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In search of the fourth dimension of space

The Galaxy Epoch

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ABSTRACT

This speculation is the logical continuation of the previous one [viXra:2006.0202]. It describes a cosmological model with a 4-sphere [*], in expansion, on the surface of which our Universe extends, but as will see, with an internal part that has its own role. The involvement of the fourth spatial dimension is unavoidable but it does not imply reintroducing the concept of an absolute space and not even that of absolute time. On the contrary, the model excludes both.

The geometry described finds an application in the calculations of Galactic Recession: those calculations are confirmed here after adding gravity.

In this subsequent formulation an explanation is sought as to why, in this "empty" space, a ray of light is bound to move on the 4-sphere surface. Then, we proceed by building a physical model, in which that empty space is filled with matter and radiation, and we try to check for any flaws. We will also show that this geometry, with its expansion mechanism, infers General Relativity.

Given the constant expansion speed hypothesized for the Universe, no redshift is due to expansion itself. Here the Cosmological redshift is Gravitational or Doppler.

In this conjecture the surface of the 4-sphere (like a kind of bubble expanding over time) goes through a continuum of states of equilibrium in which an internal pressure exerted by a radiation, in a reversible adiabatic expansion resulting from the Big Bang, balances the cohesion of the Universe whose surface tension is due solely to the pressure of the Cosmic Background Radiation.

Resulting model is an approximation for the Galaxy Epoch and it is based on the Einstein's solution for weak fields to the field equation of General Relativity. You can use this only in the context of observable Universe, during the lasts 10 billion years.

These following points can have interesting consequences:

1. this geometry infers General Relativity which, in this way, is not considered a consequence of postulates
2. there is an energy exchange between the surface and its interior so that the two sides could communicate

The implication is that what is on the surface is constrained by the speed of light c but, admitting the existence of the tachyon, what can enter the 4-sphere is not, being bound to a different space-time interval.

This last idea could be developed by linking to phenomena that violate the relativistic laws as Quantum Non-Locality.

About the pros and cons on the model:

1. Advantages: Galactic Recession and Relativity separation, Inferred and non-postulated Relativity. Possibility of admitting the tachyon if the underlying idea is developed.
2. Disadvantages: Need the internal part but not dark matter and dark energy. Weakness regarding the Covariance Principle but the effect is limited to large interstellar distances.

Finally, it is important to point out that this speculation leads to a falsifiable theory.

We briefly summarize what was previously said:

a) Our Universe lies on a 4-sphere surface $x_1^2 + x_2^2 + x_3^2 + x_4^2 = c^2t^2$ where radius is $r = ct$ with c as light velocity and t as time elapsed from Big Bang.
b) Radial velocity $v_r = c$ is constant except during the initial period.
c) Also tangent velocity $v_t = ctd\Theta/dt = c$ is constant over time. Galactic redshift is due to Doppler effect. [3]
d) Our relativistic time-like zone is a portion of space delimited, in every direction, by an arc of length $ct\Theta \; with \; \Theta = 1 \; rad$.

[*] – By 4-sphere we mean the hypersphere embedded in four-dimensional space R4 (someone call it 4-ball too); its surface is named by topologists a S^3 sphere.

STILL ASSUMPTIONS

A ray of light can travel an entire great circle and return to the starting point (much forward in time). However, we cannot detect in any way a radiation from a galaxy outside the relativistic cone of light (a bit like what happens to an observer who cannot capture any photon from a black hole). Whatever the frame of reference, only radiation emitted by objects belonging to one's own time-like zone can be detected. These photons continue to go round in circles along a geodesic. Outside the limits of the observable Universe (at angular distances Θ > 1 in every direction) the light ray, during the entire route, cannot meet anything because everything flies ahead at faster speeds. In vacuum it cannot be deflected or absorbed in any way.

Up to now, no hypothesis has been made on the "empty" space delimited by this geometry. To proceed, the fourth dimension of space is involved. We place the Big Bang at the center of the 4-sphere and assume that all the primordial *ylem* (hot plasma), launched away by a giant explosion, was blocked onto a sort of event horizon. There remained, squeezed on the surface, nothing could get out and subsequently also the scattering photons. Over time, reactions took place and cooling changed the conditions. The event horizon somehow shrank, radiation was released and expansion begun.

THE SPECIAL RELATIVITY APPROXIMATION

At "time of last scattering", after the of Recombination era [4], relic photons were released and traveled along 4-sphere's surface arcs as geodesics. This radiation has not disappeared, it is still present today as Cosmic Background Radiation (CBR) [1] providing the "vacuum" with sufficient energy and pressure that, in a homogeneous space, still provide the gravity to maintain these geodesics. The last statement derives from the equivalence between mass and energy. As we saw in the previous article, the flat space of Special Relativity enters the context of this curved surface.

From the assumptions made previously, at "time of last scattering" expansion velocity was null. In absence of relative motion, rays, started from any point on the surface, can reach any other point. Since then, expansion resumes, maintaining a constant speed.

The subsequent constancy of radial velocity $v_r = c$, hypothesized in the previous article, implies that also tangent velocity $v_t = ctd\Theta/dt = c$ does not change over time. This is valid for the whole period in which gravity has maintained these geodesics, that is, for the whole period concerned.

Let us write the geodesic equation with reference to 4-sphere geometry:

$$ct\frac{d\Theta}{dt} = c \quad or \quad \frac{d\Theta}{dt} = \frac{1}{t} \quad and \quad \frac{t_2}{t_1} = e^{\Theta} \quad for\ every\ \Theta$$

Knowing the angle Θ we can easily get the time the ray started: $t_{past} = t_{today}e^{-\Theta}$.

From the interval of flat space-time $ds^2 = -dx_1^2 - dx_2^2 - dx_3^2 + c^2dt^2$ we put $dx_1^2 + dx_2^2 + dx_3^2 = (ctd\Theta)^2$

With $ds^2 = 0$ for a light-like interval, we obtain $ctd\Theta = cdt$ that is the geodesic equation $v_t = ctd\Theta/dt = c \quad and \quad d\Theta = dt/t$.

Today the arc approximated here by a segment has a curvature of $2.40 * 10^{-4}Mpc^{-1} = 7.77 * 10^{-27}m^{-1}$: Special Relativity is a very close approximation for this curved surface.

In the context of the Principle of Equivalence, get easily the *proper coordinates* for ourselves as observer, marks a positive point for the 4-sphere hypothesis.

ON THE EDGE OF PHYSICS AND BEYOND: SPHERES, BUBBLES, WORK AND ENERGY

I find that the 4-sphere surface is an interesting entity that we can find also in the interior solution of the Schwarzschild metric, as its space-time geometry. Then, you could think that, in extreme physical conditions, fluids can settle in this geometry and that, when conditions cease, this geometry may be preserved in a following expansion.

What we add to doing here is to extend physical laws to a four-dimensional space with the aim of verifying later the results in our local frame of reference. We had already done this previously, hypothesizing a fourth dimension and then obtaining, as result, the Special Relativity of our space, not as a postulate nor as a limit of an ad hoc chosen metric.

