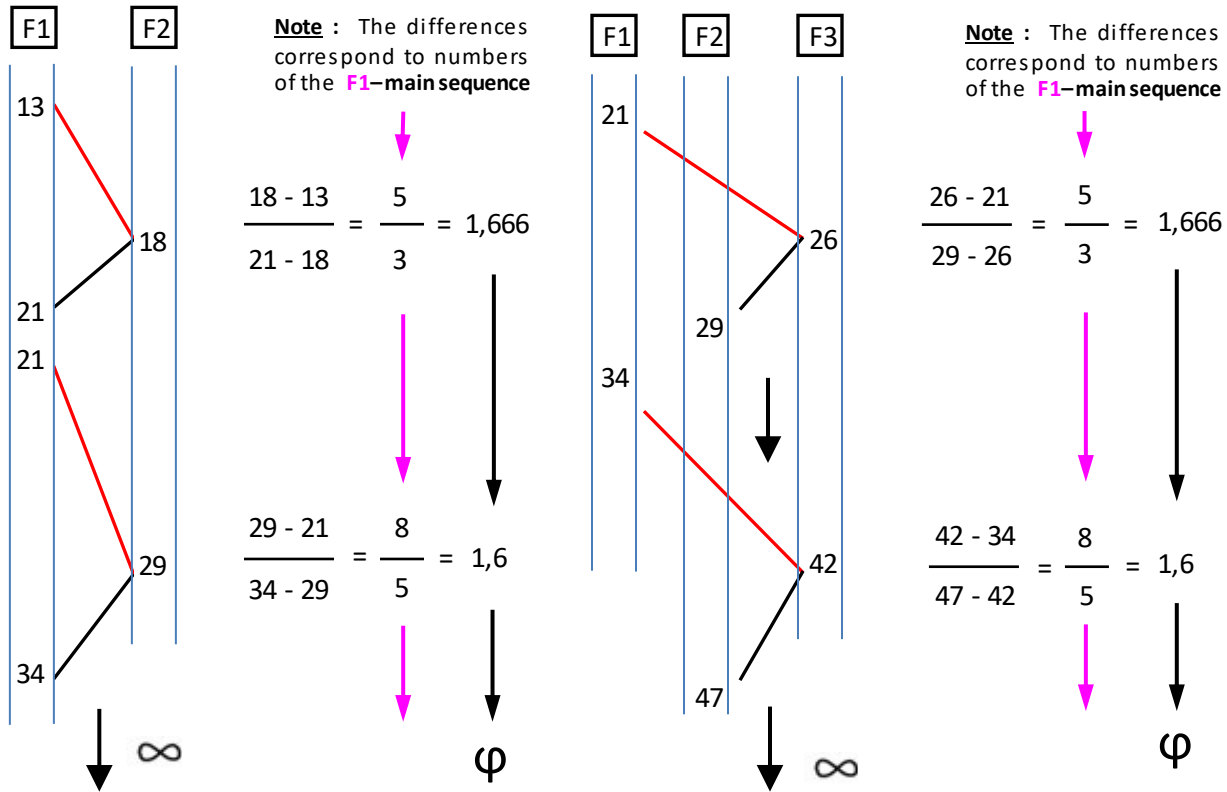
Lucas-Sequence (bottom main bows)

3 A general rule exists which connects numbers of different Fibonacci-Sequences by the golden ratio φ

→ The following two examples explain the rule which was described in general on the previous page :

The examples show how the quotient of the differences between the numbers of designated Fibonacci-Sequences (indicated by red- and black-lines in the table), is approaching the golden ratio for the number sequences progressing towards infinity.

For the examples we look at the Fibonacci Sequences **F1, F2** and **F3** (→ F2 is the Lucas-Sequence, F3 = F1 x 2)



4 Interesting properties of the Fibonacci-F1 Sequence (and other Fibonacci-Sequences) :

- The numbers of the **Fibonacci F1** – Number Sequence seem to contain all prime numbers as prime factors !
- This is not the case for all other Fibonacci-Sequences where certain prime factors are missing ! (see **Appendix**)
- And all prime factors appear periodic in defined “number-distances” in the sequence (see left side of table)
- This is the case for all Fibonacci-Sequences ! (→ These mentioned properties must be analysed in more detail !)