Current cosmology accepts an *origin* for time and, referring to the Big Bang, it speaks of a "*singularity*". We have not changed philosophy too much, here, if we replace the concept of "*singularity*" with a point in another dimension that cannot ever be reached and measured by us. About, instead, the relativity of reference frames between dimensions, there are no violations until we do not try to compare values.

The sphere and the bubble have a symmetry that lends themself to be easily generalized. We can think of the 4-sphere surface as a bubble where the cohesion force is due not to a surface tension [2] but to gravity. Because of its high discontinuity in space, mass from matter should be irrelevant for great values of r. Pressure from radiation, instead, may be essential.

In this generalization, we hypothesize a 4-sphere that expands over time because of an explosion at its center:

1. About the kinetic energy, with a constant expansion speed, $\Delta E_k = 0$.
2. Referring to the 4-sphere surface a work E_γ is done by gravity acting like a surface tension: the cohesion force of the surface is $\gamma = f(r)$.
3. We cannot be sure that transformations are adiabatic: heat could flow out from surface through some mechanism like thermal radiation or something else.
4. About the pressure gradient on the bubble Δp_{4-dim} we assumed a null external pressure so that no additional work is done by volume expansion. By analogy with the surface tension we put $\gamma dS_{4-sphere}$ for the work done by the cohesion forces. The equilibrium relation, then, could take the form: $p_{4-dim}(t) dV_{4-sphere} = \gamma(t) dS_{4-sphere}$.
5. Equilibrium is maintained in expansion. If $p_{4-dim}(t) = f(\gamma)$ then the equality must hold for every value of $r = ct$. The continuous succession of states of equilibrium over time suggests a reversible expansion.

Check the conservation of energy of the Universe is not obvious but here we are only looking for a criterion to establish which physical entities are involved in our balances and if it is possible to exclude the concept of absolute space. For this purpose, we say that energy is conserved if it holds for the sum of proper energies of all region of space.

Referring to the Galaxy epoch, after an initial period, some internal pressure p_{4-dim} left after the explosion. With reference to our Universe and considering the cohesion energy E_γ as part of its Internal Energy U_{Univ} we have:

$\Delta U_{Univ} = q - w$. Both w and q are negative, w is work done by pressure p_{4-dim}, q is the heat given up:

$$dU_{Univ} = dE_m + dE_r + \gamma dS_{4-sphere} = q - w$$

where E_m is energy from matter, E_r from radiation.

We can write:

$$dE_m + dE_r + \gamma dS_{4-sphere} = q - w \quad \text{but} \quad \gamma dS_{4-sphere} = -w \quad \text{so} \quad dE_m + dE_r = q.$$

If ρ is the density of radiation, $V = S_{4-sphere}$ and $E_r = \rho V h\nu$ (where $h\nu$ is the energy of a photon) then:

$$dE_m + dE_r = c^2 dm + (V + dV)(\rho + d\rho)(h\nu + hd\nu) - \rho V h\nu = c^2 dm + \rho V h\delta\nu + d(\rho V)h\nu$$

but $-c^2 dm = d(\rho V)h\nu$ (from the mass-energy equivalence) and the result is $\rho V h\nu = q$.

In our assumption cosmological redshift is of a gravitational type. If we exclude other causes for the redshift, from this, no proper energy is lost and no energy is anyway exchanged. We would have to ask ourselves how we should reason if the expansion of the Universe did not happen at constant speed but this is not the case.

Then:

$$dU_{Univ} = \gamma dS_{4-sphere} = -w$$

Assuming that energy is not conserved could reintroduce the concept of absolute space. However, if we accept a work w from an adiabatic expansion for the explosion, then for the energy balance it would be: $U_{4-sphere} = const$ favoring the idea, stated above, that fluids in extreme physical conditions were disposed on the surface of a 4-sphere and that particular geometry was subsequently preserved for our Universe.

Isotropy, homogeneity, circular path for radiation and lack of energy conservation are the essential conditions for this speculation.

To avoid collapsing, the cohesive force of the 4-sphere surface needs to be balanced by another force. The lack of energy conservation, is only due to the work w, and we have two possible conjectures to proceed:

a) $U_{4-sphere} = const$. A residual radiation propagates from the center of the 4-sphere, origin of the Big Bang, in a radial direction, exerting some form of pressure on the inside of the surface. The 4d state equation of its adiabatic expansion is unknown. It seems reasonable to put $U_{4-sphere} = const$.

b) $U_{Univ} = const$. Some non-directly measurable form of energy w, belonging to our Universe, opposes radiation pressure. In this way U_{Univ} is conserved and we can consider the Universe as a single physical entity.

In any case the 4-sphere surface model can survive as a curvature for space-time.

Here are some hypothetical calculations regarding choice a).

Assuming zero for variable t at the beginning of the expansion (after the last scattering), it follows ($scat$ stays for "relative to last scattering"):

1. from $p(t)dV = \gamma(t)dS$ it follows $p(t) = 3\gamma(t)/ct$
2. but $\gamma(t) = \rho/3$ where the latter is the is the expression for the pressure of a disordered radiation of density ρ
3. we put $\rho = (\rho_{scat} S_{scat}/S)/ct$ for the CBR density, decreasing with S and redshift z as $(ct)^{-4}$
4. the result is $p(t) = \rho/ct \ = p_{scat}/(ct)^5 = aV^{-5/4}$ where a is constant.

The state equation of a 3d reversible adiabatic expansion for radiation is $PV^{4/3} = const$. Here for the above 4d expansion we obtained $PV^{5/4} = const$. Accepting this result as a 4d reversible adiabatic expansion would also confirm radiation pressure as the only cohesive force γ.

We opt for choice a).

The purpose of these calculations is only to describe qualitatively, but using a language that we know, the functioning of this model. Referring to the Galaxy epoch, the 4-sphere hypothesis incudes that:

a) The surface of the 4-sphere (like a kind of bubble expanding over time) goes through a continuum of states of equilibrium in which an internal pressure by a radiation, in a reversible adiabatic expansion resulting from the Big Bang, balances the cohesion of the Universe whose surface tension is due solely to the pressure of the Cosmic Background Radiation.
b) There is no exchange of energy due to the cosmological redshift of radiation: it is of gravitational origin. Then, for the energy balance of the whole 4-sphere, it would be: $U_{4-sphere} = const$ without heat exchange between the surface and the inside.
c) It is not reintroduced the concept of absolute space.

From the macroscopic point of view, the conservation of the energy for the whole 4-sphere is an interesting conjecture but, as we will see, further developments will not be taken for granted.

Finally, let us note what these assumptions entail for the 4-sphere surface as seen from a point of belonging:

1) It is not possible to identify a privileged reference frame or to recognize its state of motion whatever it is.
2) Light propagates through empty space with a definite speed c independent of the relative speed of source and observer.

If we add to this the considerations on the Principle of Equivalence, expressed above in the paragraph on Special Relativity, we can affirm that the requirements of Relativity can be said to be satisfied and that this model infers it without the need for postulates.

ABOUT ASSUMING A METRICAL TENSOR

By relating time to the 4th spatial dimension we obtain the usual curved space-time. After this, we no longer need the equation of the surface: $x_1^2 + x_2^2 + x_3^2 + x_4^2 = c^2t^2$. As we will see later, fourth dimension of space x_4 will appear again in a mathematical context but no longer in physics.

The generic procedure to get the metric of 4-sphere curved space-time sems extremely complex in a Cartesian reference frame.

The solution is not even simplified using polar coordinates:

1. Let's choose a reference frame based on a radius $r = ct$ as time coordinate and on three angles θ, φ, ψ as space coordinates $(0, 2\pi)$. As reference points, unfortunately, we cannot choose known stars as "Alpha Ursae Minoris – Polaris" or "Delta Orionis – Mintaka" on the Orion's Belt. This because of their proximity to us.
2. The three coordinates on the surface are given by the angles θ, φ, ψ where the first two are the equivalent of Longitude and Colatitude (using zenith angle $= 90^0 -$ Latitude) and where we will call the third "Universe Height". Astronomic Celestial coordinate Declination and Right ascension are relative to our observable Universe, here Universe Colatitude and Longitude refers to the whole 4-sphere. As convention we indicate a point P as $P(\varphi, \theta, \psi)$, with Colatitude before Longitudes.
3. Let's establish a position $P_N(0,0,0)$ for the "North pole" of our 4-sphere. Since all the points on the surface are equivalent, we can choose "Ursa Major GN-108036". Then we chose a Prime Meridian $P_{M0}(undef, 0, undef)$, passing through some other known point in space (say passing through "Sculptor A2744 YD4"). Note that all points $P_{EM-}(\pi/2, 0, \ undef)$ on the Universe Equator are out of our observable Universe. A third point $P_{EM}(\pi/2, 0, \pi/2)$ is at Universe Height $\pi/2$ on the Universe Equator, at $\pi/2$ from P_N measured on Prime Meridian.

The corresponding Cartesian coordinate can be useful:

1. $x_1 = ct\sin(\psi)\sin(\varphi)\cos(\theta)$
2. $x_2 = ct\sin(\psi)\sin(\varphi)\sin(\theta)$
3. $x_3 = ct\sin(\psi)\cos(\varphi)$
4. $x_4 = ct\cos(\psi)$

Note that θ, φ are the Longitude and Colatitude of the sphere.

Also are useful the 4-vector $\boldsymbol{r} =$
$(ct\sin(\psi)\sin(\varphi)\cos(\theta), \quad ct\sin(\psi)\sin(\varphi)\sin(\theta), \quad ct\sin(\psi)\cos(\varphi), \quad ct\cos(\psi))$

and its derivatives ($t =$ const on the surface):

1. $\mathbf{r}_\theta = (-\mathrm{ct}\sin(\psi)\sin(\varphi)\sin(\theta),\ \mathrm{ct}\sin(\psi)\sin(\varphi)\cos(\theta),\ 0,\ 0)$
2. $\mathbf{r}_\varphi = (\mathrm{ct}\sin(\psi)\cos(\varphi)\cos(\theta),\ \mathrm{ct}\sin(\psi)\cos(\varphi)\sin(\theta),\ -\mathrm{ct}\sin(\psi)\sin(\varphi),\ 0)$
3. $\mathbf{r}_\psi = (\mathrm{ct}\cos(\psi)\sin(\varphi)\cos(\theta),\ \mathrm{ct}\cos(\psi)\sin(\varphi)\sin(\theta),\ \mathrm{ct}\cos(\psi)\cos(\varphi),\ -\mathrm{ct}\sin(\psi)))$

These are 4-vectors of a Euclidean space: for us, there is the inner product and the angles it defines.

The three inner products are all equal to zero: $\mathbf{r}_\theta \cdot \mathbf{r}_\varphi = \mathbf{r}_\varphi \cdot \mathbf{r}_\psi = \mathbf{r}_\theta \cdot \mathbf{r}_\psi = 0$: they are orthogonal.

Once the angle ξ between two points, P_1 *with vector* $\mathbf{r}_1$ and P_2 *with vector* $\mathbf{r}_2$, has been calculated:

$$\xi = \left|\arccos\left(\frac{1}{c^2t^2}\mathbf{r}_1 \cdot \mathbf{r}_2\right)\right|$$

you can refer to the arc of great circle $r\xi$ to simplify the reasoning on light geodesics.

Saw the variables to use, it seems hard to set up the latter relation. Space and time variables are tightly coupled: it is not at all obvious to formulate a covariant expression for this space-time interval: $ds^2 = g_{\mu\nu}dx^\mu dx^\nu$.

In a coordinate system with origin in the center of the 4-sphere and with respect to which the observer is stationary [*], we have seen that the maximum achievable speed for an object bound to the 4-sphere surface is $v_t = ctd\xi/dt = c$. The overall speed of a ray of light is not constrained by the constant *c:* it is its tangential component.

Let us consider now a solution in the form: $ds^2 = -h_{r\xi}d(r\xi)^2 + h_t c^2 dt^2$ and the differential of the product $r\xi$: $d(r\xi) = ctd\xi + c\xi dt$. To obtain the desired geodesic we must put $c\xi dt = 0$ as if the radius *r* were a constant.

In our hypothesis the only possible spatial displacement in the radial direction occurs at constant velocity: the term $c\xi$ gives the Galactic Recession. As we will see, by considering the Universe expansion as a succession of equilibrium states, the velocity $v_r = dr/dt$ does not anyway appear in the Stress-Energy tensor nor directly in our application of the Einstein's equation. Holding out Galactic Recession from calculation for the metric $g_{\mu\nu}$, the tightly coupling of variables disappears so that we can look for a solution in the form $ds^2 = -h_\xi c^2t^2d\xi^2 + h_t c^2dt^2$. Here the expansion of the Universe manifests itself through the increasing term c^2t^2. Dilation of the distance, due to expansion, can only be felt at the interstellar level.

In this speculation we have not yet talked about the Covariance principle. Although it is possible to express the same metric in other coordinates, our quantity $g_{\mu\nu}$ does not transform as a tensor: we have just defined a pseudo-tensor. This lack of generalization is the weakness of the logic plant but does not invalidate it. It is difficult to think of another representation of coordinates in which the same metric can be equally easily expressed but the use of this model is reserved for the geodesic of light.

Thus, we have variables whose differentials only partially enter the metric pseudo-tensor *because in our conjecture the radial dimension cannot be perceived in any way*. Quantities to be used are therefore cdt and $ctd\xi$ where the first describes a variation of time, the second a variation along the expanding 4-sphere arc.

Notwithstanding the equation of geodesic $ctd\xi/dt = \mathrm{c}$, the speculation deals with the use of the Doppler-type redshift for the calculation of the galactic recession. The purpose of the following analysis is to verify how much the presence of a gravitational redshift can modify our result. The idea is then to consider a sufficiently small zone of the universe where the Cartesian variable x can be merged with our arc ξ, so that $\Delta x \simeq ct\Delta\xi$, and to evaluate there the trend of the gravitational field over the last 10 billion years. Under these conditions our pseudo-tensor becomes a tensor.