Table 2 : Periodicity of the prime factors of the **Fibonacci F1** - Number Sequence :

some prime factors shown in table form											in prime factors factorized Fibonacci-Numbers		sum of digits	Fibonacci-Sequence F1					
41	37	31	29	23	19	17	13	11	7	5	3	2		repeating products	new products	F	F'	F''	Nr.
																		1	1
																		1	2
																	2	3	3
																	3	4	4
																	5	5	5
																	8	6	6
																	4	7	7
																	3	8	8
																	7	9	9
																	10	10	10
																	17	11	11
																	9	12	12
																	8	13	13
																	17	14	14
																	7	15	15
																	24	16	16
																	22	17	17
																	19	18	18
																	14	19	19
																	24	20	20

→ See some selected Fibonacci-Sequences in more detail in the **Appendix** !

5 Constant φ (Φ) defines all Fibonacci-Sequences and the square roots of all natural numbers

The asymptotic ratio of successive Fibonacci numbers leads to the Golden Ratio constant φ (or Φ)

The Fibonacci Sequences describe morphological patterns in a wide range of living organisms. It is one of the most remarkable organizing principles mathematically describing natural and manmade phenomena.

The constant φ is the positive solution of the following quadratic equation :

$$x + 1 = x^2$$

$$\rightarrow \varphi = \frac{1 + \sqrt{5}}{2} = 1.618034\dots$$

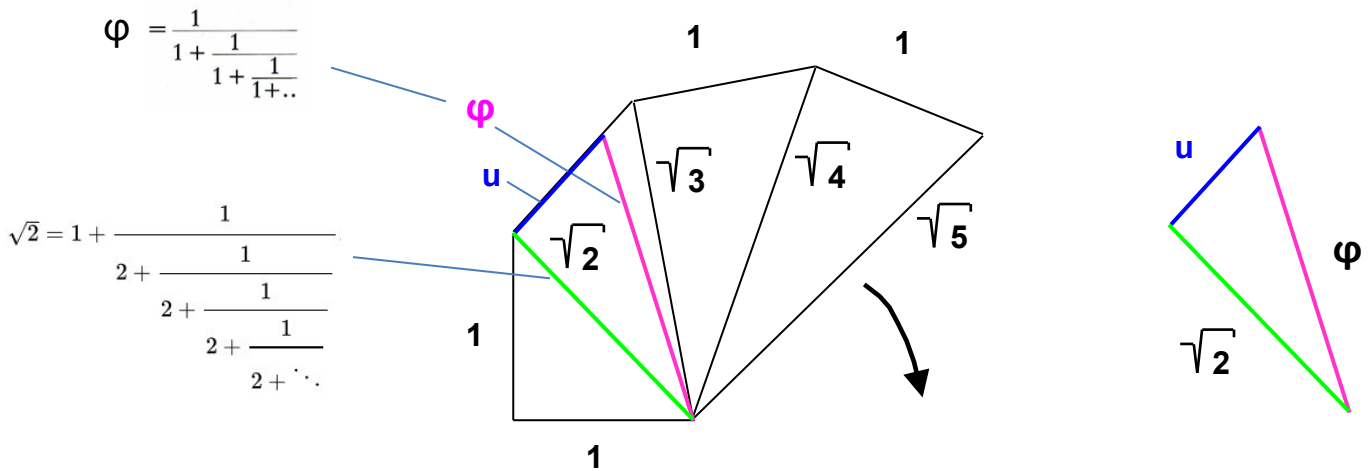


Because the value of constant φ is close to the **square root of 2** and the **square root of 3**, I draw φ into the start section of the **Square Root Spiral** :

5.1 To the discovery of an important algebraic equation regarding Constant φ (Phi)

\rightarrow This discovery indicates that constant φ and the base unit **1** form the base of mathematics and geometry.
And the distribution and structure of matter (energy) in space, is fundamentally based on constant Phi and 1

The start of the **Square Root Spiral** is shown with the constant φ drawn in :



Now we see what we can do with this arrangement of right triangles , and with the help of the **Pythagorean theorem**.

From the right triangle φ , square root of 2 & **u** follows :

$$\varphi^2 = (\sqrt{2})^2 + u^2 \quad ; \quad \text{application of the Pythagorean theorem}$$

$$\rightarrow u = \sqrt{\varphi^2 - 2} = 0,786151377\dots \quad ; \quad \text{we can calculate this value of } u \text{ with the calculator}$$

I did research with Google, and I found a study where the **constant u was expressed with an algebraic term !**

With the help of this algebraic term it was possible to find interesting new properties of constant φ !

\rightarrow see next page !