This is what will be done in the next paragraph.

[*] – You can find a discussion about coordinate transformation between inertial frames and uniformly rotating ones with also paradoxes in:

[Springer:10.1140/epjc/s10052-014-3098-6 - On Franklin's relativistic rotational transformation and its modification](Springer:10.1140/epjc/s10052-014-3098-6)

AN APPROXIMATE SOLUTION FOR THE GALAXY EPOCH
FROM EINSTEIN'S WEAK FIELDS

The very small curvature of space in our present period is the confirmation of a current weak gravitational field. We can resume the analysis with the previously described coordinates $\mathrm{dx}^{\mu} = \mathrm{ctd}\varphi,\ \mathrm{ctd}\theta,\ \mathrm{ctd}\psi,\ \mathrm{cdt}$: We look for a model that approximates an almost flat space-time in a neighborhood of any point on the surface. From this part of the whole we expect to derive the field equation for the present and to apply it back in time so that we can observe rays of light from the most distant galaxies.

We have already seen before that, for each point $P(\varphi,\theta,\psi)$, the tangents to Colatitude, Longitude and Height are orthogonal: the angles between the coordinates φ,θ,ψ are always $\pi/2$. Then the differential arc is:

$$\mathrm{c}^2\mathrm{t}^2\mathrm{d}\xi^2 = \mathrm{c}^2\mathrm{t}^2\sin^2(\psi)\mathrm{d}\varphi^2 + \mathrm{c}^2\mathrm{t}^2\sin^2(\psi)\sin^2(\varphi)\mathrm{d}\theta^2 + \mathrm{c}^2\mathrm{t}^2\mathrm{d}\psi^2$$

If the vectors $\mathbf{e}_{\varphi},\mathbf{e}_{\theta},\mathbf{e}_{\psi}$ can be assumed as an orthogonal covariant basis of this space we note that, with the 4-sphere radius $\boldsymbol{r} = \mathrm{ct}\ \mathbf{e}_{\mathrm{t}}$, the basis $\mathbf{e}_{\mathrm{t}}$ for our time coordinate is orthogonal to the previous ones too (so it had to be on the basis of the Principle of Equivalence).

For the basis $\mathbf{e}_{\varphi},\mathbf{e}_{\theta},\mathbf{e}_{\psi},\mathbf{e}_{\mathrm{t}}$, a double angle rotation on ψ and φ is function of the current values of ψ_0 and φ_0

$$\mathrm{f}_{\psi} = \sin(\psi) \quad \text{and} \quad \mathrm{f}_{\varphi} = \sin(\varphi)$$

and it is given by:

$$\mathbf{C}(\psi,\varphi) = \begin{bmatrix} f_\psi & 0 & 0 & 0 \\ 0 & f_\psi\, f_\varphi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The compound transformation $g'_{\mu\nu} = \mathbf{C}(\psi,\varphi)\mathbf{C}(\psi,\varphi)\, g_{\mu\nu}\mathbf{C}^{-1}(\psi_0,\varphi_0)\mathbf{C}^{-1}(\psi_0,\varphi_0)$ gives the metric tensor for the rotation.

All points are equivalent, to simplify we choose the point at the Universe Equator in $P_{EM}(\pi/2,0,\pi/2)$, then what remains is $c^2t^2d\xi^2 = c^2t^2(d\varphi^2 + d\theta^2 + d\psi^2)$.

Now let us solve the following field equation (we assume the cosmological constant $\Lambda = 0$):

$$\frac{8\pi G}{c^4}\, T_\mu^\nu = R_\mu^\nu - \frac{1}{2} R g_\mu^\nu$$

to get the tensor $g_{\mu\nu}$ for the interval $ds^2 = g_{\mu\nu}\, dx^\mu dx^\nu$.

The analysis begin with the Einstein's solution for weak fields $g_{\mu\nu} = h_{\mu\nu} + \eta_{\mu\nu}$ where $\eta_{\mu\nu}$ are the constant Galilean values for Special relativity and $h_{\mu\nu}$ are small correction terms. $\mathcal{E}_{0r}$ and $c^2\rho_{0m}$ are respectively the current proper energy density of radiation and matter. The surface cohesive force of this model is attributable to uniform radiation pressure p.

As an expression for volume we put $V = 2\pi^2 c^3 t^3$ for 4-sphere surface and, for the previous assumptions about gravity, $\rho_0 V_0 \simeq \rho V$ is constant over time. [*] We can, then, calculate mass (or energy) density and volume at present time. Moreover ρ_0 can be considered the density of a perfect fluid composed of a mix of matter and radiation.

Let us remember the precedent qualitative description of the 4-sphere model in which we put the relation $\Delta U_{Univ} = -w$. The latter will be used in the next calculation in which, for an infinitesimal element of volume δV, we have a work $-\delta w$ done by internal pressure so as to satisfy the relationship:

$$\frac{\partial(c^2\rho\delta V)}{\partial t} dt = \delta w - p\frac{\delta V}{\partial t} dt \qquad \textit{as measured by a local observer}$$

in which matter, in the form of discontinuities in mass distribution, has no rule. You can eliminate it from the Stress Energy tensor.

From the equilibrium condition the right member is zero, then the Stress Energy tensor for this disordered radiation is:

$$T^{\mu\mu} = 0 \quad \mu = 1,3 \ , \ T^{44} = \frac{G\mathcal{E}_r}{c^4} \quad \textit{and} \quad T_\mu^\nu = \frac{G\mathcal{E}_r}{c^4}\, \delta_\mu^\nu \ \ (\textit{where}\ \delta_\mu^\nu\ \textit{is the Kronecker delta})$$

With the quantities $h_\mu^\lambda = \eta^{\lambda\alpha}\, h_{\mu\alpha}$ *and* $h = \eta^{\lambda\alpha}\, h_{\alpha\lambda}$, the field equation result

$$\left(h_\mu^\nu - \frac{1}{2}\delta_\mu^\nu h\right) = 4\int \frac{T_\mu^\nu}{r} dV = \frac{G}{c^4 ct}\delta_\mu^\nu \int \mathcal{E}_r\, dV$$

We put $r = ct = const$ over V because the "interesting point" of the Einstein's solution, here is any point in time of the 4-sphere surface.

Integrating on V, after calculating the quantity $\mathcal{E}_{0r}V_0 = \int \mathcal{E}_r \, dV \simeq const$, we get:

$$\left(h_\mu^\nu - \frac{1}{2}\delta_\mu^\nu h\right) = \frac{4GE_r}{c^4 ct}\delta_\mu^\nu \quad \mu = \nu \qquad 0 \quad \mu \neq \nu \qquad E_r = \mathcal{E}_{0r}V_0 \simeq const$$

We can see that $h_{\mu\nu} = \eta_{\mu\nu}h_0$. Values of h_μ^λ are all equals, say to $4h_0$, and with $h = 4h_0$ then follows:

$$h_0 = -\frac{2GE_r}{c^4 ct}$$

and the space-time interval is

$$ds^2 = \eta_{\mu\nu}(1 + h_0)dx^\mu dx^\nu$$

but coordinates are isotropic, that is all points of space are equivalent, so the latter expression holds for all spatial rotations, in this case the rotation $\mathbf{C}(\psi,\varphi)\mathbf{C}(\psi,\varphi)\ \eta_{\mu\nu}(1 + h_0)$ giving:

$$ds^2 = -c^2t^2(1 + h_0)[\sin^2(\psi)d\varphi^2 + \sin^2(\psi)\sin^2(\varphi)d\theta^2 + d\psi^2] + (1 + h_0)c^2dt^2$$

The equation is valid in a sufficiently small zone of the universe where the Cartesian variable x can be merged with our arc ξ so that $\Delta x \simeq ct\Delta\xi$. You can use it to evaluate the trend of the gravitational field over the last 10 billion years. It leads to the usual light geodesic: $d\xi = dt/t$. We must conclude that Relativity is an approximation but its application has an undetectable margin of error until we operate below the large interstellar distances.

Let us do some calculation:

Calculation for h_0. (We assume that mass E_r is constant over time)

- Today energy density of CBR $\mathcal{E}_{0r} = 4.02 * 10^{-14}\ J\ m^{-3}$ [**]
- Constant over time, energy $E_r = \mathcal{E}_{0r}V = 2\pi^2 r^3 \mathcal{E}_{0r} = 1.69 * 10^{66}\ J$
- Constant $h_0 = 2.94 * 10^6\ ly$

Verification of the gravitational redshift relative to the time when ray of light started from the farthest galaxy.

- The expansion speed c is constant over time. In the absence of other factors, it means that the distance, measured from source and receiver, between two successive wave crests does not change over time. <u>There is no redshift due to the expansion itself</u>.
- In absence of a relative angle ξ, that gives the Doppler effect, the redshift is the quotient between the proper times of receiver and transmitter $(g_{44\ today}/g_{44\ early})^{1/2}$, not in relative motion with respect to each other, as for the Schwarzschild metric:

$$1 + z = \frac{\sqrt{1 - \frac{2GE_r}{c^5 t_{today}}}}{\sqrt{1 - \frac{2GE_r}{c^5 t_{early}}}}$$

For a galaxy at its maximum distance ($\xi \simeq 1$), $t_{Max} \simeq 5 * 10^9$ $years$ value is $z = 1.86 * 10^{-4}$. [***]

The latter value is the confirmation that throughout the Galaxy Epoch gravity remained negligible.

The initial assumption $\Delta x \simeq ct\Delta\xi$ was applied only in the final steps where we set the spatial terms g_{11}, g_{22}, g_{33} in our expression of ds^2. The large time interval used in this last calculation does not invalidate the entire formula but only concerns these terms of no interest in Gravitational Redshift.

With this consideration and gravity negligible throughout the Galaxy Epoch, the Einstein's model for weak fields has been correctly applied.

Accepting a negligible error, Galactic redshift can always be calculated as Doppler redshift.

If Relativity is an approximation, could the exact solution be needed?

Beyond the complexity, perhaps only a numerical solution could bring a result, I don't think we would be able to find a context in which the calculations provided by this model are better than those provided by Relativity. The distances to be treated, between any two given gravitationally unbound points, may be too large to obtain accurate measurements. I believe that, at least for now, the use of this equation is limited to justifying our calculations for the Galactic Recession in a context where General Relativity applies independently.

[*] - We assume that the mass of matter does not change from past. About the energy of radiation, its constancy, as an approximation over the range of time in question, is due to the Weak Fields hypothesis.

[**] - See later in the paragraph USING 4-SPHERE FORMULAS.

[***] - Here, for the age of the Universe, the time used $t = 1.36 * 10^{10}$ $years$ is different from the value of other models as the Lambda-CDM. [7] However, a verification regards the time elapsed from the Big Bang is possible, through a simple calculation on the observed Hubble constant:

Hubble's recessional velocity $H = 72\ Km\ s^{-1}\ Mpc^{-1}$
Calculated $\Theta_{1\ Mpc} = H/c = 2.4 * 10^{-4}\ rad$
Time elapsed from Big Bang $t_{now} = 1/c\Theta_{1\ Mpc} = 3.26 * 10^6 / c\Theta_{1\ Mpc} = 1.36 * 10^{10}\ years$
Corresponding time from Lambda-CDM $t = 1.37 * 10^{10}\ years$

USING 4-SPHERE FORMULAS

Observing the solution found for the Galaxy Epoch, one realizes that this could not be a good candidate for the complete solution: fields became too strong as radiation energy increase. Concluding, we have provided only a part of the solution and without the rest we cannot proceed furthermore with the earlier periods. We need the exact model to move on a space-time context, so we can use the physics we know.

In fact, looking at the surface equation $x_1^2 + x_2^2 + x_3^2 + x_4^2 = c^2t^2$ we can immediately see that the presence of the fourth spatial dimension x_4 leads us to a dead end. Previously we made some rough energy balance and some hypothetical calculation but nothing more. Variable x_4 cannot be used in any law of physics and, in the moment, about it is impossible to make any logical reasoning. The absence of a law on x_4 does not allow us to predict changes on the evolution of the Universe over time.

Only the surface formulas can be used:

$V = 2\pi^2c^3t^3$ $\quad M = (\rho_r + \rho_m)2\pi^2c^3t^3$ $\quad$ where ρ_r, ρ_m are the densities of radiation and matter and M is the total mass.

As an example, we calculate the mass $M_r = \rho_r 2\pi^2c^3t^3$ equivalent to the total energy of CBR and $M_m = \rho_r 2\pi^2c^3t^3$ corresponding to the total mass of matter:

$E_{avg} = 3.83k_bT = 3.83 * 1.38 * 10^{-23}JK^{-1} * 2.7K = 1.43 * 10^{-22}J$
where E_{avg} is the average energy of a photons (as a blackbody) [8]
$\mathcal{E}_r = aT^4 = 7.566 * 10^{-16}Jm^{-3}K^{-4} * 2.7^4K^4 = 4.02 * 10^{-14}Jm^{-3}$
where $a = 4\sigma/c$ is the radiation constant [5]
$\rho_r = \mathcal{E}_r/E_{avg} = 2.82 * 10^8m^{-3}$ (the number of CBR photons per cubic meter)
$M_r = \mathcal{E}_r c^{-2}2\pi^2c^3t^3 = 1.88 * 10^{49}\ Kg$
$\rho_{nH} \simeq 0.225$ hydrogen atoms m^{-3} [6]
$\rho_H = \rho_{nH}uMA/u = 0.225 * 1.00784 * 1.66 * 10^{-27} = 3.76 * 10^{-28}\ Kg\ m^{-3}$ (other sources give a value of approximately $1.50 * 10^{-33}\ Kg\ m^{-3}$)
$M_m = \rho_H 2\pi^2c^3t^3 = 1.58 * 10^{52}\ Kg$

GALACTIC REDSHIFT IN COSMOLOGICAL EPOCHS: COSMIC BACKGROUND RADIATION

The assumption that at "time of last scattering" expansion velocity was null is necessary for CBR to respect the observed value of the standard deviation in its radiation temperature: T = 2.7255 ± 0.0006K. With a tolerable deviation of 0.0002 we cannot admit the presence of any Doppler effect.