The algebraic calculation of the square roots of all natural numbers only with constant φ & 1

From **Equation (4.10)** from the study shown on the righthand side I have found the algebraic term which describes the calculated value of u :

$$\frac{\sqrt{2\sqrt{5} - 2}}{2} = 0,786151377... = u$$

From this algebraic term it follows :

$$\sqrt{\varphi^2 - 2} = \frac{\sqrt{2\sqrt{5} - 2}}{2}$$

$\rightarrow 4\varphi^2 - 8 = 2\sqrt{5} - 2$; we square both sides and transform

$$\varphi^2 = \frac{\sqrt{5} + 3}{2} ; (1) \text{ we solve for } \varphi^2$$

$$\sqrt{5} = 2\varphi^2 - 3 ; (2) \text{ we solve for } \sqrt{5}$$

Compare !



$$\varphi = \frac{\sqrt{5} + 1}{2}$$

Now we go back to the square root spiral and use the following right triangle :

$$(\sqrt{6})^2 = (\sqrt{5})^2 + 1^2 ; \text{ application of the Pythagorean theorem}$$

$$6 = (2\varphi^2 - 3)^2 + 1 ; \text{ we replace } \sqrt{5} \text{ by equation (2) and transform}$$

$$\rightarrow 3 = \frac{\varphi^4 + 1}{\varphi^2} (3) \rightarrow \sqrt{3} = \sqrt{\frac{\varphi^4 + 1}{\varphi^2}} (4) ; \text{ square root 3 expressed by } \varphi \text{ and 1 !}$$

Now we use the following right triangle :

$$(\sqrt{3})^2 = (\sqrt{2})^2 + 1^2 ; \text{ application of the Pythagorean theorem \& inserting equation (3)}$$

$$\rightarrow 2 = \frac{\varphi^4 + 1}{\varphi^2} - 1 \rightarrow 2 = \frac{\varphi^4 - \varphi^2 + 1}{\varphi^2} (5) \text{ and } \sqrt{2} = \sqrt{\frac{\varphi^4 - \varphi^2 + 1}{\varphi^2}} (6)$$

Now we insert equation (3) in equation (2) :

square root 2 expressed by φ and 1 !

$$\rightarrow \sqrt{5} = 2\varphi^2 - \frac{\varphi^4 + 1}{\varphi^2} \rightarrow \sqrt{5} = \frac{\varphi^4 - 1}{\varphi^2} ; (7) ; \text{ square root 5 expressed by } \varphi \text{ and 1 !}$$

PHASE SPACES IN SPECIAL RELATIVITY : TOWARDS ELIMINATING GRAVITATIONAL SINGULARITIES

from PETER DANENHOWER \rightarrow see weblink: <https://arxiv.org/pdf/0706.2043.pdf>

Abstract : This paper shows one way to construct phase spaces in special relativity by expanding Minkowski Space. These spaces appear to indicate that we can dispense with gravitational singularities. The key mathematical ideas in the present approach are to include a complex phase factor, such as, $e^{i\phi}$ in the Lorentz transformation and to use both the proper time and the proper mass as parameters. To develop the most general case, a complex parameter $\sigma = s + im$, is introduced, where s is the proper time, and m is the proper mass, and σ and $\sigma/|\sigma|$ are used to parameterize the position of a particle (or reference frame) in space-time-matter phase space. A new reference variable, $u = m/r$, is needed (in addition to velocity), and assumed to be bounded by 0 and $c^2/G=1$, in geometrized units. Several results are derived: The equation $E = mc^2$ apparently needs to be modified to $E^2 = (s^2c^{10})/G^2 + m^2c^4$, but a simpler (invariant) parameter is the "energy to length" ratio, which is c^4/G for any spherical region of space-time-matter. The generalized "momentum vector" becomes completely "masslike" for $u \approx 0.7861...$, which we think indicates the existence of a maximal gravity field. Thus, gravitational singularities do not occur.

Instead, as $u \rightarrow 1$ matter is apparently simply crushed into free space. In the last section of this paper we attempt some further generalizations of the phase space ideas developed in this paper.