During Recombination [*] and earlier, in the Radiation Era, pressure and energy density were so high that radiation itself were imprisoned. After the end of Recombination era, all radiation has been released. These relic photons reach us with the same redshift. Note that to reach us, a radiation emitted in the end of Recombination Era (380,000 years from Big Bang), traveled one or more full laps. [**]

We must then look for different models for specific eras. A first rough subdivision could be between Galaxy Epoch and "time of last scattering" of CBR:

$z = z(t)\ and\ \partial z/\partial\Theta = 0$ after release of relic photons
$z = z(\Theta)\ and\ \partial z/\partial t = 0$ in late matter dominated period

More specifically:

- After release of relic photons and throughout an initial period, gravity is strong and uniform, decreasing with time. It depends on matter and on strong radiation energy.
- During the Galaxy Epoch, close to a star, the uniform component of gravity, from radiation, is negligible compared to that generated by the star [***]. If gravity has changed since the light ray started, this may be due to a change in mass of the star or to some other reason.

We should say that (g is gravity):

$$z = z(\Theta, g) \ and \ \partial z/\partial t = 0$$

and, as a more reasonable assumption in the absence of other information,

$$\partial g/\partial t = 0$$

As long as the expansion speed remains constant, the redshift is not attributable to the expansion itself. During the Radiation Era, from the time of last scattering onward, the redshift is gravitational while in the Galaxy Epoch it is due to the Doppler effect. In between time it is of mixed type.

[*] - Time to the end Recombination Era is taken from Theory of Big Bang

[**] - We can calculate the angle traveled by relic photons to reach us $\Theta = 5/2\pi + 2.63$. You can use:

$$\Theta = \ln\left(\frac{t_{today}}{t_{past}}\right) \quad for\ every\ \Theta$$

[***] - The observed surface gravitational redshift of a massive neutron star is about $z = 0.4$

GALACTIC COORDINATES

The observable Universe is a volume, on the surface of the 4-sphere, delimited in the three spatial dimensions by an arc of $\Theta = 1\ rad$. In this volume we are at the center O.

Fixed the origin for the time axis t coinciding with the Big Bang, we can use three angles as a galactic coordinate system: the position of an astronomic object A can be defined by the direction of the 4-sphere arc OA and the angle λ of this one. For the direction we can adopt the usual coordinates: Right ascension α and Declination δ. About the 4-sphere arc angle, say "Arc λ", knowing the Galactic redshift *z*, you have:

$$\lambda = ((1+z)^2 - 1)/((1+z)^2 + 1)\ rad$$

Present proper distance $s = ct_{now}\lambda$

Moving on 4-sphere surface coordinates, Colatitude, Longitude and Height, is quite complicate. Maybe it needs the aid of a computer program or some more suitable mathematical method. Here we give only some tools and a way to approach the solution:

Let's recall the coordinate in the 4-sphere space **U**: $P = P(\varphi, \theta, \psi)$:

1. $x_1 = ct\sin(\psi)\sin(\varphi)\cos(\theta)$
2. $x_2 = ct\sin(\psi)\sin(\varphi)\sin(\theta)$
3. $x_3 = ct\sin(\psi)\cos(\varphi)$
4. $x_4 = ct\cos(\psi)$

The 4-vector $\boldsymbol{r} =$
$(ct\sin(\psi)\sin(\varphi)\cos(\theta), \quad ct\sin(\psi)\sin(\varphi)\sin(\theta), \quad ct\sin(\psi)\cos(\varphi), \quad ct\cos(\psi))$

and its derivatives:

1. $\mathbf{r}_\theta = (-ct\sin(\psi)\sin(\varphi)\sin(\theta), \ ct\sin(\psi)\sin(\varphi)\cos(\theta), \quad 0, \quad 0)$
2. $\mathbf{r}_\varphi = (ct\sin(\psi)\cos(\varphi)\cos(\theta), \quad ct\sin(\psi)\cos(\varphi)\sin(\theta), \quad -ct\sin(\psi)\sin(\varphi), \quad 0)$
3. $\mathbf{r}_\psi = (ct\cos(\psi)\sin(\varphi)\cos(\theta), \quad ct\cos(\psi)\sin(\varphi)\sin(\theta), \quad ct\cos(\psi)\cos(\varphi), \quad -ct\sin(\psi)))$

After converting δ using zenith angle $= 90^0 - \text{Declination}$, in the space **O** of observable Universe, for a point, $U = U(\delta, \alpha, \lambda)$:

1. $y_1 = \sin(\delta)\cos(\alpha)$
2. $y_2 = \sin(\delta)\sin(\alpha)$
3. $y_3 = \cos(\delta)$
4. $y_4 = ct\lambda$

The vector $\boldsymbol{u} = (\sin(\delta)\cos(\alpha), \quad \sin(\delta)\sin(\alpha), \quad \cos(\delta))$ (with unit length)

and its derivatives:

1. $\mathbf{u}_\alpha = (-\sin(\delta)\sin(\alpha), \quad \sin(\delta)\cos(\alpha), \quad 0,)$
2. $\mathbf{u}_\delta = (\cos(\delta)\cos(\alpha), \quad \cos(\delta)\sin(\alpha), \quad 0)$

Note that two stars can be nearby on **U** but distant on **O**: it complicates approximations.

An angle on the 4-sphere is given by:

$$\xi = \arccos\left(\frac{1}{c^2t^2}\mathbf{r}_1 \cdot \mathbf{r}_2\right)$$

while the one on the observable Universe (that is on the 4-sphere surface, between the Earth and two star) is:

$$\gamma = \arccos(\mathbf{u}_1 \cdot \mathbf{u}_2)$$

To use Right Ascension and Declination we need the formulas effective for arcs and angles on the surface. For this purpose, given three points, we can set the 4-plane that passes through them and the center of the 4-sphere. Once got it, we have a 3-sphere so to use the Sine Theorem and other tools.

Here calculations in polar coordinates are hard so let's move on to Cartesian ones:

$x_1^2 + x_2^2 + x_3^2 + x_4^2 = c^2t^2$
$x_4 = ax_1 + bx_2 + cx_3$ (where this 4-plane passes through the North Pole and the Earth).

We have $x_1^2 + x_2^2 + x_3^2 - c^2t^2 = -(ax_1 + bx_2 + cx_3)^2$.

This means that if a point belongs to the 3-plane: $ax_1 + bx_2 + cx_3 = 0$ and belongs to the 3-sphere: $x_1^2 + x_2^2 + x_3^2 = c^2t^2$ then it also belongs to the 4-sphere after we put

$$x_4 = ax_1 + bx_2 + cx_3.$$

About the steps to find the position of an unknown star $P_x(\varphi, \theta, \psi)$, variables must be chosen so that the point lies both on of the sphere and the plane. That gives a first condition $F(\varphi, \theta, \psi) = 0$. Note that parameters a, b, c, for the equation of the 3-plane, are not linearly independent but we need all them later to set x_4. [*]

For the whole procedure to be valid, we should demonstrate that the transformation preserves angles and distances between the three points in question. To avoid calculations, we see that the same is true in 3d when we intersect a sphere with a plane, passing through the center, to get a circle.

For triangulations of the 4-sphere we start getting coordinates of some points. We use our Earth, Ursa Major GN-108036, Sculptor A2744 YD4 and Piscis Austrinus BDF-3299:

1. Our Earth Us $P_0(\varphi, \theta, \psi)$ and $U_0(0, 0, 0)$
2. Ursa Major GN-108036 $z = 7.2$ $P_N(0, 0, 0)$ and $U_1(0.4863,\ 3.3003,\ 0.9707)$ - Boreal Hemisphere
3. Sculptor A2744 YD4 $z = 8.38$ $P_{EP-}(undef, 0, undef)$ and $U_2(-1.0405,\ 0.0629,\ 0.9775)$ – Austral Hemisphere
4. Piscis Austrinus BDF-3299 $z = 7.11$ $P_3(\varphi, \theta,\ \psi) = U_3(-0.9570,\ 5.8827,\ 0.9700)$ – Austral Hemisphere
5. … and so on …

We can give here the trace of a solution for our North Star Polaris. In these coordinates, it is close to the Earth:

1. Alpha Ursae Minoris – Polaris $z = 0.000055$ $U_4(0.0128,\ 0.6624,\ 0.000055)$ – Boreal Hemisphere
2. Our Earth $\mathbf{r}_0 = (a, b, c, d)$
3. Ursa Major GN-108036 $\mathbf{r}_\mathrm{N} = (0, 0, 0, ct)$
4. Sculptor A2744 YD4 $\mathbf{r}_2 = (e, 0, f, g)$

With respect to the Earth $P_0(\varphi, \theta, \psi)$, the coordinates of Alpha Ursae Minoris – Polaris are: $P_4(\varphi + x, \theta + y,\ \psi + z)$ where x, y, z are unknown.

We follow these steps:

1. Define a point P_W on the direction $P_0\, P_N$ at the same distance $P_W\, P_N = P_4\, P_N$. U_W lies on the segment $U_0\, U_N$.
2. The first condition on x, y, z comes from the sphere and plane passing through $P_0\, P_N\, P_4$
3. Calculate the angle between P_N and P_4 in **O:** $\gamma = \arccos(\mathbf{u}_\mathrm{N} \cdot \mathbf{u}_4) = 0.8788$
4. Use the Sine Theorem in the triangle $P_0\, P_W\, P_4$, right in P_W: $|\arcsin(\lambda\gamma)| = \varepsilon = 0.000048$
5. Calculate the other cathetus with the Cosine theorem: $\cos\lambda = \cos\varsigma \cos\gamma$ and $\varsigma = 0.000027$

Now we abandon the 3-sphere $x_1^2 + x_2^2 + x_3^2 = c^2t^2$ and, back to the 4-sphere equation, we can solve the displacement between $P_0\, P_4$:

1. the value $\sin(\psi)\sin(\varphi)\,\Delta\theta$ is equal to ε.
2. the value $\sin(\psi)\Delta\varphi$ is equal to ς.

[*] - Since for the North Pole we arbitrarily assumed $x_4 = 0$, it is not strange that all the points are constructed in the same way and all satisfy the condition of coplanarity on x_4. In this construction, we can reasonably think that, for every three points of the 4-sphere, passes a sphere that preserves angles and distances between them.

A BRIEF EXCURSUS:

HOW COULD BE THE PHYSICS OF THE ENTIRE 4-SPHERE

The simplicity with which, until now, you arrive at the conclusion that this model is totally consistent with all the concepts expressed by Relativity, also giving a coherent answer for Galactic Recession has a price: all the difficulties have been moved in the part that has to treat the Recombination Era.

Let us remember what entails, for our model with a constant recession speed, the very low standard deviation detected in the Cosmic Background Radiation: At the time of the "Last Scattering", when all the cosmic radiation was released, there must have been almost no expansion and the energy was the same for the whole CBR.

The idea we are trying to pursue is that an initial period of inflation was followed by a period in which expansion temporarily stopped and then resume at the "Last Scattering" (perhaps because some characteristics of the plasma, such as incompressibility, drastically change). The total time in which this expansion was not constant is small compared to the age of the Universe so we can think that the current recession speed practically coincides with its average value.

As said above, for our Universe and during the Galaxy epoch, we hypothesized a bubble that expands in the absence of an external pressure, where no heat is exchanged and the only work is done by the cohesion forces to maintain its surface. This work is balanced by pressure of radiation inside the bubble. Actually, we should consider the hypothesis of the existence of a vacuum outside the bubble completely absurd: absolute space has been excluded, so it makes no sense to speak of an external vacuum or pressure even to affirm that the latter is naught. Nevertheless, even looking at the whole thing from the point of view of our Universe, we must still conclude that the work done as the result of the expansion is null. The container of our Universe, in fact, despite having a finite volume, paradoxically has no edges or walls: Particles of matter and radiation expand freely in all directions without ever meeting any boundaries.

As regards the period concerning the Recombination and earlier, in our assumption existing plasma was disposed on the surface of a 4-sphere and, for some reason, this geometry was preserved in all subsequent states. However, the existing radiation was not bound to arrange itself in the same way and the part not absorbed by the plasma could spread progressively inside, leaving the surface.

It is, then, in these conditions that, in our speculation, we must think about the way in which the cooling, hypothesized by the theory of Big Bang, took place. If the cooling was not due to the expansion then the heat must have left our Universe.

In practice, we must demonstrate that radiation, even in the presence of extreme gravity conditions, does not arrange itself like the rest of the plasma. The conclusions follow assuming a slow diffusion of photons towards the inside of the 4-sphere so that the ratio between the amount of radiation absorbed and emitted by the plasma was affected by a progressive decrease in the concentration of photons determining a slow but continuous cooling.

Our conjecture considers General Relativity not arising from two postulates but as a consequence of the shape of the Universe and its expansion, through the stretching of its radius as $r = ct$. This certainly does not make things easier: at the time of Recombination the heat transmission took place towards the inside of the 4-sphere and it is there that we must study the phenomenon. Rejecting Relativity as an axiom leaves us in the absence of any physical law known!

However, we can consider Last Scattering as a limit situation, then, chosen a point on the surface of the 4-sphere, where $ds^2 = -h_\xi c^2 t^2 d\xi^2 + h_r c^2 dt^2$ is the interval that our conjecture applies (see above), we can always refer to Special Relativity setting $h_\xi = h_r = 1$ and having, for a ray of light:

$$0 = -r^2 d\xi^2 + dr^2 \qquad now\ 0 \le r \le ct$$

As it must have been from the assumptions made so far, the velocity of the ray of light, although not necessarily constrained to the current value c, must involve a radial component. All the radiation produced in any reaction in the Early Universe, in absence of expansion, must necessarily abandon the surface, spreading within the 4-sphere.