Now we use the following right triangle :

$$(\sqrt{6})^2 = (\sqrt{5})^2 + 1^2 \quad ; \quad \text{application of the Pythagorean theorem \& inserting equation (7)}$$

$$\rightarrow 6 = \left(\frac{\varphi^4 - 1}{\varphi^2} \right)^2 + 1 \quad \rightarrow \quad 6 = \frac{\varphi^8 - \varphi^4 + 1}{\varphi^4} \quad (8) \quad \text{and} \quad \sqrt{6} = \sqrt{\frac{\varphi^8 - \varphi^4 + 1}{\varphi^4}} \quad (9)$$

We can now continue and use the following right triangles of the square root spiral :

$$(\sqrt{7})^2 = (\sqrt{6})^2 + 1^2 \quad ; \quad \text{application of the Pythagorean theorem \& inserting equation (8)}$$

$$\rightarrow 7 = \frac{\varphi^8 + 1}{\varphi^4} \quad (10) \quad \rightarrow \quad \sqrt{7} = \sqrt{\frac{\varphi^8 + 1}{\varphi^4}} \quad (11)$$

In the same way we can now calculate all square roots of all natural numbers with the next right triangles :

$$\rightarrow 8 = \frac{\varphi^8 + \varphi^4 + 1}{\varphi^4} \quad (12) \quad \text{and} \quad \sqrt{8} = \sqrt{\frac{\varphi^8 + \varphi^4 + 1}{\varphi^4}} \quad (13)$$

$$\rightarrow 10 = \frac{\varphi^8 + 3\varphi^4 + 1}{\varphi^4} \quad (14) \quad \text{and} \quad \sqrt{10} = \sqrt{\frac{\varphi^8 + 3\varphi^4 + 1}{\varphi^4}} \quad (15)$$

$$\rightarrow 11 = \frac{\varphi^8 + 4\varphi^4 + 1}{\varphi^4} \quad (16) \quad \text{and} \quad \sqrt{11} = \sqrt{\frac{\varphi^8 + 4\varphi^4 + 1}{\varphi^4}} \quad (17)$$

$$\rightarrow 12 = \frac{\varphi^8 + 5\varphi^4 + 1}{\varphi^4} \quad (18) \quad \text{and} \quad \sqrt{12} = \sqrt{\frac{\varphi^8 + 5\varphi^4 + 1}{\varphi^4}} \quad (19)$$

From the above shown formulas (equations) I have realized **a general rule** for all natural numbers > 10 :

Note : \rightarrow The expression $(3+n)$ in the rule can be replaced by products and / or sums of the equations (3) to (13)

$$\rightarrow (10+n) = \frac{\varphi^8 + (3+n)\varphi^4 + 1}{\varphi^4} \quad (20) \quad \text{and} \quad \sqrt{(10+n)} = \sqrt{\frac{\varphi^8 + (3+n)\varphi^4 + 1}{\varphi^4}} \quad (30)$$

For $n \rightarrow \infty$

With this general formula we can express all natural numbers ≥ 10 and their square roots only with φ and 1 !

This statement is also valid for all rationals (fractions) and their square roots. This is a quite interesting discovery !!

Constant Phi (φ) which defines the structure of the Dodecahedron and Icosahedron (together with base unit 1) is a very important (space structure) constant for the real / physical world ! Please also read my following study :

\rightarrow [The Black Hole in M87 \(EHT2017\) may provide evidence for a Poincare Dodecahedral Space Universe](#)

Weblink1 to the study : <http://vixra.org/abs/1907.0348> ; alternative : Weblink2 : [Weblinkto_archive.org](#)

Constant Pi (π) can also be expressed by only using constant φ and 1 !

Viète's formula from 1593 :

$$\downarrow$$

$$\pi = \frac{2}{\sqrt{2}} \frac{2}{\sqrt{2+\sqrt{2}}} \frac{2}{\sqrt{2+\sqrt{2+\sqrt{2}}}} \dots$$

→ It is also possible to derive from Viète's formula a related formula for π that still involves nested square roots of two, but uses only one multiplication :

$$\pi = \lim_{k \rightarrow \infty} 2^k \underbrace{\sqrt{2 - \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \dots + \sqrt{2}}}}}}}_{k \text{ square roots}}$$

If we replace the number 2 in the above shown formulas by the found equation (5) where number 2 can be expressed by constant φ and 1, then we can express the constant Pi (π) also by only using the constant φ and 1 !

Replace Number 2 in the above shown formulas with this term.

$$\downarrow$$

$$\rightarrow 2 = \frac{\varphi^4 + 1}{\varphi^2} - 1 \quad \rightarrow \quad \boxed{2 = \frac{\varphi^4 - \varphi^2 + 1}{\varphi^2}} \quad (5) \quad \text{and} \quad \sqrt{2} = \sqrt{\frac{\varphi^4 - \varphi^2 + 1}{\varphi^2}} \quad (6)$$

It becomes clear that the irrationality of Pi (π) is also only based on the constant φ and 1, in the same way as the irrationality of all irrational square roots, is only based on constant φ & 1 ! Numbers don't exist ! Only φ & 1 exist !

Constant Pi (π) can now be expressed in this way, by only using constant φ and 1 :

$$\pi = \lim_{k \rightarrow \infty} \left[\frac{\varphi^4 - \varphi^2 + 1}{\varphi^2} \right]^k \underbrace{\sqrt{\frac{\varphi^4 - \varphi^2 + 1}{\varphi^2} - \sqrt{\frac{\varphi^4 - \varphi^2 + 1}{\varphi^2} + \sqrt{\frac{\varphi^4 - \varphi^2 + 1}{\varphi^2} + \dots + \sqrt{\frac{\varphi^4 - \varphi^2 + 1}{\varphi^2}}}}}_{k \text{ square roots}}$$

It becomes clear that the irrationality of Pi (π) is only based on constant φ and 1, in the same way as the irrationality of all irrational square roots, is only based on constant φ & 1 !

Natural Numbers, their square roots and irrational and transcendental constants like Pi (π) can be expressed (calculated) by only using constant φ and 1 ! This is also valid for all rationals (fractions) and their square roots.

Numbers and number-systems don't seem to exist ! They are manmade and therefore can be eliminated.

This is an interesting discovery because it allows to define most (maybe all) geometrical objects only with φ & 1 !

The result of this discovery may lead to a new base of number theory. Not numbers like 1, 2, 3,..... and constants like Pi (π) etc. are the base of Number Theory ! Only the constant φ and the base unit 1 (which shouldn't be considered as a number) form the base of mathematics and geometry. This will certainly also have an impact on Physics !

Constant φ and the base unit 1 must be considered as the fundamental „space structure constants“ of the real physical world !

In the physical world the geometries of all possible crystal-lattice-structures are fundamentally based on Phi and 1.

There probably isn't something like a base unit if we consider a „wave model“ as the base of physics and if we see the universe as one oscillating unit. In the universe everything is connected with everything. see : [Quantum Entanglement](#)

→ Please also read my 12 Conjectures on the next page (Chapter 6)

Chapter 6 :

Referring to my discovery regarding constant φ (Phi), I want to define the following 12 Conjectures :

Here the 12 conjectures : (\rightarrow you can call them **Harry K. Hahn's conjectures**)

1.) All Natural Numbers and their square roots can be expressed (calculated) by only using the mathematical constant Phi (golden mean = 1.618..) and number 1. This statement is also valid for all rationals (fractions) and their square roots

2.) All existing irrational numbers seem to be constructions out of Phi and 1.

For example the irrational transcendental constant Pi (3.1415926...) can also be expressed by only using Phi and 1 !

3.) Phi and 1 are the base units of Mathematics ! Numbers and number-systems don't exist ! They are manmade and therefore can be eliminated. In principle Mathematical Science can be carried out by only using Phi and 1, as base units.

4.) All geometrical objects, including the Platonic Solids can be described by only using constant Phi and 1.

Because all natural numbers, their square roots, rationals (fractions) and probably all irrational and all transcendental numbers too, can be expressed by only using Phi and 1.

5.) Point 4.) leads me to the conclusion that in the physical world the geometries of all possible crystal-lattice-structures are fundamentally based on Phi and 1. The more fundamental the lattice the simpler it can be expressed by Phi and 1.

6.) Point 4.) 5.) & 7.) leads me to the conclusion that on the molecular level (and probably on the atomic level too), as well as on the macroscopic (cosmic) level the distribution and structure of matter (=energy) in space, is fundamentally based on constant Phi and 1. \rightarrow **Phi represents a fundamental physical „Space Structure Constant“**

Together with Point 7.) this indicates that the curvature of spacetime at the molecular level (crystals) and at the atomic level, as well as on the macroscopic level is defined only by the "Space Structure Constant Phi" and the base unit 1.

\rightarrow This idea will help to unify General Relativity with Quantum Mechanics ! If the gravitational singularity in M87 indeed has a dodecahedral structure then gravitation, which is the geometric property of spacetime, can be described in Quantum Mechanics and at the cosmic level by the same constant duo : Phi and base unit 1 !

7.) The structure of the M87 black hole (\rightarrow **EHT2017**) indicates a dodecahedral structure. The distribution of matter in gravitational singularities therefore seems to be defined essentially by constant Phi and base unit 1 ! The largescale distribution of matter in the universe seems to be predominantly based on an order-5 Poincare-Dodecahedral-Space. \rightarrow [weblink to my study](#) (or alternatively here : <http://vixra.org/abs/1907.0348>)

Title : "EHT2017 may provide evidence for a Poincare Dodecahedral Space Universe"

8.) The natural numbers can be assigned to a defined infinite set of Fibonacci-Number Sequences.