Note that from the equation

$$\frac{8\pi G}{c^4} T^{\mu\nu} = R^{\mu\nu} - \frac{1}{2} R g^{\mu\nu} \qquad now\ c = c(t)\ could\ also\ be\ the\ expansion\ speed$$

quantity *c,* as an expansion speed does not admit a zero value, otherwise as a constant it assumes the value we know and expansion can stop. We opt for choice $c = constant$.

To proceed with our analysis, the most reasonable solution consists, then, in looking for a physics that applies to the entire 4-sphere and that is reduced to Einstein's field equation on its surface and in our reference frame. But simply setting $r = ct$ leads to no result, furthermore, imposing a discontinuity would seem preferable to avoid passages to and from the surface.

In this new model can we think of a time coordinate as the Galileo's absolute time? Its reintroduction could seem reasonable because the presence of the radial coordinate allows you to identify a privileged reference frame: *the motionless center*. But we have discarded absolute space so when we have two points in relative angular motion what is the fixed one?

We got to the point: Relativity on the surface excludes absolute space and this in turn excludes Galileo's absolute time.

Let us then think of our Universe expanding with $r = ct$ and fix the origin of our reference frame in the center. Here we have two points A and B in relative angular motion between them and with the same radial velocity c. With our reference frame rotating with A, if B emits a ray of light, its tangential speed, always at its maximum value and without being dragged by the B speed, must be equal to the radial speed c to not abandon the surface:

$$\begin{cases} v_t = |r d\xi/dt| = c \\ v_r = dr/dt \quad = c \end{cases}$$

The speed of light constrains our physics with limits, such as negative square roots, which turn out to be insurmountable, but what would change if below the surface the constraint for light were reduced to the geodesic equation:

$$\begin{cases} v_t = |r d\xi/dt| = \kappa \\ v_r = |dr/dt| \quad = \kappa \end{cases} \qquad with\ k > c$$

Then we would have the light that once exited the surface would acquire speed and would be reflected every time the radial component encounters an obstacle (in the center and on the surface). Meeting the surface, reflecting radiation would exert the pressure we previously assumed.

In this short excursus we cannot ignore the *tachyon* seen as particle which exhibits non-local [*] behaviors or as force carrier capable of mediating quantum entanglement [9].

To see how, near the surface, the motion of the tachyon could be, we choose an area of the Universe small enough to apply Special Relativity and merge that spacetime interval with a new one (only as an example to simplify the discussion)

$$ds^2 = -r^2 d\xi^2 + dr^2 + c^2 dt^2 \qquad here\ we\ want\ continuity\ dr \to 0\ to, from\ the\ surface$$

We want that, for the tachyon moving away or approaching the surface, quantities as energy and momentum tend to the relativistic values proper to particle or light. Then, two points on the surface, even in the elsewhere zone, are connected by a trajectory in which an increasing negative radial component is followed by a positive one, having the new term $dr^2 = 0$ on the surface. Note also that, if the particle is able to interact with the elements we know, then as long as it stays on the surface we must be able to detect it.

This last consideration is particularly important in the case of the neutrino whose behavior leads us to suspect (there is still insufficient evidence) that it could be a tachyon [**].

In a nutshell, the tachyon would be an object capable of penetrating the hypersphere and moving at speed > c to re-emerge elsewhere on the surface; only here we must be able to detect it. It is hard to say what all the consequences are but tachyon by its nature cannot be measured continuously, even if it never left the surface.

I did not go any further looking for (not verifiable) intervals and field equations such as to justify the arrangement of the plasma and then the resumption of expansion. If we want in future to move forward with this conjecture, we must apply this idea to a falsifiable theory by linking it to phenomena that General Relativity is not able to explain.

Among all branches of science, the best candidate for our scopes is Quantum Mechanics and maybe, within this, the phenomenon to be chosen is “Non-locality”.

Continuing the speculation could be hard and yield no results; it could be done looking for the metric tensor which governs the interior of the hypersphere. To a first approximation, a track to follow might be:

- Formulating the Einstein-Hilbert action in these 4 spatial dimensions
- Assuming a reasonable Stress and Energy tensor for disordered radiation
- Checking the desired solution for weak fields to obtain a space-time interval
- Verifying the equilibrium of the 4-sphere: it must be $PV^{5/4} = const$ for the state equation of a 4d reversible adiabatic expansion for radiation (you can start from $TdS = dQ = 0$).

Once the new space-time has been defined, the connection with Quantum Field Theory and Non-locality might begin.

[*] - Non-locality [10] can also be explained assuming the existence of compactified higher dimensions as in the following article:

MDPI 2076-3417/9/24/5406; Quantum Correlations and Quantum Non-Locality: A Review and a Few New Ideas

[**] - This article, on the other hand, examines the non-local aspects of the neutrino in Quantum Field Theory and under the assumption of the existence of the "Preferred Frame":

[arXiv:2103.13982] - Quantum Field Theory of Space-like Neutrino

CURIOSITIES AND FEATURES OF THE MODEL

A ray of light, which travels the most recent circle and reaches us after a rotation of 2π, had an age of 25.4 million years when started. In that period and before no stars still exist. No images may overlap, nor ghost images exist and we never could ask ourselves if the ray had traveled an arc Θ or a Θ + 2nπ one.

From what can be deduced from this geometry, what belongs to our universe is bound to remain on the surface of the 4-sphere and therefore anything that moves cannot exceed the speed of light *c*. If we admit the existence of the *tachyon*, we must conclude that it cannot be considered constrained to the surface and that it can enter inside. This also applies to quantum phenomenon of Non-locality in which it has been shown that information on the state of a quantum object is transmitted at a speed greater than that of light.

I wanted to present this model even if incomplete, limiting its scope to what, in these hypotheses, could be studied with General Relativity: Galaxy Epoch and the last 10 billion years. In my opinion, the model fully explains the isotropy and homogeneity of the Universe, as well as it provides a circular path for CBR and radiation in general. It is also totally consistent with all the concepts expressed by Relativity, giving a coherent answer for the most distant galaxies: *In this geometry, at all times, galaxies never cross the relativistic light cone.* Galactic recession with its superluminal motion does not enter the Einstein's equation. *From this model the principle of relativity and the recession mechanism arise together separately.*

Accepting the 4th spatial dimension does not imply reintroducing the concept of an absolute space and not even that of absolute time, observed Relativity excludes them both. The attempt to associate the local reality with its possible representation in R^n was dictated by the desire to go deeper into the field of Ontology.

References from Wikipedia:

[1] - Cosmic background radiation

[2] - Surface tension

[3] - Redshift

[4] - Recombination (cosmology)

[5] - Stefan-Boltzmann constant

[6] - Friedmann equations

[7] - Lambda-CDM model

[8] - Planck's law

[9] - Quantum entanglement

[10] - Quantum nonlocality