9.) This infinite set of Fibonacci-Number Sequences, and the numbers contained in these sequences, are connected to each other by a complex precisely defined spatial network based on constant Phi. (\rightarrow **see table in Appendix A**). For the progressing Fibonacci-Sequences towards infinity, the connections between the numbers approach constant Phi.

\rightarrow **see explanation in Chapter 2 and 3 and in Appendix A**

10.) Constant Phi (golden mean = 1.618..) must be a fundamental constant of the final equation(s) of the universal mathematical and physical theory. (\rightarrow It may be the only irrational constant that appears in the(se) equation(s))

11.) The number-5-oscillation (\rightarrow the numbers divisible by 5) in the two number sequences $6n+5$ (Sequence 1) and $6n+1$ (Sequence 2), with $n=(0,1,2,3,...)$, defines the distribution of the prime numbers and non-prime-numbers. The number-5-oscillation defines the starting point and the wave length of defined non-prime-number-oscillations in these Sequences 1+2 (SQ1 & SQ2). (Note : the combination of the two sequences SQ1 & SQ2 is considered here)

\rightarrow weblink to my study : <https://arxiv.org/abs/0801.4049> (or alternatively here : <http://vixra.org/abs/1907.0355>)

For a quick overview please see **pages 15 to 18** in this study : [weblink to the study](#) : **"EHT2017 may provide evidence..."**

12.) The importance of the number-5-oscillation for the distribution of primes and non-primes is a further indication for the conjecture that the largescale structure of the universe seems to be predominantly (mainly) based on an order-5 Poincare-Dodecahedral-Space structure. \rightarrow The space structure of the universe seems to be based essentially on the **5.Platonic Solid : the Dodecahedron** (\rightarrow consisting of 12 regular pentagonal faces, three faces meeting at each vertex)

The time will show if my Conjectures are correct !

References :

Symmetry in Plants - by Roger V. Jean & Denis Barabe (1998) – University Quebec, CA - ISBN No. : 981-02-2621-7
Weblink (Google Books) : https://books.google.de/books/about/Symmetry_In_Plants.html?id=2fbsCgAAQBAJ&redir_esc=y

Changes in phyllotactic pattern structure in Pinus mugo due to changes in altitude

Study to Fibonacci pattern variation in Pinus Mugo by Dr. Iliya Iv. Vakarelov, University of Forestry, Bulgaria (1982-1994)
From the book „Symmetry in Plants“ by Roger V. Jean and Denis Barabe, Universities of Quebec and Montreal, Canada
(Part I. Chapter 9 , pages 213-229), ISBN : 981-02-2621-7 , Weblinks: [Weblink_1](#) ; [Weblink_2](#) (Google Books)

Other studies which indicate phyllotactic pattern variability (with a noticeable distribution pattern) within the same species → in all probability depending mainly on environmental factors :

Aberrant phyllotactic patterns in cones of some conifers : a quantitative study - by Veronika Fierz

Weblink: [Aberrant phyllotactic patterns in cones of some conifers](#) (researchgate.net)

Novel Fibonacci and non-Fibonacci structure in the Sunflower - by J. Swinton, E. Ochu & Others

https://www.researchgate.net/publication/303354855_Novel_Fibonacci_and_non-Fibonacci_structure_in_the_sunflower; [Weblink2](#)

A study which indicates that far-red & infrared radiation with wave-lengths > 750 nm is the trigger for phyllotactic-pattern formation & bud-induction :

Red Light Affects Flowering under long days in a Short-day Strawberry Cultivar

by Fumiomi Takeda & D. Michael Glenn - USDA-ARS, Appalachian Fruit Research Station (USA), Kearneysville, WV 2543 0
– publication: HortScience 43(7):2245-2247.2008 - Weblinks to study: [Weblink 1](#), [Weblink 2](#)

To the importance of constant Phi (φ) for the physical world , and studies regarding the Square Root Spiral :

Phase Spaces in Special Relativity : Towards eliminating Gravitational Singularities

by Peter Danenhower , Weblink: <https://arxiv.org/pdf/0706.2043.pdf>

Microscope Images indicate that Water Clusters are the cause of Phyllotaxis - by Harry K. Hahn

<https://archive.org/details/microscope-images-indicate-that-water-clusters-are-the-cause-of-phyllotaxis>

alternative weblink: <https://vixra.org/abs/2005.0118>

The Black Hole in M87 (EHT2017) may provide evidence for a Poincare Dodecahedral Space Universe - by Harry K. Hahn

<https://archive.org/details/TheBlackHoleInM87EHT2017MayProvideEvidenceForAPoincareDodecahedralSpaceUniverse/page/n1>

alternative Weblink: <http://vixra.org/abs/1907.0348>

The golden ratio Phi (φ) in Platonic Solids: <http://www.sacred-geometry.es/?q=en/content/phi-sacred-solids>

The Ordered Distribution of Natural Numbers on the Square Root Spiral - by Harry K. Hahn

<http://front.math.ucdavis.edu/0712.2184> PDF : <http://arxiv.org/pdf/0712.2184>

The Distribution of Prime Numbers on the Square Root Spiral - by Harry K. Hahn

<http://front.math.ucdavis.edu/0801.1441> PDF : <http://arxiv.org/pdf/0801.1441>

Appendix B.) : Fibonacci – Sequence – Tables F1 ; F2 ; F6 ; F8

Note: The numbers of the Fibonacci F1 – Number Sequence seem to contain all prime numbers as prime factors ! and all prime factors appear periodic in defined “number-distances“ in the sequence (see left side of table)

Table 2 : Periodicity of some of the prime factors of the numbers of the **Fibonacci F1 - Number Sequence :**

some prime factors shown in table form													in prime factors factorized Fibonacci-Numbers		sum of digits	Fibonacci-Sequence F1			
41	37	31	29	23	19	17	13	11	7	5	3	2	repeating products	new products		F	F'	F''	Nr.
															1			1	
															1			2	
															2	1		3	
															3	1		4	
															5	2	1	5	
													2 ³		8	3		6	
															4	5	2	7	
									7		3				3	7		8	
						17						2			7	17		9	
								11		5					10	11		10	
													3 ² 2 ⁴	2x2x2	17	34	21	11	
															9	144	55	12	
			29				13								8	233	89	13	
										5	2				17	377	144	14	
									7		3				7	610	233	15	
															24	987	377	16	
															22	1597	610	17	
						19	17					2 ³			19	2584	987	18	
															14	4181	1597	19	
	37														24	6765	2584	20	
41								11	5	3					20	10946	4181	21	
															17	17711	6765	22	
															28	28657	10946	23	
															27	46368	17711	24	
			23						7	3 ²	2 ⁵		2x2x2x2x3x3x3	2x7x23	19	75025	28657	25	
										5 ²					19	121393	46368	26	
															29	196418	75025	27	
						17					2				21	317811	121393	28	
			29				13				3				23	514229	196418	29	
															17	832040	317811	30	
		31						11	5		2 ³		2x5x61x	2x2x11x31	31	1346269	514229	31	
															30	2178309	832040	32	
									7		3		3x7x47x	2207	34	3524578	1346269	33	
															37	5702887	2178309	34	
															35	9227465	3524578	35	
															27	14930352	5702887	36	
															35	24157817	9227465	37	
															44	39088169	14930352	38	
															43	63245986	24157817	39	
															24	102334155	39088169	40	
															31	165580141	63245986	41	
			29				13				2 ³		2x13x421x	2x2x29x211	46	267914296	102334155	42	
															41	433494437	165580141	43	
															33	701408733	267914296	44	
															29	1134903170	433494437	45	
															35	1836311903	701408733	46	
															37	2971215073	1134903170	47	
				23					7	3 ²	2 ⁶		2x2x2x2x2x3x3x7x23x	2x47x1103	54	4807526976	1836311903	48	

Note : all prime numbers are marked in yellow and all numbers not divisible by 2, 3 or 5 are marked in orange

Table 3 : Periodicity of some of the prime factors of the numbers of the **Fibonacci F2 (Lucas) - Number Sequence :**

some prime factors shown in table form												in prime factors factorized Fibonacci-Numbers		sum of digits	Fibonacci-Sequence F2 (Lucas-Sequence)				
41	37	31	29	23	19	17	13	11	7	5	3	2	repeating products		new products	L	L'	L''	No.
															1			1	
															3			2	
												2^2		2x2	4	1		3	
															7	3		4	
															4	11	4	1	5
															9	18	7	3	6
															11	29	11	4	7
															11	47	18	7	8
															13	76	29	11	9
															6	123	47	18	10
															19	199	76	29	11
															7	322	123	47	12
															8	521	199	76	13
															15	843	322	123	14
															14	1364	521	199	15
															11	2207	843	322	16
															16	3571	1364	521	17
															27	5778	2207	843	18
															25	9349	3571	1364	19
															16	15127	5778	2207	20
															23	24476	9349	3571	21
															21	39603	15127	5778	22
															26	64079	24476	9349	23
															20	103682	39603	15127	24
															28	167761	64079	24476	25
															21	271443	103682	39603	26
															22	439204	167761	64079	27
															25	710647	271443	103682	28
															29	1149851	439204	167761	29
															36	1860498	710647	271443	30
															20	3010349	1149851	439204	31
															38	4870847	1860498	710647	32
															40	7881196	3010349	1149851	33
															24	12752043	4870847	1860498	34
															28	20633239	7881196	3010349	35
															34	33385282	12752043	4870847	36
															26	54018521	20633239	7881196	37
															33	87403803	33385282	12752043	38
															23	141422324	54018521	20633239	39
															38	228826127	87403803	33385282	40
															34	370248451	141422324	54018521	41
															54	599074578	228826127	87403803	42
															43	969323029	370248451	141422324	43
															52	1568397607	599074578	228826127	44
															41	2537720636	969323029	370248451	45
															30	4106118243	1568397607	599074578	46
															62	6643838879	2537720636	969323029	47
															47	10749957122	4106118243	1568397607	48
															46	17393796001	6643838879	2537720636	49
															39	28143753123	10749957122	4106118243	50

Note : all prime numbers are marked in yellow and all numbers not divisible by 2, 3 or 5 are marked in orange

Table 4 : Periodicity of some of the prime factors of the numbers of the **Fibonacci F6 - Number Sequence :**

Periodicity of the prime factors 2 - 41 shown in table form												in prime factors factorized Fibonacci-(F6)-Numbers	Fibonacci-F6 Sequence				
41	37	31	29	23	19	17	13	11	7	5	3	2		F6	F6'	F6''	Nr.
														1			1
														4			2
														5	1		3
														9	4		4
									7					14	5	1	5
														23	9	4	6
														37	14	5	7
														60	23	9	8
										5	3			97	37	14	9
														157	60	23	10
														254	97	37	11
														411	157	60	12
									7	5				665	254	97	13
														1076	411	157	14
														1741	665	254	15
														2817	1076	411	16
														4558	1741	665	17
														7375	2817	1076	18
														11933	4558	1741	19
														19308	7375	2817	20
														31241	11933	4558	21
														50549	19308	7375	22
														81790	31241	11933	23
														132339	50549	19308	24
														214129	81790	31241	25
														346468	132339	50549	26
														560597	214129	81790	27
														907065	346468	132339	28
														1467662	560597	214129	29
														2374727	907065	346468	30
														3842389	1467662	560597	31
														6217116	2374727	907065	32
														10059505	3842389	1467662	33
														16276621	6217116	2374727	34
														26336126	10059505	3842389	35
														42612747	16276621	6217116	36
														68948873	26336126	10059505	37
														111561620	42612747	16276621	38
														180510493	68948873	26336126	39
														292072113	111561620	42612747	40
														472582606	180510493	68948873	41
														764654719	292072113	111561620	42
														1237237325	472582606	180510493	43
														2001892044	764654719	292072113	44
														3239129369	1237237325	472582606	45
														5241021413	2001892044	764654719	46
														8480150782	3239129369	1237237325	47
														13721172195	5241021413	2001892044	48
														22201322977	8480150782	3239129369	49
														35922495172	13721172195	5241021413	50
														58123818149	22201322977	8480150782	51
														94046313321	35922495172	13721172195	52
														152170131470	58123818149	22201322977	53
														246216444791	94046313321	35922495172	54
														398386576261	152170131470	58123818149	55

Note : all prime numbers are marked in yellow and all numbers not divisible by 2, 3 or 5 are marked in orange

Table 5 : Periodicity of some of the prime factors of the numbers of the **Fibonacci F8 - Number Sequence :**

Periodicity of the prime factors 2 - 41 shown in table form												in prime factors factorized Fibonacci-(F8)-Numbers	Fibonacci-F8 Sequence						
41	37	31	29	23	19	17	13	11	7	5	3	2	sum of digits	F8	F8'	F8''	Nr.		
???			???			prime factor is missing													1
						prime factor is missing													2
						prime factor is missing													3
																			4
																			5
																			6
																			7
																			8
																			9
																			10
																			11
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																			44
																		45	

Note : all prime numbers are marked in yellow and all numbers not divisible by 2, 3 or 5 are marked in orange